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Distance Related Graph Invariants in Triangulations and Quadrangulations of the Sphere

by

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Dedication

To Edward Patrick Henson Sr. for teaching me the value of hard work and for being a catalyst for my love of mathematics.

Acknowledgments

First and foremost, I am forever indebted to both Éva Czabarka and László Székely for providing me with the guidance, knowledge and encouragement to complete this degree. You two are the best advisors any graduate student could ask for, and have rightfully earned the departmental name of "Mom" and "Dad". I would also like to thank the additional members of my committee: Joshua Cooper, Linyuan Lu, George McNulty and Karl Gregory for your patience and cooperation, especially during my comprehensive examinations.

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Abstract

The Wiener index of a connected graph is the sum of the distances between all unordered pairs of vertices. I provide asymptotic upper bounds and sharp lower bounds for the Wiener index of simple triangulations and quadrangulations with given connectivity. Additionally, I make conjectures for the extremal triangulations and quadrangulations which maximize the Wiener index based on computational evidence. If $\overline{\sigma}(v)$ denotes the arithmetic mean of the distances from v to all other vertices of G, then the remoteness and proximity of G are defined as the largest and smallest value of $\overline{\sigma}(v)$ over all vertices v of G, respectively. I give sharp upper bounds on the remoteness and asymptotic upper bounds on the proximity of simple triangulations and quadrangulations of given order and connectivity.

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Chapter 1

INTRODUCTION

1.1 Motivation, Related Work and Summary

First discovered in 1947, see [44], the Wiener index of a connected graph is the sum of the distances between all unordered pairs of distinct vertices. The Wiener index was observed to be related to the boiling point of certain alkanes. Numerous other chemical applications were discovered later, for one such example, refer to [41]. The systematic study of the mathematical properties of the Wiener index began with classical papers by Doyle and Graver, [18], Entringer, Jackson and Snyder, [21] and Plesník, [40]. Finding bounds for the Wiener index has been a topic of great interest, the most general upper bounds are attained by paths, see [18], [33] and [40]. Many results bound the Wiener index of graphs with fixed graph parameters, for example, [4], [14] and [32] consider the minimum degree, [15] and [23] consider the connectivity, while [24] considers the maximum degree. For more recent results regarding the Wiener index, see [16], [27], [30], [35], [31], [28] and [38].

If one considers trees, on a fixed number of vertices, it is known that the star graph minimizes the Wiener index ([21]), while the path graph maximizes it ([33]). Upper and lower bounds for the Wiener index of binary trees was studied in [24].

In this dissertation, we focus on triangulations and quadrangulations, which have a fixed number of edges. This makes them an interesting class of graphs when studying distance related graph invariants. Additionally, triangulations and quadrangulations have wide reaching applications, such as robotics, see [19] and [37]. Within the last

year, there have been several results regarding the Wiener index in triangulations and quadrangulations, see [9], [11], [22], [25] and [29], some of these results will be analyzed in Chapters 1 and 2.

Aside from the Wiener index, I also provide upper bounds for the proximity and remoteness of triangulations and quadrangulations, which are the smallest and largest values of $d(v, G)$, over all vertices $v \in G$. Upper bounds for the proximity and remoteness were proven in [2] and [46]. Results when fixing the minimum degree can be found in [13]. Upper bounds for the remoteness of maximally planar graphs were recently provided by [9] and [11], the latter will be discussed in Chapter 1. In Chapter 3, the best known upper bounds for the proximity of triangulations and quadrangulations will be presented [10]. Coincidentally, the extremal triangulations and quadrangulations which are conjectured maximize the Wiener index happen to maximize the remoteness, and are conjectured to maximize the proximity, these structures will appear in Chapters 1 and 3.

1.2 Background Definitions and Theorems

I would like to take the time to thank the authors of both [5] and [17]. Without these books, exploring the field of graph theory would have been a much more daunting task. The following definitions provide a summary of what will be needed in this dissertation, but one should consult [5] and [17] for more details and examples. Where obvious, abbreviated notation may be used, for example, if there is only one graph G being considered, I may use E rather than $E(G)$ to denote the edges in G. Also, unless otherwise stated, all graphs will be assumed to be finite, simple and connected.

Definition 1.1. A graph is an ordered pair (V, E), where $E \subseteq V^2$. We refer to V as the set of vertices, and E as the set of edges. $V(G)$ represents the vertices of G, while $E(G)$ denotes the edges within G.

Definition 1.2. The order of a graph G is the cardinality of $V(G)$, while the size of G is the cardinality of $E(G)$.

Definition 1.3. Two vertices a and b are said to be adjacent if $(a, b) \in E$. Any vertex b that is adjacent to a is called a neighbor of a. The set of all neighbors of a is called the neighborhood of a, denoted by $N(a)$.

Definition 1.4. The complete graph on n vertices, denoted by K_n, is the graph where all distinct pairs of n vertices are adjacent.

Definition 1.5. Two graphs G_1 and G_2 are isomorphic, written $G_1 \cong G_2$, if there exists a bijection $\phi : V_1 \to V_2$, such that for every edge $ab \in E_1, \phi(a)\phi(b) \in E_2$.

Definition 1.6. A function f is called a graph invariant if $f(G_1) = f(G_2)$ for any isomorphic graphs G_1 and G_2.

Definition 1.7. G_1 is a subgraph of G_2, denoted $G_1 \subseteq G_2$, if $V_1 \subseteq V_2$ and $E_1 \subseteq E_2$.

Definition 1.8. The induced subgraph, $G \setminus X$, is the subgraph obtained by deleting all vertices in $X \subseteq V$ from V.

Definition 1.9. A subgraph $G_1 \subseteq G_2$ is said to be a spanning subgraph if $V_1 = V_2$.

Definition 1.10. A graph G, with a given graph property P, is said to be edge-maximal if adding any additional edge to G destroys P.

Definition 1.11. The degree of a vertex a, denoted by $d(a)$, is the cardinality of $N(a)$.

Definition 1.12. The maximum degree of a graph G, $\Delta(G)$, is the largest vertex degree. Similarly, the minimum degree of a graph G, $\delta(G)$, is the smallest vertex degree.

Definition 1.13. The average degree of a graph G, denoted by $d(G)$, is the sum of all degrees in G, divided by the number of vertices in G.

Definition 1.14. A non-empty graph is called a path if $V = \{a_0, \ldots, a_k\}$ and $E = \{a_i a_{i+1} | i = 0, \ldots, k\}$, where all a_i are distinct, such a path is denoted $P = a_0 \ldots a_k$. The cardinality of E is called the length of the path.

Definition 1.15. A non-empty graph is called a cycle if it is a path $a_0 \ldots a_k$ with exactly one additional edge $a_0 a_k$. The length of a cycle C is equal to the number of vertices in C, which is equal to the number of edges in C.

Definition 1.16. The length of the smallest cycle in a graph G is called the girth of G, denoted $g(G)$.

Definition 1.17. The distance between two vertices $a, b \in V$, $d(a, b)$, is the length of the shortest path with endpoints a and b. If no such path exists, we say $d(a, b) = \infty$.

Definition 1.18. The Wiener index of a graph G, denoted $W(G)$, is the sum of the distances between all unordered pairs of vertices.

Definition 1.19. The distance between a vertex and a graph, $d(v, G)$, is the sum of distances $d(u, v)$ for all $u \in V$.

Definition 1.20. Given a vertex $v \in V$, the eccentricity, denoted by $e(v)$, is the largest distance from v to any other vertex in G. The largest eccentricity is called the diameter of G, while the smallest eccentricity is called the radius of G.

Definition 1.21. The set of vertices distance $0 \leq i \leq e(v)$ from v are called the distance i neighbors of v. This set is denoted by $N_i(v)$ and has cardinality $n_i(v)$.

Definition 1.22. A graph G is said to be connected if all pairs of vertices in G are joined by a path. If G is not connected, it is called disconnected.

Definition 1.23. Maximally connected subgraphs of G are called the components of G.

Definition 1.24. Given a connected graph G, a cutset is a subset V_1 of V, such that $G \setminus V_1$ is disconnected.

Definition 1.25. A connected graph G is said to be κ-connected if $|V| > \kappa$ and removing any $\kappa - 1$ vertices from G does not disconnect it.

Definition 1.26. A graph is said to be acyclic if it contains no cycles.

Definition 1.27. A tree, T, is an edge maximal connected acyclic graph.

Definition 1.28. A graph G is said be be bipartite if it can be partitioned into two sets A and B, written $G = (A, B)$, such that every edge in G has one endpoint in A and one endpoint in B.

Definition 1.29. A minor of a graph G is any graph which can be obtained by a sequence of vertex deletions, edge deletions, and edge contractions.

Definition 1.30. A simple graph is a graph that has no loops or multi-edges. A loop is an edge were both endpoints are a single vertex. A multi-edge is a collection of two or more edges with identical endpoints.

Definition 1.31. A polygon is a subset of $\mathbb{R}^2$ which is the union of a finite amount of straight line segments.

Definition 1.32. A drawing of a graph G is a graph $G' \cong G$, represented in the plane.

Definition 1.33. A graph is planar if it can be drawn in the plane in a way where there are no edge crossings.

Definition 1.34. A triangulation is an edge maximal planar graph. Equivalently, all faces of G are bounded by 3-cycles, or triangles.

Definition 1.35. A quadrangulation is an edge maximal bipartite planar graph. Equivalently, all faces of G are bounded by 4-cycles, or quadrangles.

Definition 1.36. If G_1 and G_2 are graphs, the graph sum, $G_1 + G_2$, is the graph which contains all vertices from $V_1 \cup V_2$ and has edges v_1v_2 for all $v_1 \in V_1$ and $v_2 \in V_2$.

Definition 1.37. Given a graph G, if $A, B \subseteq V$, then $m(A, B)$ denotes the number of edges between A and B.

Theorem 1. *If a graph G has at least two vertices, then $\kappa(G) \leq \delta(G)$.*

Theorem 2. *Euler's Formula states that if G is a planar graph, drawn in the plane without edge-crossings, then $n-e+f = 2$ where $n = |V|$, $e = |E|$ and f is the number of faces of G.*

Corollary 1. *A triangulation with n vertices has $3n - 6$ edges and $2n - 4$ faces.*

Corollary 2. *A quadrangulation with n vertices has $2n - 4$ edges and $n - 2$ faces.*

Theorem 3. *A graph G is planar if and only if it contains no K_5 or $K_{3,3}$ minor.*

Chapter 2

Upper Bounds on the Wiener Index and Remoteness

2.1 Summary of the Results

A natural class of study for the Wiener index is planar graphs. However, as the maximum Wiener index of graphs is attained by a path, it makes sense to consider more restricted classes of planar graphs, like simple triangulations and quadrangulations. [9] and [11] independently investigated the maximum Wiener index of triangulations and presented the same simple triangulation of order n (see Figures 2.5, 2.6, 2.7) with Wiener index

$$W(T_n) = \frac{1}{3}\binom{n+2}{3} - \frac{1}{3}\left\lfloor \frac{n+2}{3} \right\rfloor = \begin{cases} \frac{n^3}{18} + \frac{n^2}{6} & \text{if } n = 3k \\ \frac{n^3}{18} + \frac{n^2}{6} - \frac{2}{9} & \text{if } n = 3k+1 \\ \frac{n^3}{18} + \frac{n^2}{6} - \frac{1}{9} & \text{if } n = 3k+2, \end{cases} \tag{2.1}$$

which they conjectured to be optimal (see Figures 2.5, 2.6, 2.7). [Note that this sequence is present in the On-Line Encyclopedia of Integer Sequences [43] under A014125, which is the bisection of A001400. The displayed closed form is due to Bruno Berseli [43].] My collaborators and I, [20], announced that this conjecture is asymptotically true before the paper [9] was submitted. Che and Collins [9] verified this conjecture for simple triangulations of order not exceeding 10. Using a computer, we verified this conjecture for simple triangulations of order not exceeding 18, see

Table 1 in [11]. Very recently, Debarun Ghosh, Ervin Győri, Addisu Paulos, Nika Salia, Oscar Zamora verified this conjecture [26].

In this chapter we prove a *generalization* of this conjecture *asymptotically*. Note that every simple triangulation is 3-connected, but a simple triangulation cannot be 6-connected because of the number of edges. Similarly, quadrangulations must be 2-connected, but cannot be 4-connected. Our main theorem, Theorem 5, proves that for any $3 \leq \kappa \leq 5$, the Wiener index of any κ-connected simple triangulation of order n is at most $\frac{1}{6\kappa}n^3 + O(n^{5/2})$. We also prove in Theorem 6 that for any $2 \leq \kappa \leq 3$, the Wiener index of any κ-connected simple quadrangulation of order n is at most $\frac{1}{6\kappa}n^3 + O(n^{5/2})$.

We provide constructions matching the upper bounds of Theorems 5 and 6 for the maximum Wiener index of triangulations and quadrangulations of given connectivity. We do more, as we exhibit triangulations and quadrangulations following patterns by the residue of the order n modulo κ, which we conjecture as realizers of the maximum Wiener index. Our conjectures are based on extensive computations. We next detail these conjectures.

We constructed 4-connected simple triangulations with Wiener index

$$W(T_n^4) = \begin{cases} \frac{n^3}{24} + \frac{n^2}{4} + \frac{n}{3} - 2 & \text{if } n = 4k+2 \\ \frac{n^3}{24} + \frac{n^2}{4} + \frac{5n}{24} - 1 & \text{if } n = 4k+3 \\ \frac{n^3}{24} + \frac{n^2}{4} + \frac{n}{3} - 2 & \text{if } n = 4k \\ \frac{n^3}{24} + \frac{n^2}{4} + \frac{5n}{24} - \frac{3}{2} & \text{if } n = 4k+1, \end{cases} \tag{2.2}$$

(see Figures 2.8, 2.9, 2.10, 2.11). This proves that Theorem 5 is also asymptotically tight for $\kappa = 4$. Furthermore, we conjecture that the repetition of the obvious pattern in these figures provide the extremal triangulations. Using a computer, we verified this conjecture for simple triangulations of order not exceeding 22, see Table 2.2.

We constructed 5-connected simple triangulations with Wiener index

$$W(T_n^5) = \begin{cases} \frac{n^3}{30} + \frac{3n^2}{10} - \frac{23n}{15} + \frac{168}{5} & \text{if } n = 5k+2 \\ \frac{n^3}{30} + \frac{3n^2}{10} - \frac{23n}{15} + 31 & \text{if } n = 5k+3 \\ \frac{n^3}{30} + \frac{3n^2}{10} - \frac{23n}{15} + \frac{161}{5} & \text{if } n = 5k+4 \\ \frac{n^3}{30} + \frac{3n^2}{10} - \frac{23n}{15} + 32 & \text{if } n = 5k \\ \frac{n^3}{30} + \frac{3n^2}{10} - \frac{23n}{15} + \frac{156}{5} & \text{if } n = 5k+1, \end{cases} \tag{2.3}$$

(see Figures 2.12, 2.13, 2.15, 2.16, 2.17). This proves that Theorem 5 is also asymptotically tight for $\kappa = 5$. Furthermore, we conjecture that the repetition of the obvious pattern in these figures provide the extremal triangulations. We arrived to these conjectures using a computer and also some guesswork regarding the pattern. Therefore these conjectures for the 5-connected case are less supported with computational evidence than other conjectures in this chapter, as we were able to do the computation only up to the order 32, see Table 2.3. The issue here, is that the pattern slowly develops, and orders following the same pattern differ by 5, therefore we do not have sufficiently many data points to have a very convincing conjecture.

We are indebted to Paul Kainen, who after hearing about our triangulation results, asked whether we can prove similar results for simple quadrangulations. Recall that any simple quadrangulation is 2-connected, but no simple quadrangulation is 4-connected. We conjecture that the maximum Wiener index of a simple quadrangulation of order n is

$$W(Q_n) = \begin{cases} \frac{n^3}{12} + \frac{7n}{6} - 2 & \text{if } n = 2k \\ \frac{n^3}{12} + \frac{11n}{12} - 1 & \text{if } n = 2k+1, \end{cases} \tag{2.4}$$

based on Figures 2.18, 2.19. Furthermore, we conjecture that the repetition of the obvious pattern in these figures provide the extremal quadrangulations. Using a computer, we verified this conjecture for simple quadrangulations of order not exceeding 20, see Table 2.4. The authors of [29] were recently able to verify this conjecture.

We conjecture that the maximum Wiener index of a 3-connected simple quadrangulation of order n is

$$W(Q_n^3) = \begin{cases} \frac{n^3}{18} + \frac{n^2}{3} - \frac{17n}{6} + \frac{206}{9} & \text{if } n = 3k + 14 \\ \frac{n^3}{18} + \frac{n^2}{3} - \frac{17n}{6} + 20 & \text{if } n = 3k + 15 \\ \frac{n^3}{18} + \frac{n^2}{3} - \frac{17n}{6} + \frac{184}{9} & \text{if } n = 3k + 16, \end{cases} \tag{2.5}$$

based on Figures 2.20, 2.21, 2.22. Furthermore, we conjecture that the repetition of the obvious pattern in these figures provide the extremal quadrangulations. Using a computer, we verified this conjecture for simple quadrangulations of order not exceeding 28, see Table 2.5.

Section 2.4 contains the conjectures stated so far in the form of drawings for some fixed order, but with emphasis on the general pattern: the red colored part is the repeated pattern. Even more, we conjecture based on computational evidence that those drawings not only provide the maximum Wiener index, but for sufficiently large n they are unique with this property.

We remark here that the result of [26] described by formula (2.1) does not hold for *non-simple* triangulations. For the construction of non-simple triangulations with asymptotically larger Wiener indices, see Figure 2.1. In fact, we conjecture that these constructions are optimal for non-simple triangulations. The non-simple quadrangulation on Figure 2.2 has a larger Wiener index than conjectured best simple quadrangulation on Figure 2.18, but the difference is not in the leading term.

Che and Collins noted [9] that the *minimum* Wiener index of a general triangulation of order n is a trivial problem, as Euler's formula determines the number of edges, and there are constructions, in which every pair of vertices are at most distance two. The situation is analogous for quadrangulations. For minimizers, see Figure 2.3. In Chapter 2, we will explore the minimum Wiener index of triangulations and quadrangulations with higher connectivity.

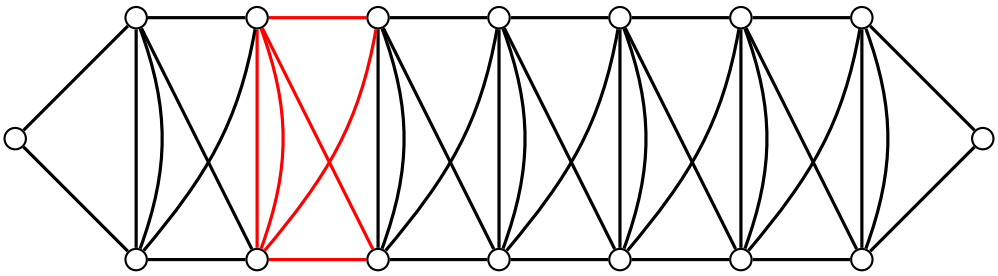

Figure 2.1: A non-simple triangulation with larger Wiener index. $W(T'_n) = \frac{n^3}{12} + \frac{2n}{3} - 1$ for even n.

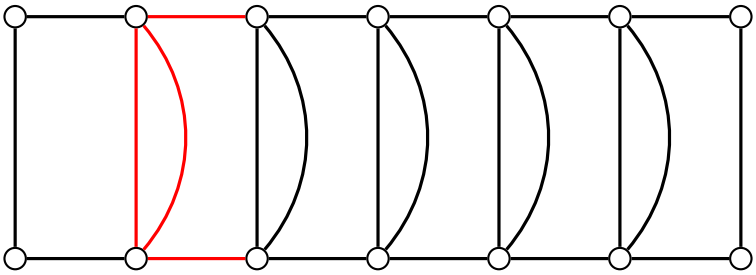

Figure 2.2: A non-simple quadrangulation with larger Wiener index. $W(Q'_n) = \frac{n^3}{12} + \frac{n^2}{4} - \frac{n}{3}$ for even n. For this sequence, see A131423 [43].

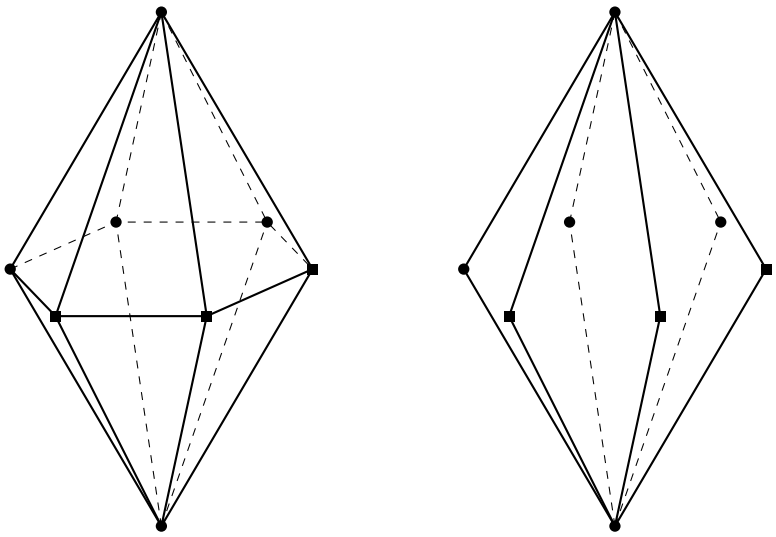

Figure 2.3: Minimum Wiener index simple triangulations and quadrangulations.

There is second research direction of this chapter, in addition to the Wiener index. We give bounds on the total distance $\sigma(v)$ and the average distance $\overline{\sigma}(v)$ of a vertex v, defined as the sum and the average, respectively, of the distances from v to all other vertices. Bounds on $\sigma(v)$ were obtained, for example, in [3] [21] and [46]. Of particular interest is the maximum value over all $v \in V(G)$ of $\overline{\sigma}(v)$ in a graph G, usually referred to as the *remoteness* $\rho(G)$, of G. It was shown by Zelinka [46] and, independently, by Aouchiche and Hansen [2] that the remoteness is at most $\frac{n}{2}$. For graphs of given minimum degree δ these bounds were improved in [13] by a factor of about $\frac{3}{\delta+1}$. For more recent results remoteness see, for example, [12], and [45].

We give sharp upper bounds on remoteness of triangulations and quadrangulations with given connectivity in Corollary 3 and Proposition 2. These bounds are sharp by Figures 2.5 through 2.12 and Figures 2.14 through 2.22. It is not difficult to compute the distances on those figures from the black vertex to the remaining vertices and show that the sum of distances from the black vertex meets the upper bound for remoteness. Details will be provided in section 2.5. Our results show that the maximum remoteness among triangulations and quadrangulations of prescribed connectivity κ is achieved on the graphs that are conjectured to maximize the Wiener index, *except for 5-connected triangulations of order* $n = 5k + 3$. There are, however, numerous realizations of the maximum of remoteness in all classes that we investigate, except among quadrangulations.

2.2 Upper Bounds on the Remoteness

In this section we present bounds on the remoteness of triangulations and quadrangulations. A sharp upper bound on the remoteness of a triangulation of given order was given by Che and Collins [9]. We give corresponding bounds for 4-connected and 5-connected triangulations, as well as for 2-connected and 3-connected quadrangulations.

We begin by stating a sharp bound on the distance of an arbitrary vertex in a κ-connected graph of given order due to Favaron, Kouider and Mahéo [23], from which we will derive some of our bounds.

Proposition 1. *[23] Let G be a κ-connected graph of order n, and x an arbitrary vertex of G. Then*

$$\sigma(x) \leq \lfloor \frac{n+\kappa-1}{\kappa} \rfloor (n-1-\frac{\kappa}{2}\lfloor \frac{n-1}{\kappa} \rfloor).$$

Every simple triangulation is 3-connected, and every simple quadrangulation is 2-connected. Proposition 1 yields the following sharp bounds for the remoteness of 3-connected and 4-connected triangulations and 2-connected quadrangulations.

Corollary 3. *(a) [9] If G is a simple triangulation of order n, then*

$$\rho(G) \leq \frac{n+2}{6} + \varepsilon_n,$$

where $\varepsilon_n = 0$ if $n \equiv 1 \pmod 3$, and $\varepsilon_n = \frac{1}{3(n-1)}$ if $n \equiv 0, 2 \pmod 3$.

(b) If G is a 4-connected triangulation of order n, then

$$\rho(G) \leq \frac{n+3}{8} + \varepsilon_n,$$

where $\varepsilon_n = 0$ if $n \equiv 1 \pmod 4$, $\varepsilon_n = \frac{3}{8(n-1)}$ if $n \equiv 0, 2 \pmod 4$, and $\varepsilon_n = \frac{1}{2(n-1)}$ if $n \equiv 3 \pmod 4$.

(c) If G is a simple quadrangulation of order n, then

$$\rho(G) \leq \frac{n+1}{4} + \varepsilon_n,$$

where $\varepsilon_n = 0$ if $n \equiv 1 \pmod 2$, and $\varepsilon_n = \frac{1}{4(n-1)}$ if $n \equiv 0 \pmod 2$. □

Proposition 1 also yields good bounds for the remoteness of 5-connected triangulations and 3-connected quadrangulations. These bounds are however not sharp for

all values of n. In order to obtain sharp bounds we need some additional terminology and results from [1].

Let v be a fixed vertex of a connected plane graph and $i \in \mathbb{N}$ with $i < e(v)$. We say that a vertex $w \in N_i(v)$ is *active* if it has a neighbor in $N_{i+1}(v)$.

Lemma 1. *[1] Let G be a 3-connected plane graph, v a vertex of G and $i \in \mathbb{N}$ with $1 \leq i \leq e(v) - 1$. For every active vertex $w \in N_i(v)$ there exist two other active vertices $w', w'' \in N_i(v)$ such that w and w' share a face of G, and w and w'' also share a face of G.*

Lemma 2. *(a) Let G be a 5-connected simple triangulation, v a vertex of G and $d = e_G(v)$. If $n_{d-1}(v) = 5$, then $n_d(v) = 1$.*
(b) Let G be a 3-connected simple quadrangulation, v a vertex of G and $d = e_G(v)$. If $n_{d-1}(v) = 3$, then $n_d(v) = 1$. If $n_{d-2} = 3$ and $n_{d-1} = 4$, then $n_d(v) \neq 1$.

Proof. (a) Assume that G is a 5-connected simple triangulation, v is a vertex of G, and $n_{d-1} = 5$, where d is the eccentricity of v. This implies that N_{d-1} is a minimum cutset of G. Hence, since G is a triangulation, N_{d-1} induces a cycle C of length 5. We first show that

$$\text{the vertices in } N_d \text{ are all inside } C, \text{ or all outside } C. \tag{2.6}$$

Suppose not. Then there exist vertices $a, b \in N_d$ such that a is inside C, and b is outside C. Since G is 5-connected, there exist a (v, N_{d-1})-fan F_v, an (a, N_{d-1})-fan F_a, and a (b, N_{d-1})-fan F_b. Any two of these three fans share only the vertices of N_{d-1}. Indeed, other than vertices in N_{d-1}, fan F_v contains only vertices in $\bigcup_{i=0}^{d-1} N_i$, while fan F_a contains only vertices in $N_{d-1} \cup N_d$ that are inside C, while fan F_b contains only vertices in $N_{d-1} \cup N_d$ that are outside C. Now contracting the vertices in $F_a - N_{d-1}$, the vertices in $F_b - N_{d-1}$, and the vertices in $F_v - N_{d-1}$ to three single vertices yields a graph that contains $3K_1 + C_5$ as a subgraph. Hence G contains $3K_1 + C_5$ as a

minor. Contracting three consecutive vertices of the 5-cycle shows that this implies that G contains $K_3 + 3K_1$ as a minor, which contradicts the planarity of G. This contradiction proves (2.6).

By (2.6) we may assume that all vertices of N_d are inside the cycle C. Since every vertex of N_d is adjacent to some vertex of N_{d-1}, the subgraph $G[N_d]$ is outerplanar. Hence

$$m(G[N_d]) \leq \begin{cases} 0 & \text{if } n_d = 1, \\ 1 & \text{if } n_d = 2, \\ 2n_d - 3 & \text{if } n_d \geq 3. \end{cases} \tag{2.7}$$

We now bound the sum of the degrees of the vertices in N_d. Let H be the plane graph obtained from $G[N_{d-1} \cup N_d]$ by adding a new vertex z in the outer face of C and joining it to all five vertices of C. Then H has order $n(H) = 1+n_{d-1}+n_d = n_d+6$. Since H is a plane graph we have $m(H) \leq 3n(H) - 6 \leq 3n_d + 12$. At least 10 edges of H are incident with z or belong to C, and are thus not incident with any vertex of N_d, so they don't contribute to the sum of the degrees of vertices in N_d. Since the edges of $G[N_d]$ contribute two to the sum of the degrees of vertices in N_d, we have

$$\sum_{x \in N_d} \deg_G(x) \leq (m(H) - 10) + m(G[N_d]) \leq \begin{cases} (3n_d + 2) + 0 & \text{if } n_d = 1, \\ (3n_d + 2) + 1 & \text{if } n_d = 2, \\ (3n_d + 2) + (2n_d - 3) & \text{if } n_d \geq 3. \end{cases}$$

It is easy to verify that this implies $\sum_{x \in N_d} \deg_G(x) < 5n_d$ whenever $n_d > 1$. But since G is 5-connected, every vertex of G has degree at least five. Hence we conclude that $n_d = 1$, which proves (a).

(b) Let G be a 3-connected simple quadrangulation, v a vertex of G, and $d = e(v)$. To prove the first statement, assume that $n_{d-1} = 3$. Let $N_{d-1}(v) = \{w, w', w''\}$. Since G is a quadrangulation and thus bipartite, the set $\{w, w, w''\}$ is independent in G. Since G is 3-connected, the vertices w, w', w'' have a neighbor in N_d and are thus active. By Lemma 1, w and w' share a face, and so do w and w'', as well as w' and

w''. Hence we can add edges ww', ww'' and ww'' to G to obtain a plane graph (but not a quadrangulation). Let C be the cycle consisting of the edges $ww', w'w'', w''w$. A proof similar to that in (a) shows that the vertices of N_d are all inside C, or all outside C. Without loss of generality we assume the former. We now bound the sum of the degrees of the vertices in N_d.

Let H be the plane graph obtained from $G[N_{d-1} \cup N_d] + E(C)$ by adding a new vertex z in the outer face of C and joining it to all three vertices of C. Since G is a quadrangulation, the only faces of H of length three are the six faces that have one of the three edges of C on their boundary. Let H' be the plane graph $H - E(C) = G[N_{d-1} \cup N_d]$. Then $n(H') = n_{d-1} + n_d + 1 = n_d + 4$ and, since H' has only faces of length at least four, $m(H') \leq 2n(H') - 4 = 2n_d + 4$.

Exactly three edges of H are incident with z and are thus not incident with any vertex of N_d. Since G is bipartite, $G[N_d]$ contains no edges. Hence

$$\sum_{x \in N_d} \deg_G(x) = (m(H') - 3) \leq 2n_d + 1.$$

This implies $\sum_{x \in N_d} \deg_G(x) < 3n_d$ whenever $n_d > 1$. But since G is 3-connected, every vertex of G has degree at least three. Hence we conclude that $n_d = 1$, which proves the first statement of (b).

To prove the second statement of (b) assume that $n_{d-2} = 3$ and $n_{d-1} = 4$. Suppose to the contrary that $n_d = 1$.Let $N_{d-2} = \{w, w', w''\}$. The same arguments as in the proof of the first statement of (b) show that we can add the edges $ww', ww'', w'w''$ to G to obtain a plane graph, that these three edges form a cycle C, and that the vertices in $N_{d-1} \cup N_d$ are either all inside C or all outside C, without loss of generality the former. Let H be the plane graph obtained from $G[N_{d-2} \cup N_{d-1} \cup N_d] + E(C)$ by adding a new vertex z in the outer face of C and joining it to all three vertices of C. Since G is a quadrangulation, the only faces of H of length three are the six faces that have one of the three edges of C on their boundary. Let H' be the plane graph $H - E(C) = G[N_{d-2} \cup N_{d-1} \cup N_d]$. Then $n(H') = n_{d-2} + n_{d-1} + n_d + 1 = 9$

and, since H' has only faces of length at least four, $m(H') \leq 2n(H') - 4 = 14$. Exactly three edges of H' are incident with z and thus not incident with vertices in N_{d-1}. Since G is bipartite, no edge joins two vertices of N_{d-1}, and so we have $\sum_{x \in N_{d-1}} \deg_G(x) \leq 11 < 3n_{d-1}$. Therefore, N_{d-1} contains a vertex of degree less than three in G, which contradicts G being 3-connected. The second statement of (b) follows. □

For the remaining proofs of this section we define the function F which assigns to a finite sequence $X = (x_0, x_1, \ldots, x_k)$ of integers the value $F(X) = \sum_{i=0}^{k} i x_i$. So if v is a vertex of eccentricity d in a connected graph G, then $\sigma(v) = \sum_{i=0}^{d} i n_i(v) = F(n_0, n_1, \ldots, n_d)$.

Proposition 2. *(a) Let G be a 5-connected triangulation of order n. Then*

$$\rho(G) \leq \frac{n+4}{10} + \varepsilon_n,$$

where $\varepsilon_n = -\frac{3}{5(n-1)}$ if $n \equiv 0 \pmod 5$, $\varepsilon_n = -\frac{1}{n-1}$ if $n \equiv 1 \pmod 5$, $\varepsilon_n = \frac{2}{5(n-1)}$ if $n \equiv 2 \pmod 5$, and $\varepsilon_n = -\frac{2}{5(n-1)}$ if $n \equiv 3, 4 \pmod 5$.

(b) If G is a 3-connected quadrangulation of order n, then

$$\rho(G) \leq \frac{n+2}{6} + \varepsilon_n,$$

where $\varepsilon_n = -\frac{5}{3(n-1)}$ if $n \equiv 0 \pmod 3$, $\varepsilon_n = -\frac{1}{n-1}$ if $n \equiv 1 \pmod 3$, and $\varepsilon_n = \frac{1}{3(n-1)}$ if $n \equiv 2 \pmod 3$.

Proof. (a) It suffices to show that for an arbitrary vertex v of G we have

$$\sigma(v) \leq \frac{n^2+3n}{10} + \varepsilon'_n,$$

where $\varepsilon'_n = -10$ if $n \equiv 0 \pmod 5$, $\varepsilon'_n = -14$ if $n \equiv 1 \pmod 5$, $\varepsilon'_n = 0$ if $n \equiv 2 \pmod 5$, and $\varepsilon'_n = -8$ if $n \equiv 3, 4 \pmod 5$.

Fix $v \in V(G)$ and let $d = \mathrm{e}(v)$. Then

$$\sigma(v) = \sum_{i=0}^{d} i n_i = F(n_0, n_1, \ldots, n_d).$$

All n_i are positive integers, $n_0 = 1$ and $\sum_{i=0}^{d} n_i = n$. Since G is 5-connected we also have $n_i \geq 5$ for all $i \in \{1, 2, \ldots, d-1\}$. To bound $F(n_0, n_1, \ldots, n_d)$ from above we assume that n is fixed, and that $d' \in \mathbb{N}$ and $X_{max}(n) = (n'_0, n'_1, \ldots, n'_{d'})$ maximise the function F among all integers d and sequences X that satisfy these constraints. We first note that $n'_1 = n'_2 = \cdots = n'_{d-1} = 5$. Indeed, if $n'_i \geq 5$ for some i with $1 \leq i \leq d'-1$, then decreasing n'_i by 1 and increasing n'_{i+1} by 1 yields a new sequence X' that satisfies the above constraints and for which $F(X') = F(X_{max}(n)) + 1$, contradicting the choice of $X_{max}(n)$. Also, if $n'_{d'} > 5$, then decreasing $n'_{d'}$ by 1, appending a new entry $n'_{d'+1} = 1$ at the end and increasing d' by 1 yields a sequence that satisfies the requirement but whose F-value is greater, again a contradiction to the choice of $X_{max}(n)$. Therefore, if q and r are positive integers with $1 \leq r \leq 5$ such that $n - 1 = 5q + r$, then the unique sequence maximising F subject to the above constraints is

$$X_{max}(n) = (1, 5, 5, \ldots, 5, r),$$

where the entry 5 appears exactly q times. If $r \neq 1$, then it is easy to see that the unique sequence with the second largest F-value satisfying the constraints is the sequence

$$X'_{max}(n) = (1, 5, 5, \ldots, 5, 6, r-1),$$

where the entry 5 appears exactly $q - 1$ times.

Case 1: $n \equiv 2 \pmod 5$.

Then $F(n_0, n_1, \ldots, n_d) \leq F(X_{max}(n)) = \frac{1}{10}(n^2 + 3n)$, as desired.

Case 2: $n \equiv 0, 1, 3, 4 \pmod 5$.

Then $(n_0, n_1, \ldots, n_d) \neq X_{max}(n)$ since otherwise, if $(n_0, n_1, \ldots, n_d) = X_{max}(n)$, then $n_{d-1} = 5$ and $n_d = r \neq 1$, contradicting Lemma 2(a). Therefore, $F(n_0, n_1, \ldots, n_d) \leq F(X'_{max}(n))$, and a simple calculation shows that $F(X'_{max})(n)$ is the claimed upper bound on $\sigma(v)$.

(b) The proof of (b) is analogous to that of (a), with only two differences: The

condition $n_i \geq 5$ for all $i \in \{1, 2, \ldots, d-1\}$ in (a) is replaced by $n_i \geq 3$ for all $i \in \{1, 2, \ldots, d-1\}$. Also, Lemma 2(b) implies that for $n \equiv 1, 2 \pmod 3$ we have $(n_0, n_1, \ldots, n_d) \neq X_{max}(n)$ and so $F(n_0, n_1, \ldots, n_d) \leq F(X_{max}(n)')$, while for $n \equiv 0 \pmod 3$ Lemma 2(b) implies that $(n_0, n_1, \ldots, n_d) \neq X_{max}(n), X_{max}(n)'$ and thus $F(n_0, n_1, \ldots, n_d) < F(X_{max}(n)')$. □

2.3 Upper bounds on the Wiener index

In this section we present asymptotically sharp upper bounds on the Wiener index of simple triangulations and simple quadrangulations, and improved bounds for simple 4-connected and 5-connected triangulations as well as simple 3-connected quadrangulations.

In the statements and proofs of our results we use the following notation. If S is a separating cycle of a plane graph G, then we denote the set of vertices inside S by A, and the set of vertices outside S by B. We often use S also for the set of vertices on this cycle, and we further let $a := |A|$, $b := |B|$ and $s := |S|$. The following separator theorem by Miller is an important tool for the proof of our bounds.

Theorem 4. *([36]) If G is a 2-connected plane graph of order n whose faces have length at most ℓ, then G has a separating cycle S of length at most $2\sqrt{2\lfloor \ell/2 \rfloor n}$, such that $a, b \leq \frac{2}{3}n$.*

We now define a plane graph which will be used in the proof of the main result of this section.

Definition 1. *For $p \in \mathbb{N}$ with $p \geq 3$ let F_p be the plane graph constructed as follows. Let $C = u_0, u_1, \ldots, u_{p-1}, v_0$ be a cycle of length p. Inside C we add a cycle $C' = v_0, v_1, \ldots, v_{2p-1}v_0$ of length $2p$ and edges u_iv_{2i-1}, u_iv_{2i}, u_iv_{2i+1} for $i = 0, 1, \ldots, p-1$, with indices taken modulo p for the u_i and modulo $2p$ for the v_i. Inside C' we add a cycle $C'' = w_0, w_1, \ldots, w_{2p-1}, w_0$ of length $2p$ and edges v_iw_i, v_iw_{i+1} for $i =$*

$0.1, \ldots, 2p-1$, *with all indices taken modulo* $2p$. *Inside* C'' *we add a new vertex* z *and join it to every vertex of* C''. *The graph* F_4 *is shown in Figure 2.4.*

We define F'_p *to be a plane graph with the same vertex and edge set as* F_p, *but with the cycle* C' *outside the cycle* C, *the cycle* C'' *outside the cycle* C', *and* z *lying in the unbounded face whose boundary is* C''.

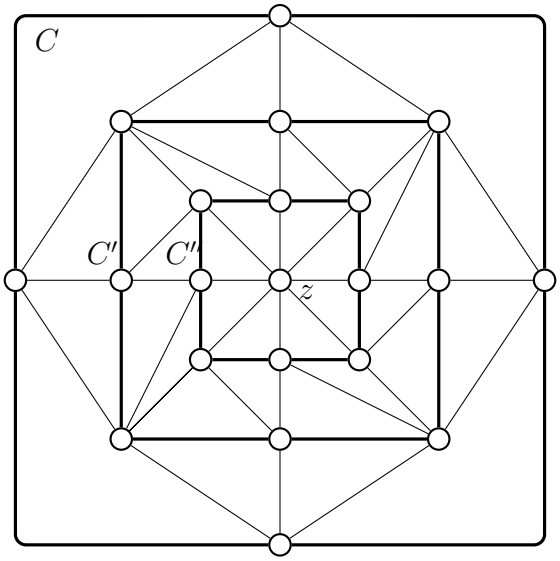

Figure 2.4: The graph F_s for $s = 4$. The s-cycle C and $2s$-cycles C' and C'' drawn with thick lines.

Lemma 3. *Let* F_p *be the graph defined in Definition 1 above.*

(a) $\kappa(F_p) \geq 5$ *for* $p \geq 3$.

(b) If $u \in V(F_p)$ *and* $M \subseteq V(C)$ *with* $|M| \leq 5$, *then* F_p *contains a* (u, M)*-fan.*

(c) If $M_1, M_2 \subseteq V(C)$ *are two sets with* $|M_1 = |M_2| \leq 5$, *then* F_p *contains a set of* $|M_1|$ *disjoint paths from* M_1 *to* M_2.

Proof. (a) It is easy to verify that any two vertices of F_p are joined by five internally disjoint paths, hence F_p is 5-connected.

(b) and (c) follow directly from F_p being 5-connected. □

Theorem 5. *Let $\kappa \in \{3,4,5\}$. Then there exists a constant C such that*

$$W(G) \leq \frac{1}{6\kappa} n^3 + Cn^{5/2}$$

for every κ-connected simple triangulation of order n.

Proof. Our proof is by induction on n. Define $C := \max\{C_1, C_2\}$, where C_1 is the smallest real x for which the inequality $W(G) \leq \frac{1}{6\kappa} n^3 + xn^{5/2}$ holds for all κ-connected simple triangulations G of order at most 10^4, and C_2 is the smallest real x for which $8.1 + 0.76x \leq x$ holds. We prove by induction on n that for all simple triangulations G of order n,

$$W(G) \leq \frac{1}{6\kappa} n^3 + Cn^{5/2}, \tag{2.8}$$

Now (2.8) holds for all $n \leq 10^4$ by the choice of C. Let $n > 10^4$. By our induction hypothesis we may assume that (2.8) holds for all graphs of order less than n.

Since G is 2-connected, it follows by Theorem 4 that G contains a separating cycle $S = t_0 t_1 \dots t_{s-1} t_0$ with $a, b \leq \frac{2}{3} n$, where A, B, a, b, s are as in Theorem 4 and above it. Let H be the simple triangulation obtained from the plane graph $G - A$ as follows. We first delete all edges between non-consecutive vertices of S that run inside the cycle S. Inside S we insert the graph F_s by identifying the cycles S and C, specifically $t_i \in S$ with $u_i \in V(F_s)$ for $i = 0, 1, \dots, s-1$. Clearly, H is a simple triangulation of order $b + 5s + 1$. Similarly, let K be the simple triangulation of order $a + 5s + 1$ obtained from the plane graph $G - B$ by deleting all edges between non-consecutive vertices of S that run outside the cycle S and inserting F'_s (a copy of F_s) into the unbounded face, bounded by the vertices of S, by identifying $t_i \in S$ with $u_i \in V(F'_s)$ for $i = 0, 1, \dots, s-1$.

For an illustration, see Figure 2.4. We claim that

$$H \text{ and } K \text{ are } \kappa\text{-connected.} \tag{2.9}$$

We prove (2.9) only for H, the proof for K is analogous. Let u, v be two arbitrary vertices of H. It suffices to show that there exist κ internally disjoint (u, v)-paths in

H. First assume that both, u and v, are in $V(F_s)$, then it follows from Lemma 3(a) and $\kappa \leq 5$ that there are κ internally disjoint (u,v)-paths in F_s, and thus in H. Now assume that exactly one of the two vertices, say u, is in $V(F_s)$. Fix a vertex $a \in A$. It follows from the κ-connectedness of G that in G there exist κ internally disjoint (a,v)-paths $P_1, P_2, \ldots, P_\kappa$. For $i = 1, 2, \ldots, \kappa$ let a_i be the last vertex of P_i on C, and let P_i' be the (a_i, v)-section of P_i. By Lemma 3(b), F_s contains a $(u, \{a_1, \ldots, a_\kappa\})$-fan F. Then F together with $P_1', \ldots, P_\kappa'$ yields a collection of κ internally disjoint (u,v)-paths in H. Finally assume that both, u and v, are not in $V(F_s)$. Then it follows from the κ-connectedness of G that there exists internally disjoint (u,v)-paths $P_1, P_2, \ldots, P_\kappa$ in G. For those paths, $P_1, \ldots, P_k$ say, that contain a vertex of $V(F_s)$, let a_i and a_i' be the first and last vertex, respectively, of P_i in $V(F_s)$. Let $M = \{a_1, \ldots, a_k\}$ and $M' = \{a_1', \ldots, a_k'\}$. By Lemma 3(c), F_s contains k disjoint paths $Q_1, \ldots, Q_k$ from M to M'. Then the (u, a_i)-sections and the (a_i', v)-sections of the paths P_i together with $Q_1, \ldots, Q_k$ and the paths $P_{k+1}, \ldots, P_\kappa$ form a collection of κ internally disjoint (u,v)-paths in H. This proves (2.9).

The two graphs H and K have exactly the vertices in $V(F_s)$ in common. We now bound the Wiener index of G in terms of the Wiener indices of, and the total distance of Z in H and K.

$$\begin{aligned} W(G) &< \sum_{\{x,y\} \subseteq B \cup V(F_s)} d_G(x,y) + \sum_{\{x,y\} \subseteq A \cup V(F_s)} d_G(x,y) + \sum_{x \in A,\ y \in B} d_G(x,y) \\ &< \binom{n}{2} \frac{s}{2} + \sum_{\{x,y\} \subseteq B \cup V(F_s)} d_H(x,y) \\ &\quad + \sum_{\{x,y\} \subseteq A \cup V(F_s)} d_K(x,y) + \sum_{x \in A,\ y \in B} d_H(x,z) + d_K(z,y). \end{aligned} \tag{2.10}$$

Indeed, for any two vertices x and y of G that are both in $A \cup V(F_s)$, we have $d_G(x,y) \leq d_H(x,y) + \frac{s}{2}$ since a shortest (x,y)-path in H either contains only vertices in $B \cup S$, in which case it is also a path in G, or it contains vertices in $V(F_s) - S$, in which case replacing the segment between the first and last occurrence of a vertex

in $V(F_s) - S$ in the path by a segment of the cycle S that contains at most $s/2$ vertices yields an (x, y)-path in G. Similarly, if x and y are both in $B \cup V(F_s)$, then $d_G(x, y) \leq d_K(x, y) + \frac{s}{2}$. Finally, if $x \in A$ and $y \in B$, then we can obtain an (x, y)-path in G from the concatenation of an (x, z)-path in H and a (z, y)-path in K by replacing z with a segment of S containing at most $s/2$ vertices. This proves (2.10).

We now bound each of the terms in (2.10). Since H and K are κ-connected simple triangulations of order $b + 5s + 1$ and $a + 5s + 1$, respectively, we have by induction

$$\sum_{\{x,y\} \subseteq B \cup V(F_s)} d_H(x, y) = W(H) \leq \frac{1}{6\kappa}(b + 5s + 1)^3 + C(b + 5s + 1)^{5/2}, \tag{2.11}$$

and

$$\sum_{\{x,y\} \subseteq A \cup V(F_s)} d_K(x, y) = W(K) \leq \frac{1}{6\kappa}(a + 5s + 1)^3 + C(a + 5s + 1)^{5/2}. \tag{2.12}$$

It follows from Corollary 3(a)-(c) that $\sigma(v) \leq \frac{1}{2\kappa}n^2 + \frac{\kappa-2}{2\kappa}n + \frac{\kappa-3}{2\kappa}$ for every vertex v of a κ-connected triangulation of order n. Hence $\sigma(z, H) \leq \frac{1}{2\kappa}(a+5s+1)^2 + \frac{\kappa-2}{2\kappa}(a + 5s + 1) + \frac{\kappa-3}{2\kappa}$ and $\sigma(z, K) \leq \frac{1}{2\kappa}(b + 5s + 1)^2 + \frac{\kappa-2}{2\kappa}(b + 5s + 1) + \frac{\kappa-3}{2\kappa}$. Hence

$$\begin{aligned}
\sum_{x \in A,\ y \in B} \Big(d_H(x, z) + d_K(z, y)\Big) &= b \sum_{x \in A} d_H(x, z) + a \sum_{y \in B} d_K(z, y) \\
&< b\,\sigma(z, H) + a\,\sigma(z, K) \\
&\leq \frac{b}{2\kappa}\Big[(a + 5s + 1)^2 + (\kappa - 2)(a + 5s + 1) + \kappa - 3\Big] \\
&\quad + \frac{a}{2\kappa}\Big[(b + 5s + 1)^2 + (\kappa - 2)(b + 5s + 1) + \kappa - 3\Big],
\end{aligned}$$

and since $a < a + 5s + 1$ and $b < b + 5s + 1$,

$$\begin{aligned}
&\sum_{x \in A,\ y \in B} d_H(x, z) + d_K(z, y) \\
&< \frac{1}{2\kappa}(a + 5s + 1)^2(b + 5s + 1) + \frac{1}{2\kappa}(a + 5s + 1)(b + 5s + 1)^2 \\
&\quad + \frac{\kappa - 2}{\kappa}(a + 5s + 1)(b + 5s + 1) + \frac{\kappa - 3}{2\kappa}(a + b).
\end{aligned} \tag{2.13}$$

Hence we obtain from (2.10), (2.11), (2.12). amd (2.13),

$$\begin{aligned} W(G) &< \frac{1}{6\kappa}(a+5s+1)^3 + C(a+5s+1)^{5/2} + \frac{1}{6\kappa}(b+5s+1)^3 + C(b+5s+1)^{5/2} \\ &\quad + \frac{1}{2\kappa}(a+5s+1)^2(b+5s+1) + \frac{1}{2\kappa}(a+5s+1)(b+5s+1)^2 + \binom{n}{2}\frac{s}{2} \\ &\quad + \frac{\kappa-2}{\kappa}(a+5s+1)(b+5s+1) + \frac{\kappa-3}{2\kappa}(a+b) \\ &= \frac{1}{6\kappa}(a+b+10s+2)^3 + C\Big[(a+5s+1)^{5/2} + (b+5s+1)^{5/2}\Big] \\ &\quad + \frac{\kappa-2}{\kappa}(a+5s+1)(b+5s+1) + \frac{\kappa-3}{2\kappa}(a+b) + \binom{n}{2}\frac{s}{2}. \end{aligned} \tag{2.14}$$

We bound the terms of the right hand side of (2.14) separately. We make use of the facts that $a+b+s=n$, and that by Theorem 4 in conjunction with $n > 10^4$ we have $s \le 2^{3/2}n^{1/2} < 0.03n - 1$. We bound the first term of (2.14) by $(a+b+10s+2)^3 = (n+9s+2)^3 \le (n+9\cdot 2^{3/2}n^{1/2}+2)^2$. To bound the second term note that the real function $f(x) = x^{5/2}$ is concave up and that $a, b \le \frac{2}{3}n$ by Theorem 4, which implies that $(a+5s+1)^{5/2} + (b+5s+1)^{5/2}$ is maximised if $a = \frac{2}{3}n$ and $b = \frac{1}{3}n - s$ (or vice versa). Therefore, $(a+5s+1)^{5/2}+(b+5s+1)^{5/2} \le (\frac{2}{3}n+5s+1)^{5/2}+(\frac{1}{3}n+4s+1)^{5/2} \le (\frac{2}{3}n+0.15n)^{5/2} + (\frac{1}{3}n+0.12n)^{5/2} = \Big((\frac{2}{3}+0.15)^{5/2} + (\frac{1}{3}+0.12)^{5/2}\Big)n^{5/2} < 0.76n^{5/2}$. To bound the third term note that $\frac{\kappa-2}{\kappa} < 1$, $a+5s+1 < n$ and $b+5s+1 < n$, so $\frac{\kappa-2}{\kappa}(a+5s+1)(b+5s+1) < n^2$. To bound the fourth term note that $\frac{\kappa-3}{2\kappa} < 1$ and $a+b < n$, so $\frac{\kappa-3}{2\kappa}(a+b) < n$. Finally, $\binom{n}{2} < \frac{1}{2}n^2$, and so we bound the fifth term by $\binom{n}{2}\frac{s}{2} < 2^{-1/2}n^{5/2}$. In total we obtain from (2.14),

$$\begin{aligned} W(G) &< \frac{1}{6\kappa}(n+9\cdot 2^{3/2}n^{1/2}+2)^3 + 0.76\,Cn^{5/2} + n^2 + n + 2^{-1/2}n^{5/2} \\ &= \frac{1}{6\kappa}n^3 + \Big(\frac{13}{\kappa} + 0.76\,C + 1 + 2^{-1/2}\Big)n^{5/2} + \frac{338}{\kappa}n^2 + \frac{8788}{3\kappa}n^{3/2}. \end{aligned}$$

Since $n \ge 10^4$, we have $\frac{338}{\kappa}n^2 + \frac{8788}{3\kappa}n^{3/2} < 2n^{5/2}$. Also, $\frac{13}{\kappa} + 0.76\,C + 1 + 2^{-1/2} < 6.1 + 0.76\,C$, and so

$$\begin{aligned} W(G) &< \frac{1}{6\kappa}n^3 + \Big(8.1 + 0.76\,C\Big)n^{5/2} \\ &\le \frac{1}{6\kappa}n^3 + Cn^{5/2} \end{aligned}$$

since C satisfies $8.1 + 0.76\,C \leq C$. The theorem follows. □

The following bound on the Wiener index of simple quadrangulations is proved in a similar way. The only difference is that a slightly modified version Q_p of the plane graph F_p is used in the proof. For an even p with $p \geq 4$ let Q_p be the plane graph obtained from a cycle $C = u_0, u_1, \ldots, u_{p-1}, u_0$ of length p, inside which we add a cycle $C' = v_0, v_1, \ldots, v_{p-1}, v_0$ of length p and edges $u_i v_i$ for $i = 0, 1, \ldots, p-1$, inside which we add a vertex z and joint it to all v_i with i even. It is easy to verify that a 3-connected quadrangualation with the insertion of Q_p stays 3-connected. Apart from this difference, the proof of Theorem 6 follows closely that of Theorem 5, hence we omit the proof.

Theorem 6. *Let $\kappa \in \{2, 3\}$. Then there exists a constant C such that*

$$W(G) \leq \frac{1}{6\kappa} n^3 + C n^{5/2}$$

for every κ-connected simple quadrangulation G of order n. □

The leading coefficients in the bounds in Theorems 5 and 6 are optimal, as shown by the graphs in Figures 2.5 through 2.13 and Figures 2.15 through 2.20.

2.4 Computational Results and Conjectures

This section contains numerous figures and tables summarizing months of computer searches. None of this would have been possible without the help provided by Plantri, a program that generates triangulations and quadrangulation on numerous surfaces. For each category of problem (triangulations, 4-connected triangulations, 5-connected triangulations, quadrangulations and 3-connected quadrangulations) there is a table, which summarizes the largest Wiener index and remoteness found for a given order in that category, along with "Count", telling how many graphs attain the optimal value. Note that remoteness in this section is *not normalized* to keep the calculations in the

domain of integers. In other words, in the Tables we show $(n-1)\rho(G)$ under the name of "Remoteness". Our Wiener index findings match those of [9] for triangulations. The number of isomorphism classes that our code searched matches the numbers in [6], [8], [7], [34], [42], verifying that the values that the search provides are in fact maximal. In each figure below, red edges represent the repeating pattern and the black node marks a vertex which maximizes the remoteness. The computational evidence suggests that for sufficiently large order, the maximum Wiener index is uniquely realized in every category, while remoteness is not, except for quadrangulations.

Although the results of [26] made the Wiener index rows of Table 2.1 obsolete for $n \geq 9$, we still include it to show the multiplicity of maximizers up to $n = 8$ and the remoteness results.

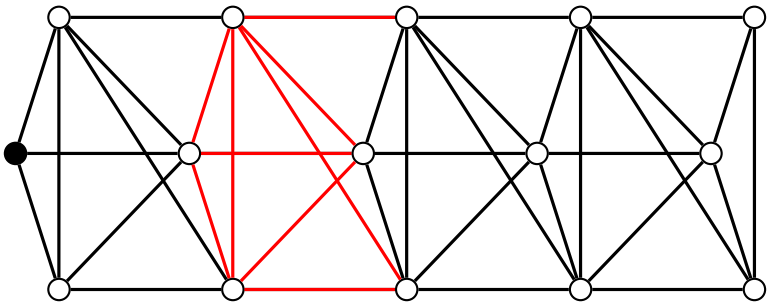

Figure 2.5: A triangulation T_n on $n = 3k$ vertices, which maximizes the Wiener index [26] and the remoteness.

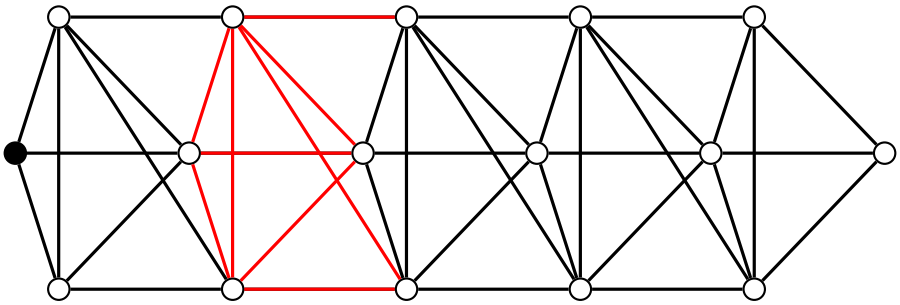

Figure 2.6: A triangulation T_n on $n = 3k + 1$ vertices, which maximizes the Wiener index [26] and the remoteness.

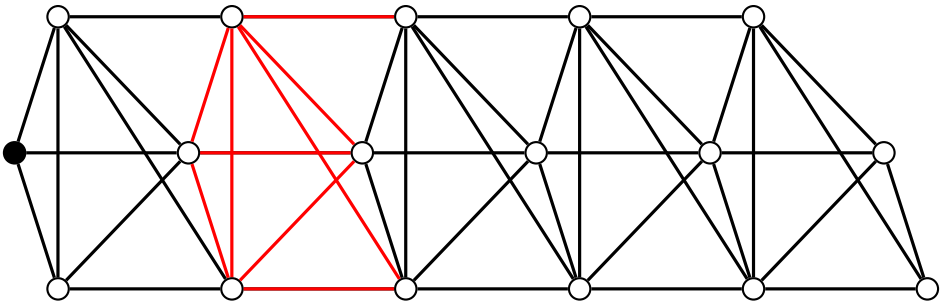

Figure 2.7: A triangulation T_n on $n = 3k + 2$ vertices, which maximizes the Wiener index [26] and the remoteness.

Table 2.1: A summary of the largest Wiener Index and remoteness among all triangulations on n vertices, and a count for how many isomorphism classes attain this value.

Order	Wiener index	Count	Remoteness	Count
4	6	1	3	1
5	11	1	5	1
6	18	2	7	1
7	27	5	9	4
8	39	2	12	2
9	54	1	15	4
10	72	1	18	17
11	94	1	22	7
12	120	1	26	25
13	150	1	30	107
14	185	1	35	35
15	225	1	40	171
16	270	1	45	743
17	321	1	51	217
18	378	1	57	1199

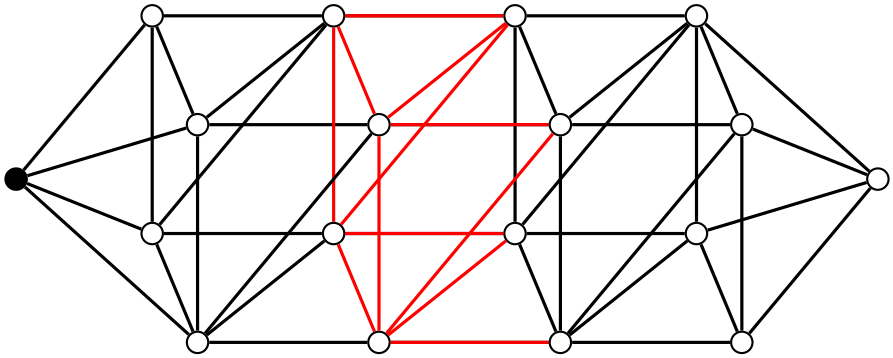

Figure 2.8: A 4-connected triangulation T_n^4 on $n = 4k + 2$ vertices, which maximizes the remoteness and is conjectured to maximize the Wiener index.

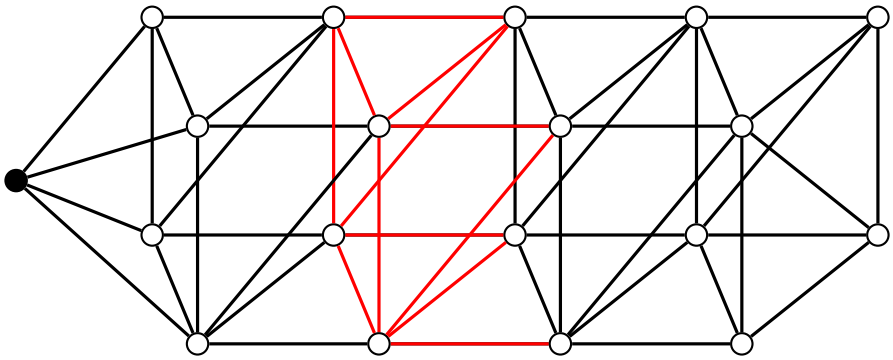

Figure 2.9: A 4-connected triangulation T_n^4 on $n = 4k + 3$ vertices, which maximizes the remoteness and is conjectured to maximize the Wiener index.

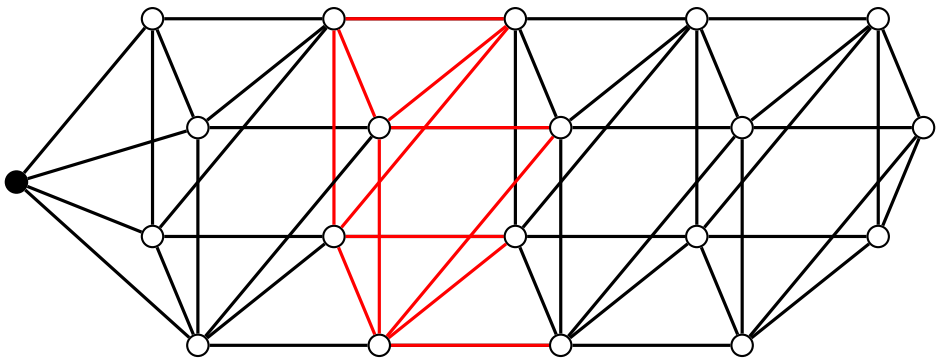

Figure 2.10: A 4-connected triangulation T_n^4 on $n = 4k$ vertices, which maximizes the remoteness and is conjectured to maximize the Wiener index.

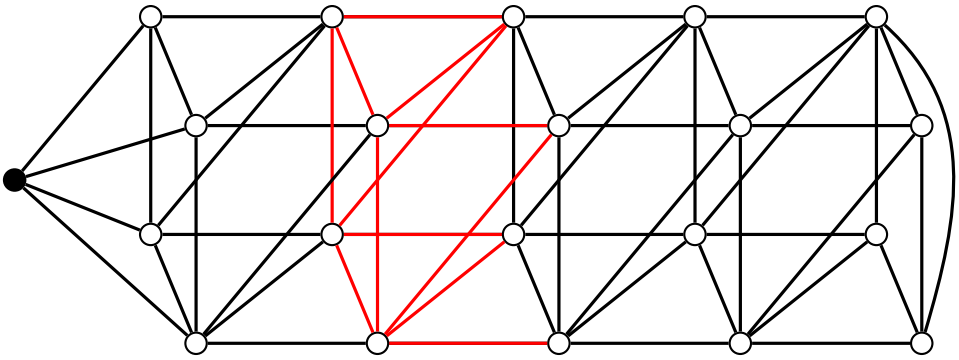

Figure 2.11: A 4-connected triangulation T_n^4 on $n = 4k + 1$ vertices, which maximizes the remoteness and is conjectured to maximize the Wiener index.

Table 2.2: A summary of the largest Wiener Index and remoteness among all 4-connected triangulations on n vertices, and a count for how many isomorphism classes attain this value.

Order	Wiener Index	Count	Remoteness	Count
6	18	1	6	1
7	27	1	8	1
8	38	2	10	2
9	51	4	12	4
10	68	1	15	4
11	87	1	18	6
12	110	1	21	16
13	135	1	24	50
14	166	1	28	24
15	199	1	32	66
16	238	1	36	186
17	279	1	40	653
18	328	1	45	250
19	379	1	50	879
20	438	1	55	2599
21	499	1	60	9429
22	570	1	66	3313

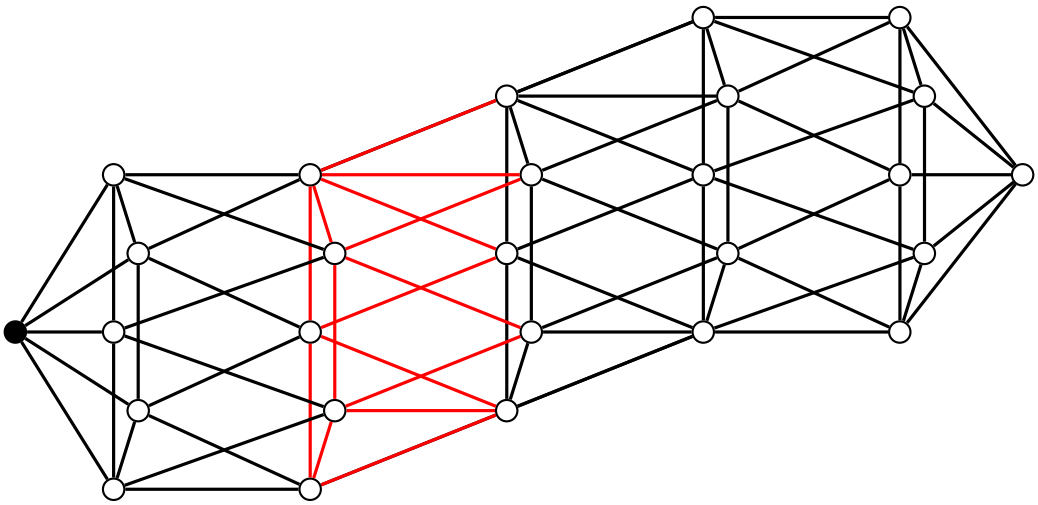

Figure 2.12: A 5-connected triangulation T_n^5 on $n = 5k+2$ vertices, which maximizes the remoteness and is conjectured to maximize the Wiener Index.

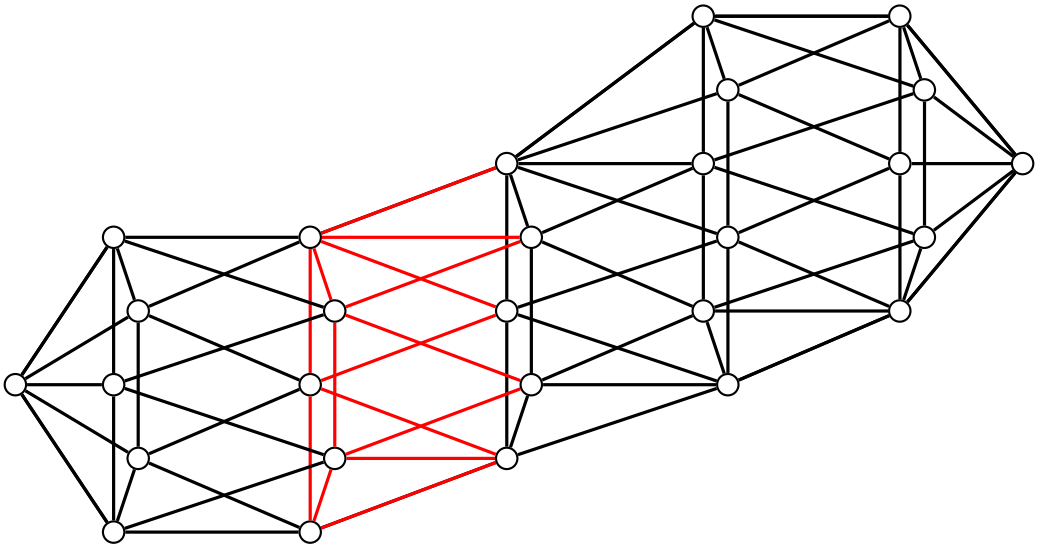

Figure 2.13: A 5-connected triangulation T_n^5 on $n = 5k+3$ vertices, which is conjectured to maximize the Wiener Index.

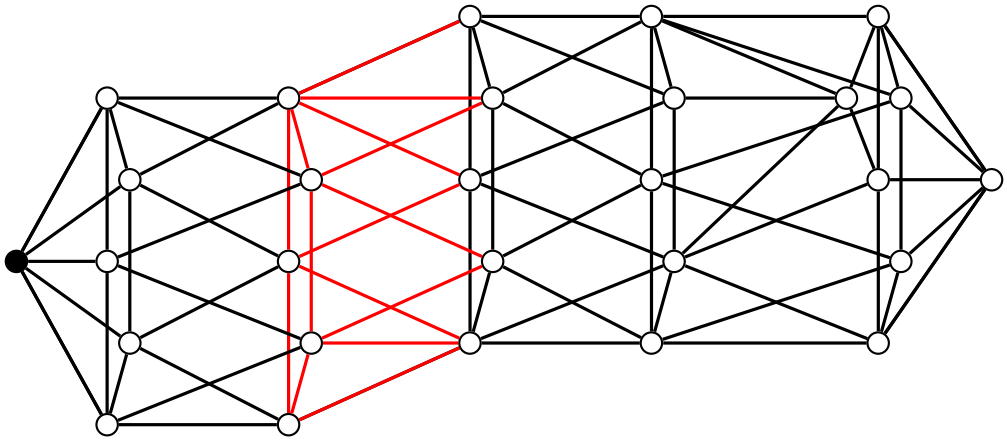

Figure 2.14: A 5-connected triangulation T_n^5 on $n = 5k + 3$ vertices which maximizes the remoteness.

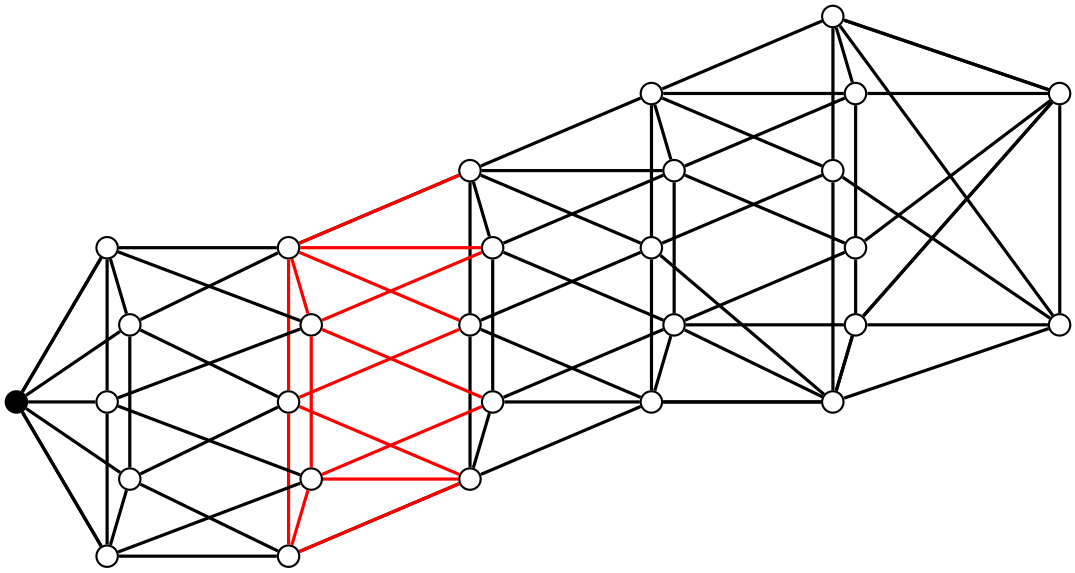

Figure 2.15: A 5-connected triangulation T_n^5 on $n = 5k + 4$ vertices, which maximizes the remoteness and is conjectured to maximize the Wiener Index.

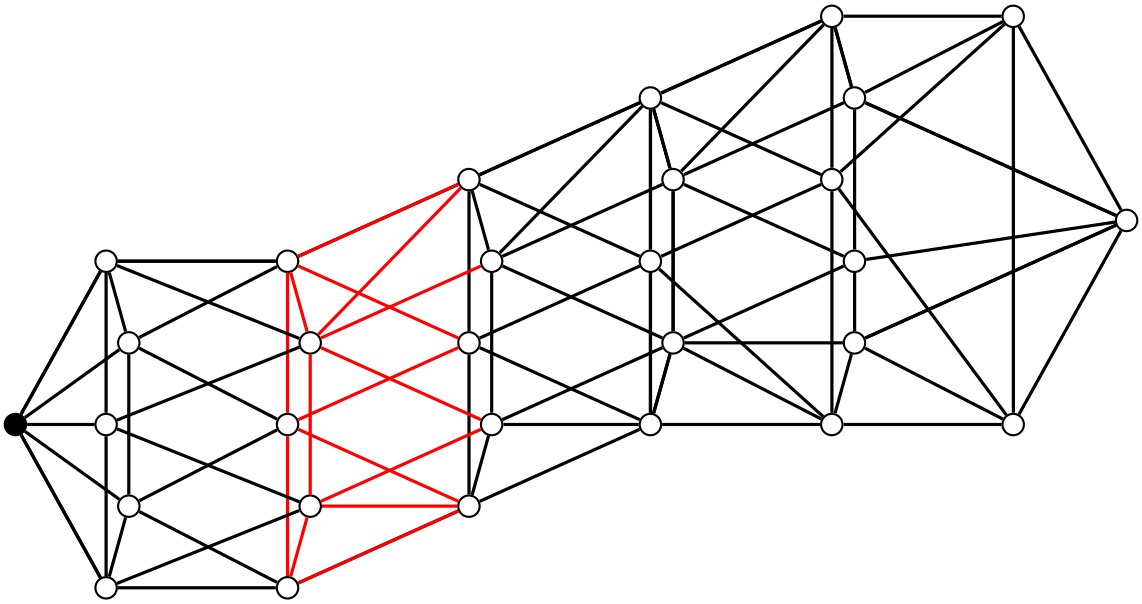

Figure 2.16: A 5-connected triangulation T_n^5 on $n = 5k$ vertices, which maximizes the remoteness and is conjectured to maximize the Wiener Index.

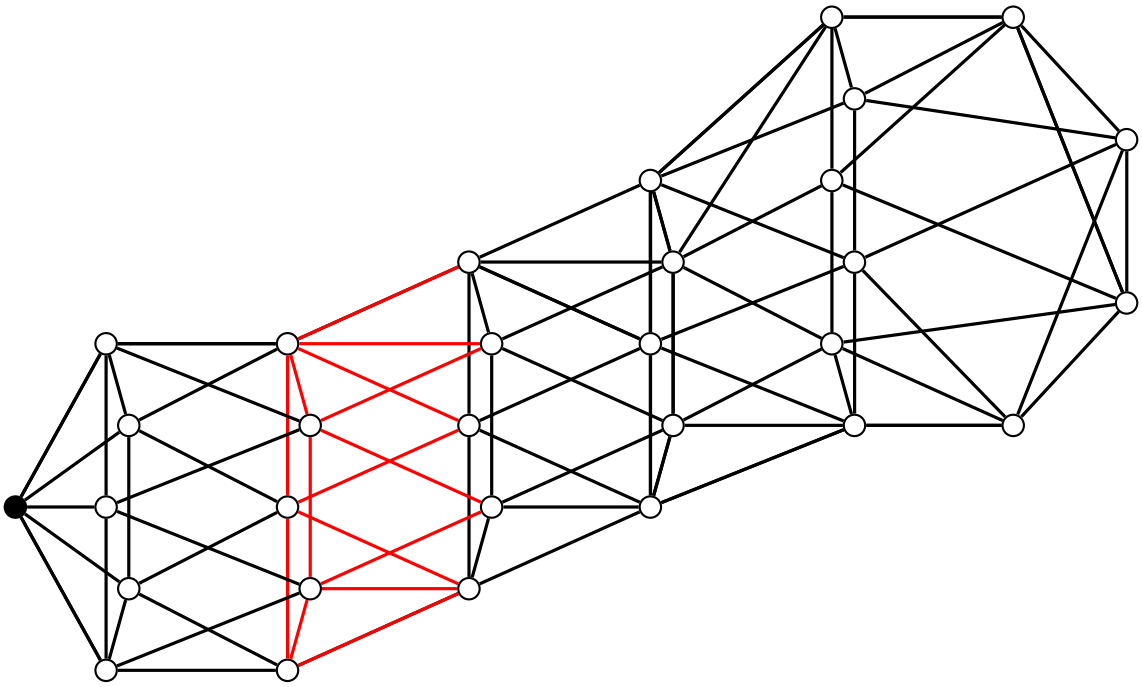

Figure 2.17: A 5-connected triangulation T_n^5 on $n = 5k + 1$ vertices which maximizes the remoteness and is conjectured to maximize the Wiener index.

Table 2.3: A summary of the largest Wiener Index and remoteness among all 5-connected triangulations on n vertices, and a count for how many isomorphism classes attain this value.

Order	Wiener index	Count	Remoteness	Count
12	108	1	18	1
13	—	0	—	0
14	159	1	23	1
15	189	1	26	1
16	222	2	29	1
17	259	1	34	1
18	300	1	37	1
19	342	1	41	2
20	391	1	45	4
21	444	1	49	9
22	500	2	55	4
23	560	1	59	11
24	630	1	64	36
25	702	1	69	66
26	780	1	74	193
27	867	1	81	39
28	955	1	86	240
29	1053	1	92	805
30	1156	1	98	1470
31	1265	1	104	4327
32	1384	1	112	763

Although the results of [29] made the Wiener index rows of Table 2.4 obsolete for $n \geq 8$, we still include it to show the multiplicity of maximizers up to $n = 7$ and the remoteness results.

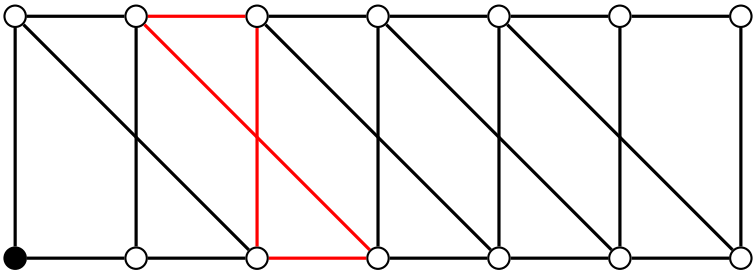

Figure 2.18: A quadrangulation Q_n on $n = 2k$ vertices, which maximizes the remoteness and is conjectured to maximize the Wiener index.

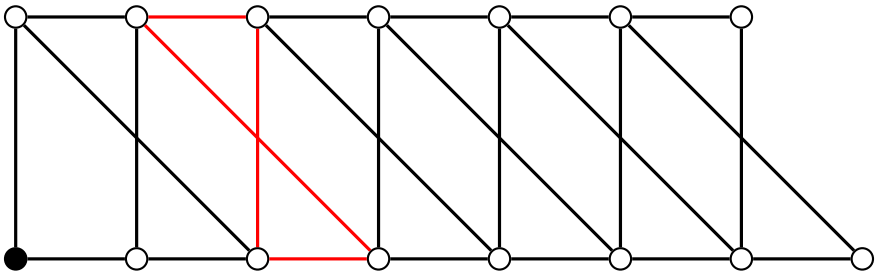

Figure 2.19: A quadrangulation Q_n on $n = 2k + 1$ vertices, which maximizes the remoteness and is conjectured to maximize the Wiener index.

Table 2.4: A summary of the largest Wiener Index and remoteness among all quadrangulations on n vertices, and a count for how many isomorphism classes attain this value.

Order	Wiener Index	Count	Remoteness	Count
4	8	1	4	1
5	14	1	6	1
6	23	1	9	1
7	34	2	12	1
8	50	1	16	1
9	68	1	20	1
10	93	1	25	1
11	120	1	30	1
12	156	1	36	1
13	194	1	42	1
14	243	1	49	1
15	294	1	56	1
16	358	1	64	1
17	424	1	72	1
18	505	1	81	1
19	588	1	90	1
20	688	1	100	1

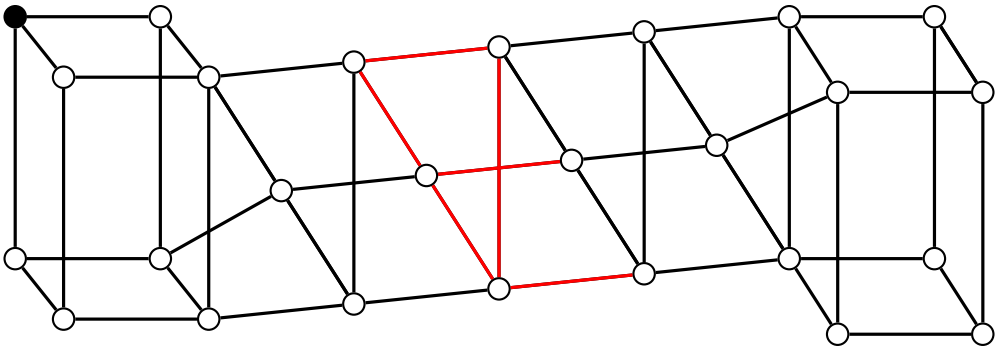

Figure 2.20: A 3-connected quadrangulation Q_n^3 on $n = 3k + 14$ vertices, which maximizes the remoteness and is conjectured to maximize the Wiener index.

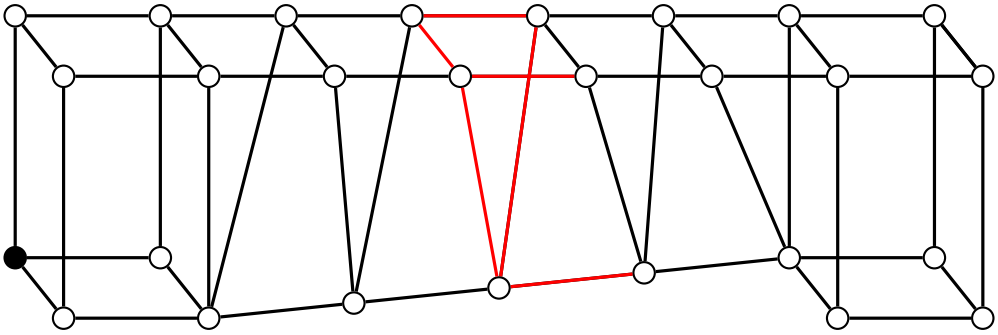

Figure 2.21: A 3-connected quadrangulation Q_n^3 on $n = 3k + 15$ vertices, which maximizes the remoteness and is conjectured to maximize the Wiener index.

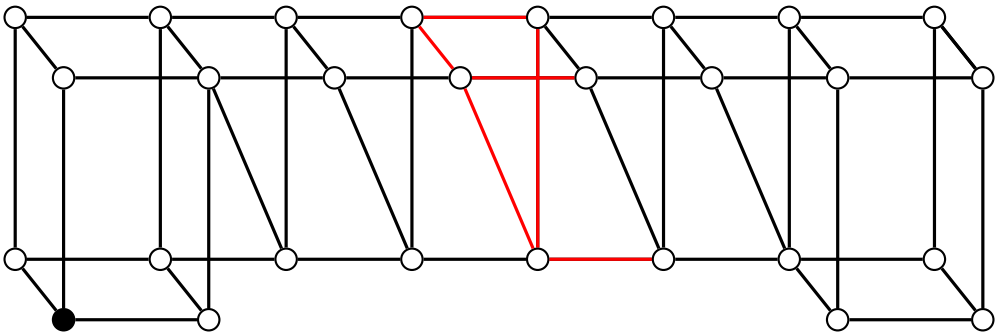

Figure 2.22: A 3-connected quadrangulation Q_n^3 on $n = 3k + 16$ vertices which maximizes the remoteness and is conjectured to maximize the Wiener index.

Table 2.5: A summary of the largest Wiener Index and remoteness among all 3-connected quadrangulations on n vertices, and a count for how many isomorphism classes attain this value.

Order	Wiener index	Count	Remoteness	Count
8	48	1	12	1
9	—	0	—	0
10	83	1	17	1
11	106	1	22	1
12	136	1	24	2
13	164	1	29	2
14	201	1	35	2
15	240	1	38	6
16	288	2	44	7
17	344	1	51	5
18	401	1	55	26
19	468	1	62	33
20	544	1	70	22
21	622	1	75	136
22	711	1	83	172
23	810	1	92	97
24	912	1	98	729
25	1026	1	107	923
26	1151	1	117	505
27	1280	1	124	3930
28	1422	1	134	4959

2.5 Remoteness Recursion Solutions

This section contains the worked out recursions for equations of the remoteness of the graphs in figures 2.5 through 2.12 and 2.14 through 2.22. Rather than $\rho(G)$, we find $\rho^*(G) = (n-1)\rho(G)$. It should be noted that the variable k will be used to denote the number of layers of each structure. Adding another copy of the red pattern in the figures from the previous section would be equivalent to increasing k by 1. Let r_k denote the remoteness on the given structure with k layers.

First, for triangulations, we have,

$$\rho^*(T_n) = \begin{cases} \frac{n^2}{6} + \frac{n}{6} & \text{if } n = 3k \\ \frac{n^2}{6} + \frac{n}{6} - \frac{1}{3} & \text{if } n = 3k+1 \\ \frac{n^2}{6} + \frac{n}{6} & \text{if } n = 3k+2 \end{cases}$$

Here, the layers represent the repeating 3-cycles. If $n = 3k$, we have,

$$r_{k+1} = r_k + 3k + 2 \text{ with } r_4 = 26$$
$$\implies r_k = \frac{1}{2}(3k^2 + k) \implies r_n = \frac{n^2}{6} + \frac{n}{6}$$

If $n = 3k+1$, we have,

$$r_{k+1} = r_k + 3k + 3 \text{ with } r_4 = 30$$
$$\implies r_k = \frac{1}{2}(3k^2 + 3k) \implies r_n = \frac{n^2}{6} + \frac{n}{6} - \frac{1}{3}$$

If $n = 3k+2$, we have,

$$r_{k+1} = r_k + 3k + 4 \text{ with } r_4 = 35$$
$$\implies r_k = \frac{1}{2}(3k^2 + 5k + 2) \implies r_n = \frac{n^2}{6} + \frac{n}{6}$$

Next, we consider 4-connected triangulations,

$$\rho^*(T_n^4) = \begin{cases} \frac{n^2}{8} + \frac{n}{4} & \text{if } n = 4k+2 \\ \frac{n^2}{8} + \frac{n}{4} + \frac{1}{8} & \text{if } n = 4k+3 \\ \frac{n^2}{8} + \frac{n}{4} & \text{if } n = 4k+4 \\ \frac{n^2}{8} + \frac{n}{4} - \frac{3}{8} & \text{if } n = 4k+5 \end{cases}$$

Here, the layers represent the repeating 4-cycles. If $n = 4k+2$, we have,

$$r_{k+1} = r_k + 4k + 5 \text{ with } r_4 = 45$$

$$\implies r_k = 2k^2 + 3k + 1 \implies r_n = \frac{n^2}{8} + \frac{n}{4}$$

If $n = 4k+3$, we have,

$$r_{k+1} = r_k + 4k + 6 \text{ with } r_4 = 50$$

$$\implies r_k = 2k^2 + 4k + 2 \implies r_n = \frac{n^2}{8} + \frac{n}{4} + \frac{1}{8}$$

If $n = 4k+4$, we have,

$$r_{k+1} = r_k + 4k + 7 \text{ with } r_4 = 55$$

$$\implies r_k = 2k^2 + 5k + 3 \implies r_n = \frac{n^2}{8} + \frac{n}{4}$$

If $n = 4k+5$, we have,

$$r_{k+1} = r_k + 4k + 8 \text{ with } r_4 = 60$$

$$\implies r_k = 2k^2 + 6k + 4 \implies r_n = \frac{n^2}{8} + \frac{n}{4} - \frac{3}{8}$$

Now, we consider 5-connected triangulations,

$$\rho^*(T_n^5) = \begin{cases} \frac{n^2}{10} + \frac{3n}{10} & \text{if } n = 5k+2 \\ \frac{n^2}{10} + \frac{3n}{10} - \frac{4}{5} & \text{if } n = 5k+3 \\ \frac{n^2}{10} + \frac{3n}{10} - \frac{4}{5} & \text{if } n = 5k+4 \\ \frac{n^2}{10} + \frac{3n}{10} - 1 & \text{if } n = 5k \\ \frac{n^2}{10} + \frac{3n}{10} - \frac{7}{5} & \text{if } n = 5k+1, \end{cases}$$

Here, the layers represent the repeating 5-cycles. If $n = 5k+2$, we have,

$$r_{k+1} = r_k + 5k + 6 \text{ with } r_4 = 55$$

$$\implies r_k = \frac{1}{2}(5k^2 + 7k + 2) \implies r_n = \frac{n^2}{10} + \frac{3n}{10}$$

If $n = 5k+3$, we have,

$$r_{k+1} = r_k + 5k + 7 \text{ with } r_4 = 59$$

$$\implies r_k = \frac{1}{2}(5k^2 + 9k + 2) \implies r_n = \frac{n^2}{10} + \frac{3n}{10} - \frac{4}{5}$$

If $n = 5k+4$, we have,

$$r_{k+1} = r_k + 5k + 8 \text{ with } r_4 = 64$$

$$\implies r_k = \frac{1}{2}(5k^2 + 11k + 4) \implies r_n = \frac{n^2}{10} + \frac{3n}{10} - \frac{4}{5}$$

If $n = 5k+5$, we have,

$$r_{k+1} = r_k + 5k + 6 \text{ with } r_4 = 69$$

$$\implies r_k = \frac{1}{2}(5k^2 + 13k + 6) \implies r_n = \frac{n^2}{10} + \frac{3n}{10} - 1$$

If $n = 5k+6$, we have,

$$r_{k+1} = r_k + 5k + 6 \text{ with } r_4 = 74$$

$$\implies r_k = \frac{1}{2}(5k^2 + 15k + 8) \implies r_n = \frac{n^2}{10} + \frac{3n}{10} - \frac{7}{5}$$

Now, we consider equation general quadrangulations,

$$\rho^*(Q_n) = \begin{cases} \frac{n^2}{4} & \text{if } n = 2k \\ \frac{n^2}{4} - \frac{1}{4} & \text{if } n = 2k+1 \end{cases}$$

Here, the layers represent the repeating 1-paths. If $n = 2k$, we have,

$$r_{k+1} = r_k + 2k + 1 \text{ with } r_5 = 25$$
$$\implies r_k = k^2 \implies r_n = \frac{n^2}{4}$$

If $n = 2k + 1$, we have,

$$r_{k+1} = r_k + 5k + 7 \text{ with } r_5 = 30$$
$$\implies r_k = k^2 + k \implies r_n = \frac{n^2}{4} - \frac{1}{4}$$

Finally, we consider equation 3-connected quadrangulations,

$$\rho^*(Q_n^3) = \begin{cases} \frac{n^2}{6} + \frac{n}{6} & \text{if } n = 3k + 14 \\ \frac{n^2}{6} + \frac{n}{6} - 2 & \text{if } n = 3k + 15 \\ \frac{n^2}{6} + \frac{n}{6} - \frac{4}{3} & \text{if } n = 3k + 16 \end{cases}$$

Here, the layers represent the repeating 2-paths. If $n = 3k + 14$, we have,

$$r_{k+1} = r_k + 3k + 16 \text{ with } r_5 = 145$$
$$\implies r_k = \frac{1}{2}(3k^2 + 29k + 70) \implies r_n = \frac{n^2}{6} + \frac{n}{6}$$

If $n = 3k + 15$, we have,

$$r_{k+1} = r_k + 3k + 17 \text{ with } r_5 = 153$$
$$\implies r_k = \frac{1}{2}(3k^2 + 31k + 76) \implies r_n = \frac{n^2}{6} + \frac{n}{6} - 2$$

If $n = 3k + 16$, we have,

$$r_{k+1} = r_k + 3k + 18 \text{ with } r_5 = 164$$

$$\implies r_k = \frac{1}{2}(3k^2 + 33k + 88) \implies r_n = \frac{n^2}{6} + \frac{n}{6} - \frac{4}{3}$$

2.6 Wiener Index Recursion Solutions

This section contains the worked out recursions for equations 2.1, 2.2, 2.3, 2.4 and 2.5. It should be noted that the variable k will be used to denote the number of layers of each structure. Adding another copy of the red pattern in the figures from the previous section would be equivalent to increasing k by 1. In each class of problems, the first case to be verified will have an explanation as to what each line is counting. The subsequent cases are similar, hence the explanation is omitted. Let w_k and r_k denote the Wiener index and remoteness on the given structure with k layers, respectively.

Let us first begin with equation (2.1),

$$W(T_n) = \frac{1}{3}\binom{n+2}{3} - \frac{1}{3}\left\lfloor \frac{n+2}{3} \right\rfloor = \begin{cases} \frac{n^3}{18} + \frac{n^2}{6} & \text{if } n = 3k \\ \frac{n^3}{18} + \frac{n^2}{6} - \frac{2}{9} & \text{if } n = 3k+1 \\ \frac{n^3}{18} + \frac{n^2}{6} - \frac{1}{9} & \text{if } n = 3k+2 \end{cases}$$

Here, the layers represent the repeating 3-cycles. Without loss of generality, suppose the C_3 is appended to the left of the existing structure. If $n = 3k$, we have,

$$\overbrace{w_{k+1} = w_k + 3}^{\text{Wiener index of added } C_3}$$

$$\overbrace{+3\sum_{i=1}^{k} i}^{\text{Distance between topmost added vertex and the previous structure}}$$

$$\overbrace{+\sum_{i=1}^{k} i + 2\sum_{i=1}^{k}(i+1)}^{\text{Distance between added middlemost vertex and the previous structure}}$$

$$\text{Distance between bottommost added vertex and the previous structure}$$

$$\overbrace{+2\sum_{i=1}^{k} i + \sum_{i=1}^{k}(i+1)}$$

$$= w_k + \frac{9k^2}{2} + \frac{15k}{2} + 3 \text{ with } w_4 = 120$$

$$\implies w_k = \frac{3}{2}(k^3 + k^2)$$

$$\implies w_n = \frac{n^3}{18} + \frac{n^2}{6}$$

If $n = 3k + 1$, we have,

$$w_{k+1} = w_k + 3$$

$$+3\sum_{i=1}^{k} i + (k+1)$$

$$+\sum_{i=1}^{k} i + 2\sum_{i=1}^{k}(i+1) + (k+1)$$

$$+2\sum_{i=1}^{k} i + \sum_{i=1}^{k}(i+1) + (k+1)$$

$$= w_k + \frac{9k^2}{2} + \frac{21k}{2} + 6 \text{ with } w_4 = 150$$

$$\implies w_k = \frac{3}{2}k(k+1)^2$$

$$\implies w_n = \frac{n^3}{18} + \frac{n^2}{6} - \frac{2}{9}$$

and if $n = 3k + 2$, we have,

$$w_{k+1} = w_k + 3$$

$$+3\sum_{i=1}^{k} i + 2(k+1)$$

$$+\sum_{i=1}^{k} i + 2\sum_{i=1}^{k}(i+1) + (k+1) + (k+2)$$

$$+2\sum_{i=1}^{k} i + \sum_{i=1}^{k}(i+1) + 2(k+1)$$

$$= w_k + \frac{9k^2}{2} + \frac{27k}{2} + 10 \text{ with } w_4 = 185$$

$$\implies w_k = \frac{1}{2}(3k^3 + 9k^2 + 8k + 2)$$

$$\implies w_n = \frac{n^3}{18} + \frac{n^2}{6} - \frac{1}{9}$$

Next, consider equation 2.2,

$$W(T_n^4) = \begin{cases} \frac{n^3}{24} + \frac{n^2}{4} + \frac{n}{3} - 2 & \text{if } n = 4k+2 \\ \frac{n^3}{24} + \frac{n^2}{4} + \frac{5n}{24} - 1 & \text{if } n = 4k+3 \\ \frac{n^3}{24} + \frac{n^2}{4} + \frac{n}{3} - 2 & \text{if } n = 4k \\ \frac{n^3}{24} + \frac{n^2}{4} + \frac{5n}{24} - \frac{3}{2} & \text{if } n = 4k+1 \end{cases}$$

Here, the layers represent the repeating 4-cycles. Without loss of generality, suppose the C_4 is added just after the leftmost cap vertex. If $n = 4k+2$, we have,

$$\overbrace{w_{k+1} = w_k - r_k + r_{k+1} + 8}^{\text{Remove cap, add new } C_4\text{, add cap back and the Wiener index of added } C_4}$$

$$\overbrace{+\sum_{i=1}^{k} i + 2\sum_{i=1}^{k}(i+1) + \sum_{i=1}^{k}(i+2) + (k+1)}^{\text{Distance between topmost added vertex and the previous structure}}$$

$$\overbrace{+2\sum_{i=1}^{k} i + 2\sum_{i=1}^{k}(i+1) + (k+1)}^{\text{Distance between second topmost added vertex and the previous structure}}$$

$$\overbrace{+2\sum_{i=1}^{k} i + 2\sum_{i=1}^{k}(i+1) + (k+1)}^{\text{Distance between second bottommost added vertex and the previous structure}}$$

$$\overbrace{+3\sum_{i=1}^{k} i + \sum_{i=1}^{k}(i-1) + 2 + (k+1)}^{\text{Distance between bottommost added vertex and the previous structure}}$$

$$= w_k + 8k^2 + 24k + 18 \text{ with } w_2 = 68$$

$$\implies w_k = \frac{2k}{3}(4k^2 + 12k + 11)$$

$$\implies w_n = \frac{n^3}{24} + \frac{n^2}{4} + \frac{n}{3} - 2$$

If $n = 4k+3$, we have,

$$w_{k+1} = w_k - r_k + r_{k+1} + 8$$

$$+\sum_{i=1}^{k} i + 2\sum_{i=1}^{k}(i+1) + \sum_{i=1}^{k}(i+2) + (k+1) + (k+2)$$

$$+2\sum_{i=1}^{k} i + 2\sum_{i=1}^{k}(i+1) + 2(k+1)$$

$$+2\sum_{i=1}^{k} i + 2\sum_{i=1}^{k}(i+1) + 2(k+1)$$

$$+4\sum_{i=1}^{k} i + 1 + 2(k+1)$$

$$= w_k + 8k^2 + 28k + 24 \text{ with } w_2 = 87$$

$$\implies w_k = \frac{1}{3}(8k^3 + 30k^2 + 34k + 9)$$

$$\implies w_n = \frac{n^3}{24} + \frac{n^2}{4} + \frac{5n}{24} - 1$$

If $n = 4k + 4$, we have,

$$w_{k+1} = w_k - r_k + r_{k+1} + 8$$

$$+\sum_{i=1}^{k} i + 2\sum_{i=1}^{k}(i+1) + \sum_{i=1}^{k}(i+2) + (k+1)$$

$$+2\sum_{i=1}^{k} i + 2\sum_{i=1}^{k}(i+1) + (k+1)$$

$$+2\sum_{i=1}^{k} i + 2\sum_{i=1}^{k}(i+1) + (k+1)$$

$$+3\sum_{i=1}^{k} i + \sum_{i=1}^{k}(i-1) + (k+1)$$

$$= w_k + 8k^2 + 32k + 32 \text{ with } w_2 = 110$$

$$\implies w_k = \frac{2}{3}(4k^3 + 18k^2 + 26k + 9)$$

$$\implies w_n = \frac{n^3}{24} + \frac{n^2}{4} + \frac{n}{3} - 2$$

If $n = 4k + 5$, we have,

$$w_{k+1} = w_k - r_k + r_{k+1} + 8$$

$$+\sum_{i=1}^{k} i + 2\sum_{i=1}^{k}(i+1) + \sum_{i=1}^{k}(i+2) + (k+1) + 3(k+2)$$

$$+2\sum_{i=1}^{k} i + 2\sum_{i=1}^{k}(i+1) + 2(k+1) + 2(k+2)$$
$$+2\sum_{i=1}^{k} i + 2\sum_{i=1}^{k}(i+1) + 2(k+1) + 2(k+2)$$
$$+3\sum_{i=1}^{k} i + \sum_{i=1}^{k}(i-1) + 4(k+1)$$
$$= w_k + 8k^2 + 36k + 40 \text{ with } w_2 = 135$$
$$\implies w_k = \frac{1}{3}(8k^3 + 42k^2 + 70k + 33)$$
$$\implies w_n = \frac{n^3}{24} + \frac{n^2}{4} + \frac{5n}{24} - \frac{3}{2}$$

Now, we consider equation 2.3,

$$W(T_n^5) = \begin{cases} \frac{n^3}{30} + \frac{3n^2}{10} - \frac{23n}{15} + \frac{168}{5} & \text{if } n = 5k+2 \\ \frac{n^3}{30} + \frac{3n^2}{10} - \frac{23n}{15} + 31 & \text{if } n = 5k+3 \\ \frac{n^3}{30} + \frac{3n^2}{10} - \frac{23n}{15} + \frac{161}{5} & \text{if } n = 5k+4 \\ \frac{n^3}{30} + \frac{3n^2}{10} - \frac{23n}{15} + 32 & \text{if } n = 5k \\ \frac{n^3}{30} + \frac{3n^2}{10} - \frac{23n}{15} + \frac{156}{5} & \text{if } n = 5k+1, \end{cases}$$

Here, the layers represent the repeating 5-cycles. Without loss of generality, suppose the C_5 is added just after the leftmost cap and existing 5-cycle. If $n = 5k + 2$, we have,

Remove cap, add new C_5, add cap back and the Wiener index of added C_5
$$\overbrace{w_{k+1} = w_k + r_{k+1} - r_k + 15}$$
Distance between added C_5 and left most C_5
$$\overbrace{+9 + 9 + 9 + 9 + 9}$$
Distance between added C_5 and next to C_5 to the right
$$\overbrace{+17 + 20 + 21 + 23 + 27}$$
Modified distance from leftmost C_5 to the original structure
$$\overbrace{+5(5(k-3)+1) - 20 - 20 - 21 - 22 - 23 + 26 + 26 + 29 + 31 + 33}$$
Distance between added C_5 and rightmost C_5 and right cap
$$\overbrace{+19 + 20 + 21 + 21 + 23 + 30(k-4)}$$
Distance between topmost added vertex to all remaining vertices
$$\overbrace{+5\sum_{i=3}^{k-2} i}$$

$$\overbrace{+3\sum_{i=3}^{k-2} i + 2\sum_{i=3}^{k-2}(i+1)}^{\text{Distance between second topmost added vertex to all remaining vertices}}$$

$$\overbrace{+2\sum_{i=3}^{k-2} i + 3\sum_{i=3}^{k-2}(i+1)}^{\text{Distance between middle added vertex to all remaining vertices}}$$

$$\overbrace{+2\sum_{i=3}^{k-2} i + 2\sum_{i=3}^{k-2}(i+1) + \sum_{i=3}^{k-2}(i+2)}^{\text{Distance between second bottommost added vertex to all remaining vertices}}$$

$$\overbrace{+\sum_{i=3}^{k-2} i + 2\sum_{i=3}^{k-2}(i+1) + 2\sum_{i=3}^{k-2}(i+2)}^{\text{Distance between bottommost added vertex to all remaining vertices}}$$

$$= w_k + \frac{25k^2}{2} + \frac{75k}{2} + 17 \text{ with } w_5 = 867$$

$$\implies w_k = \frac{1}{6}(25k^3 + 75k^2 + 2k + 192)$$

$$\implies w_n = \frac{n^3}{30} + \frac{3n^2}{10} - \frac{23n}{15} + \frac{168}{5}$$

If $n = 5k + 3$, we have,

$$w_{k+1} = w_k + r_{k+1} - r_k + 15$$

$$+9 + 9 + 9 + 9 + 9$$

$$+17 + 20 + 21 + 23 + 27$$

$$+5(5(k-3)+2) - 20 - 20 - 21 - 22 - 23 + 26 + 26 + 29 + 31 + 33$$

$$+19 + 21 + 22 + 23 + 25 + 30(k-4)$$

$$+5\sum_{i=3}^{k-2} i$$

$$+3\sum_{i=3}^{k-2} i + 2\sum_{i=3}^{k-2}(i+1)$$

$$+2\sum_{i=3}^{k-2} i + 3\sum_{i=3}^{k-2}(i+1)$$

$$+2\sum_{i=3}^{k-2} i + 2\sum_{i=3}^{k-2}(i+1) + \sum_{i=3}^{k-2}(i+2)$$

$$+\sum_{i=3}^{k-2} i + 2\sum_{i=3}^{k-2}(i+1) + 2\sum_{i=3}^{k-2}(i+2)$$

$$+(k-2)+(k-1)+(k-1)+k+k$$

$$= w_k + \frac{25k^2}{2} + \frac{85k}{2} + 25 \text{ with } w_5 = 955$$

$$\implies w_k = \frac{25k^3}{6} + 15k^2 + \frac{35k}{6} + 30$$

$$\implies w_n = \frac{n^3}{30} + \frac{3n^2}{10} - \frac{23n}{15} + 31$$

If $n = 5k + 4$, we have,

$$w_{k+1} = w_k + r_{k+1} - r_k + 15$$

$$+9+9+9+9+9$$

$$+17+20+21+23+27$$

$$+5(5(k-3)+3)-20-20-21-22-23+26+26+29+31+33$$

$$+26+28+31+30+34+40(k-4)$$

$$+5\sum_{i=3}^{k-2} i$$

$$+3\sum_{i=3}^{k-2} i + 2\sum_{i=3}^{k-2}(i+1)$$

$$+2\sum_{i=3}^{k-2} i + 3\sum_{i=3}^{k-2}(i+1)$$

$$+2\sum_{i=3}^{k-2} i + 2\sum_{i=3}^{k-2}(i+1) + \sum_{i=3}^{k-2}(i+2)$$

$$+\sum_{i=3}^{k-2} i + 2\sum_{i=3}^{k-2}(i+1) + 2\sum_{i=3}^{k-2}(i+2)$$

$$= w_k + \frac{25k^2}{2} + \frac{95k}{2} + 34 \text{ with } w_5 = 1053$$

$$\implies w_k = \frac{25k^3}{6} + \frac{35k^2}{2} + \frac{37k}{3} + 33$$

$$\implies w_n = \frac{n^3}{30} + \frac{3n^2}{10} - \frac{23n}{15} + \frac{161}{5}$$

If $n = 5k + 5$, we have,

$$w_{k+1} = w_k + r_{k+1} - r_k + 15$$

$$+9 + 9 + 9 + 9 + 9$$

$$+20 + 17 + 23 + 21 + 27$$

$$+5(5(k-3)+4) - 20 - 21 - 20 - 23 - 22 + 26 + 29 + 26 + 33 + 31$$

$$+34 + 30 + 36 + 34 + 39 + 45(k-4)$$

$$+3\sum_{i=3}^{k-2} i + 2\sum_{i=3}^{k-2}(i+1)$$

$$+5\sum_{i=3}^{k-2} i$$

$$+2\sum_{i=3}^{k-2} i + 2\sum_{i=3}^{k-2}(i+1) + \sum_{i=3}^{k-2}(i+2)$$

$$+2\sum_{i=3}^{k-2} i + 3\sum_{i=3}^{k-2}(i+1)$$

$$+\sum_{i=3}^{k-2} i + 2\sum_{i=3}^{k-2}(i+1) + 2\sum_{i=3}^{k-2}(i+2)$$

$$= w_k + \frac{25k^2}{2} + \frac{105k}{2} + 44 \text{ with } w_5 = 1156$$

$$\implies w_k = \frac{25k^3}{6} + 20k^2 + \frac{119k}{6} + 36$$

$$\implies w_n = \frac{n^3}{30} + \frac{3n^2}{10} - \frac{23n}{15} + 32$$

If $n = 5k + 6$, we have,

$$w_{k+1} = w_k + r_{k+1} - r_k + 15$$

$$+9 + 9 + 9 + 9 + 9$$

$$+17 + 20 + 21 + 23 + 27$$

$$+5(5(k-3)+5) - 20 - 20 - 21 - 22 - 23 + 26 + 26 + 29 + 31 + 33$$

$$+37+34+41+40+46+50(k-4)$$
$$+5\sum_{i=3}^{k-2} i$$
$$+3\sum_{i=3}^{k-2} i+2\sum_{i=3}^{k-2}(i+1)$$
$$+2\sum_{i=3}^{k-2} i+3\sum_{i=3}^{k-2}(i+1)$$
$$+2\sum_{i=3}^{k-2} i+2\sum_{i=3}^{k-2}(i+1)+\sum_{i=3}^{k-2}(i+2)$$
$$+\sum_{i=3}^{k-2} i+2\sum_{i=3}^{k-2}(i+1)+2\sum_{i=3}^{k-2}(i+2)$$
$$= w_k+\frac{25k^2}{2}+\frac{115k}{2}+55 \text{ with } w_5=1265$$
$$\implies w_k=\frac{25k^3}{6}+\frac{45k^2}{2}+\frac{85k}{3}+40$$
$$\implies w_n=\frac{n^3}{30}+\frac{3n^2}{10}-\frac{23n}{15}+\frac{156}{5}$$

Now, we consider equation 2.4,

$$W(Q_n)=\begin{cases}\frac{n^3}{12}+\frac{7n}{6}-2 & \text{if } n=2k\\ \frac{n^3}{12}+\frac{11n}{12}-1 & \text{if } n=2k+1\end{cases}$$

Here, the layers represent the repeating 2-paths. Without loss of generality, suppose the P_2 is added to the left of the structure. If $n=2k$, we have,

$$w_{k+1}=w_k$$
$$\overbrace{+\sum_{i=1}^{k} i+\sum_{i=1}^{k-1} i+2}^{\text{Distance between topmost added vertex to all vertices}}$$
$$\overbrace{+\sum_{i=1}^{k+1} i+\sum_{i=1}^{k} i}^{\text{Distance between bottommost added vertex to all remaining vertices}}$$
$$= w_k+2k^2+2k+3 \text{ with } w_4=93$$
$$\implies w_k=\frac{2k^3}{3}+\frac{7k}{3}-2$$

$$\implies w_n = \frac{n^3}{12} + \frac{7n}{6} - 2$$

If $n = 2k + 1$, we have,

$$\begin{aligned} w_{k+1} &= w_k \\ &+2\sum_{i=1}^{k} i + 2 \\ &+2\sum_{i=1}^{k+1} i \\ &= w_k + 2k^2 + 4k + 4 \text{ with } w_4 = 120 \\ &\implies w_k = \frac{2k^3}{3} + k^2 + \frac{7k}{3} \\ &\implies w_n = \frac{n^3}{12} + \frac{11n}{12} - 1 \end{aligned}$$

Finally, we consider equation 2.5,

$$W(Q_n^3) = \begin{cases} \frac{n^3}{18} + \frac{n^2}{3} - \frac{17n}{6} + \frac{206}{9} & \text{if } n = 3k + 14 \\ \frac{n^3}{18} + \frac{n^2}{3} - \frac{17n}{6} + 20 & \text{if } n = 3k + 15 \\ \frac{n^3}{18} + \frac{n^2}{3} - \frac{17n}{6} + \frac{184}{9} & \text{if } n = 3k + 16 \end{cases}$$

Here, the layers represent the repeating 3-paths. Without loss of generality, suppose the P_3 is added immediately before the right cube. If $n = 3k + 14$, we have,

$$w_{k+1} = w_k + \overbrace{4 + (10)(7 + 3(k-1))}^{\text{Modified distance from vertices to the left of the added } P_3 \text{ to the original structure, Wiener index of } P_3}$$

$$\overbrace{31 + 27 + 23}^{\text{Distance between added } P_3 \text{ and right cube}}$$

$$\overbrace{+21(k-3) + 26 + 29 + 32}^{\text{Distance between added } P_3 \text{ and left cube}}$$

$$\overbrace{+2\sum_{i=1}^{k-1} i + \sum_{i=1}^{k-1}(i+1)}^{\text{Distance between topmost added vertex and all remaining non-cube vertices}}$$

$$\overbrace{+\sum_{i=1}^{k-1} i + 2\sum_{i=1}^{k-1}(i+1)}^{\text{Distance between middle added vertex and all remaining non-cube vertices}}$$

$$\overbrace{+\sum_{i=1}^{k-1} i + \sum_{i=1}^{k-1}(i+1) + \sum_{i=1}^{k-1}(i+2)}^{\text{Distance between bottommost added vertex and all remaining non-cube vertices}}$$

$$= w_k + \frac{1}{2}(9k^2 + 105k + 286) \text{ with } w_1 = 344$$

$$\implies w_k = \frac{3k^3}{2} + 24k^2 + \frac{235k}{2} + 201$$

$$\implies w_n = \frac{n^3}{18} + \frac{n^2}{3} - \frac{17n}{6} + \frac{206}{9}$$

If $n = 3k + 15$, we have,

$$w_{k+1} = w_k + 4 + (10)(8 + 3(k-1))$$

$$31 + 27 + 23$$

$$+24(k-3) + 32 + 36 + 36$$

$$+2\sum_{i=1}^{k-1} i + \sum_{i=1}^{k-1}(i+1)$$

$$+\sum_{i=1}^{k-1} i + 2\sum_{i=1}^{k-1}(i+1)$$

$$+\sum_{i=1}^{k-1} i + \sum_{i=1}^{k-1}(i+1) + \sum_{i=1}^{k-1}(i+2)$$

$$= w_k + \frac{1}{2}(9k^2 + 111k + 322) \text{ with } w_1 = 401$$

$$\implies w_k = \frac{3k^3}{2} + \frac{51k^2}{2} + 134k + 240$$

$$\implies w_n = \frac{n^3}{18} + \frac{n^2}{3} - \frac{17n}{6} + 20$$

If $n = 3k + 16$, we have,

$$w_{k+1} = w_k + 4 + (10)(9 + 3(k-1))$$

$$31 + 27 + 23$$

$$+27(k-3) + 39 + 40 + 43$$

$$+2\sum_{i=1}^{k-1} i + \sum_{i=1}^{k-1}(i+1)$$

$$+\sum_{i=1}^{k-1} i + 2\sum_{i=1}^{k-1}(i+1)$$

$$+\sum_{i=1}^{k-1} i + \sum_{i=1}^{k-1}(i+1) + \sum_{i=1}^{k-1}(i+2)$$

$$= w_k + \frac{1}{2}(9k^2 + 117k + 360) \text{ with } w_1 = 468$$

$$\implies w_k = \frac{3k^3}{2} + 27k^2 + \frac{303k}{2} + 288$$

$$\implies w_n = \frac{n^3}{18} + \frac{n^2}{3} - \frac{17n}{6} + \frac{184}{9}$$

Chapter 3

Lower Bounds on the Wiener Index

3.1 Survey of Earlier Results on Triangulations and Quadrangulations

As stated in the previous chapter, the problem of minimizing the Wiener index for triangulations and quadrangulations is almost trivial. We begin by considering these cases, as well as 4-connected triangulations. Additionally, we will show that the minimizer for the Wiener index over all quadrangulations is unique.

Theorem 7 ([11])**.** *Assume $n \geq 6$. The graph T_n^4 in Figure 3.1 minimizes the Wiener index among all triangulations of order n and is 4-connected. Consequently, it minimizes the Wiener index among all 4-connected n-vertex triangulations.*

(**Remark:** T_5^4 is the only triangulation on 5 vertices, but it is not 4-connected. Red color in the figures indicate patterns to be repeated as n increases.)

Proof. A triangulation contains $3n-6$ edges, thus there are exactly $3n-6$ pairs of vertices at distance 1 apart. If we can make sure that every remaining pair of vertices are at distance 2 apart, then we have a triangulation of Wiener index $2(\binom{n}{2} - (3n-6)) + (3n-6) = n^2 - 4n + 6$, and this is clearly the minimum possible Wiener index. This is the case with T_n^4. Furthermore, it is easy to see that T_n^4 is 4-connected for all $n \geq 6$. □

It should be noted that these two classes, general triangulations and 4-connected triangulations, are the only classes studied in this paper, which fail to produce unique

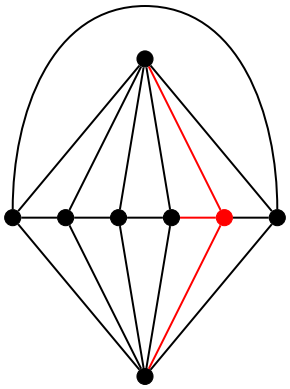

Figure 3.1: The graph T_n^4 that minimizes the Wiener index among all general and among all 4-connected triangulations of order n.

structures to minimize the Wiener index. See Table 3.1 for the number of isomorphism classes which minimize the Wiener index among all triangulations up to 18 vertices, and all 4-connected triangulations up to 22 vertices.

Theorem 8 ([9],[11])**.** *Assume $n \geq 4$. The complete bipartite graph $K_{2,n-2}$ minimizes the Wiener index among all quadrangulations.*

Proof. A quadrangulation contains $2n-4$ edges, thus exactly $2n-4$ pairs of vertices are at distance 1 apart. If we can make sure that every remaining pair of vertices are at distance 2 apart, we have a quadrangulation of Wiener index $2(\binom{n}{2} - (2n-4)) + (2n-4) = n^2 - 3n + 4$. This is the case with the quadrangulation $K_{2,n-2}$. Clearly this is the least possible Wiener index of a quadrangulation. □

Theorem 9. *Assume $n \geq 4$. Up to isomorphism, the graph $K_{2,n-2}$ is the unique minimizer of the Wiener index among all quadrangulations of order n.*

Proof. Let Q be a quadrangulation of order n that has the same Wiener index as $K_{2,n-2}$, i.e. every non-adjacent pair of vertices are at distance 2. As quadrangulations

Table 3.1: A summary of how many isomorphism classes, on n vertices, attain the minimum Wiener index for general and 4-connected triangulations.

Order	Gen Tri Count	4-Conn Tri Count
4	1	0
5	1	0
6	2	1
7	5	1
8	12	2
9	36	4
10	99	6
11	255	10
12	614	10
13	1532	14
14	3908	15
15	10727	19
16	31242	21
17	96725	25
18	311735	27
19	-	32
20	-	34
21	-	39
22	-	42

are 2-connected, the minimum degree $\delta := \delta(Q) \geq 2$. Let v be a vertex of Q with $d(v) = \delta$, and let $u_1, \ldots, u_\delta$ be the neighbors of v. The remaining $n - \delta - 1$ vertices are at distance 2 from v. As quadrangulations are bipartite, these $n - \delta - 1$ vertices can only be adjacent to $u_1, \ldots, u_\delta$, and have degree at least δ. Thus we get that $Q \cong K_{\delta,n-\delta}$, and so Q contains $K_{\delta,\delta}$ as a subgraph. Since Q is planar, we get $\delta = 2$ and $Q \cong K_{2,n-2}$. □

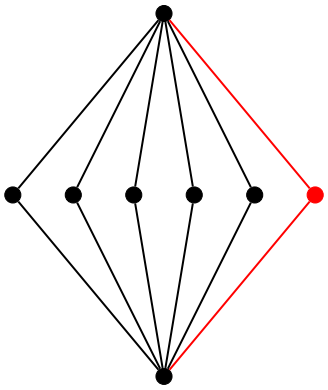

Figure 3.2: The graph $K_{2,n-2}$ which minimizes the Wiener index among all quadrangulations of order n.

3.2 Minimum Wiener Index of 3-Connected Quadrangulations

First, [39] states that every 5-connected triangulation contains a spanning 3-connected quadrangulation. While this result wasn't used directly, it provided motivation to start with 3-connected quadrangulations before 5-connected triangulations. Comparison of Figures 3.5 and 3.8 gives an illustration for even order.

To start, I define a graph structure which we will use extensively in the remainder of this section. Let v be a vertex of a 3-connected quadrangulation G. We define the *sunflower graph* S_v around v (in the planar drawing of G), as v connected to its neighbors $u_1, \ldots, u_d$ (listed in the cyclic order of the drawing), and different vertices $w_1, \ldots, w_d$ where w_i is connected to u_i and u_{i+1} (see Figure 3.3).

First we need to show that such a graph, with distinct vertices, exists in the drawing. We will also need a special property of the sunflower graph.

Lemma 4. *Assume that Q is a drawing of a 3-connected quadrangulation. Then, for any vertex v, Q contains a sunflower graph S_v with $2d(v) + 1$ distinct vertices.*

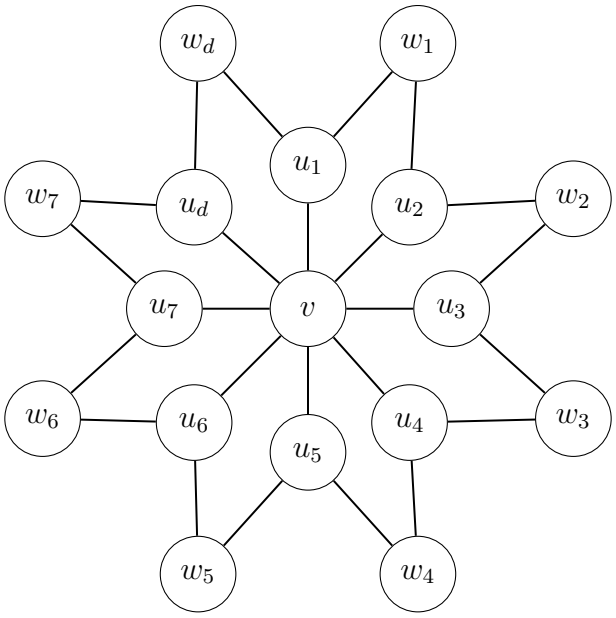

Figure 3.3: The sunflower graph S_v around v.

Furthermore, the region $\mathcal{R}_v$ bounded by the cycle $u_1w_1 \dots u_{k-1}w_{k-1}$ that contains v contains no vertices or edges that are not in S_v.

Proof. We know $\delta(Q) \geq 3$ by the 3-connectedness. Label the neighbors of v by $u_1, \dots, u_d$, in their planar cyclic order around v. For each pair of successive neighbors u_i and u_{i+1} (indices taken modulo d) let $w_i \neq v$ be their common neighbor that completes the face f_i that has u_i, v, u_{i+1} on its boundary. This means in particular, that the interior of f_i has no vertices or edges. If y is a neighbor of u_i and $y \notin \{v, w_{i-1}, w_i\}$ then y must lie between w_{i-1} and w_i in the planar cyclic order around w_i, in particular, as $d(w_i) \geq 3$, $w_{i-1} \neq w_i$. As Q is bipartite, $u_i \neq w_j$. We will show that each of the w_i's must be distinct. As $\mathcal{R}_v$ is the union of the faces f_i, this finishes the proof. Assume that $w_i = w_j$ for some $j \neq i$, then we already have that $j \notin \{i-1, i+1\}$ and the vertices $u_i, u_{i+1}, u_j, u_{j+1}$ are all different. We consider two regions of the planar drawing of Q: $\mathcal{R}_1$ is bounded by the 4-cycle $u_{i+1}vu_jw_i$ and does not contain the vertex u_i, and $\mathcal{R}_2$ is bounded by the 4-cycle $u_ivu_{j+1}w_i$ and does not contain the vertex u_{i+1}. Thus the faces bounded by $u_ivu_{i+1}w_i$ and $u_jvu_{j+1}w_j$ are

disjoint from $\mathcal{R}_1$ and $\mathcal{R}_2$. The neighbors of u_i that differ from v and w_i must lie in $\mathcal{R}_2$ and the neighbors of u_j that differ from v and w_i must lie in $\mathcal{R}_1$. Hence $\{v, w_i\}$ separates u_i from u_j (See Figure 3.4), contradicting the fact that Q is 3-connected. □

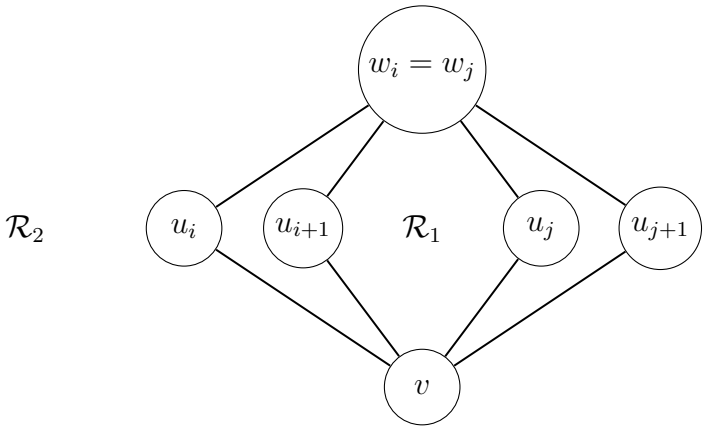

Figure 3.4: 2-element cutset in S_v when $w_i = w_j$.

Lemma 5. *If G is a 3-connected quadrangulation with bipartite vertex classes A, B, then*

$$\Delta(G) \leq \min\{|A| - 1, |B| - 1\}.$$

Moreover, if $|B| < |A|$, then for all $v \in A$ we have $d(v) \leq |B| - 2$.

Proof. Let v be a vertex with degree $\Delta = \Delta(G)$. Assume $v \in B$. As the sunflower S_v is a subgraph of the planar drawing of G, we have $1 + \Delta \leq |B|$ and $\Delta \leq |A|$. We are done unless $|B| > |A| = \Delta$, which we assume from now on. As $A = N(v)$, all neighbors of the vertices of B lie in $N(v)$, in particular, every w_i has at least 3 neighbors in A. For each i let k_i be the largest positive integer such that w_i has no neighbors in the set $\{u_{i-t} : 1 \leq t \leq k_i - 1\}$ and $\{u_{i+1+t} : 1 \leq t \leq k_i - 1\}$. Since for $k = 1$ the sets $\{u_{i-t} : 1 \leq t \leq k - 1\}$ and $\{u_{i+1+t} : 1 \leq t \leq k - 1\}$ are empty, such a positive integers exist, they have an upper bound from the fact that w_i has at least 3 neighbors in A, and for the largest such integer k_i we have that at least one

of u_{i-k_i}, u_{i+1+k_i} is a neighbor of w_i that is different from u_i, u_{i+1}. Chose i_0 such that $k = k_{i_0}$ is minimal amongst the k_i. By renumbering the u_i if necessary and changing the direction of the cyclic order we can assume that $i_0 = 1$ and w_1 is connected to u_1, u_2, u_{2+k} but none of u_{1-t}, u_{2+t} for all $1 \leq t \leq k-1$. Let $\mathcal{R}$ be the region of the sphere bounded by the 4-cycle $u_2 v u_{2+k} w_1$ that does not contain u_1. Consider w_2. By the definition of w_2 and k, w_2 lies in $\mathcal{R}$ and it has at least one neighbor u_j that does not lie in $\mathcal{R}$. The edge $w_2 u_j$ must cross the boundary of $\mathcal{R}$, which contradicts the planarity of G. Thus, we have $\Delta(G) \leq \min\{|A|-1, |B|-1\}$, as claimed.

For the second part, assume $|B| < |A|$, that is, $n > 2|B|$. We already know that $\Delta(G) \leq |B| - 1$. Assume that A contains a vertex v of degree $|B| - 1$. Since $|A| = n - |B|$ and all other vertices of A have degree at least 3, we have that $2n - 4 \geq (|B| - 1) + 3(n - |B| - 1) = 3n - 2|B| - 4$, so $n \leq 2|B|$, a contradiction. □

Lemma 6. *The number of pairs of vertices at distance exactly 2 apart in a 3-connected quadrangulation G of order n is at most*

$$\sum_v \binom{d(v)}{2} - 2(n-2).$$

This count is exact precisely when G has no non-facial 4-cycles.

Proof. Euler's Formula gives us that any quadrangulation on n vertices has $n-2$ faces. The number of 2-paths in G is equal to $\sum_v \binom{d(v)}{2}$. This sum, however, overcounts the number of pairs of vertices distance 2 apart. In a 3-connected quadrangulation, two faces cannot share two consecutive edges from their boundaries. Thus, for each face, we are double counting the two pairs of vertices distance 2 apart, and so we may safely subtract $2(n-2)$. There are pairs of vertices which we have double counted even after the substraction precisely when there are non-facial 4-cycles. □

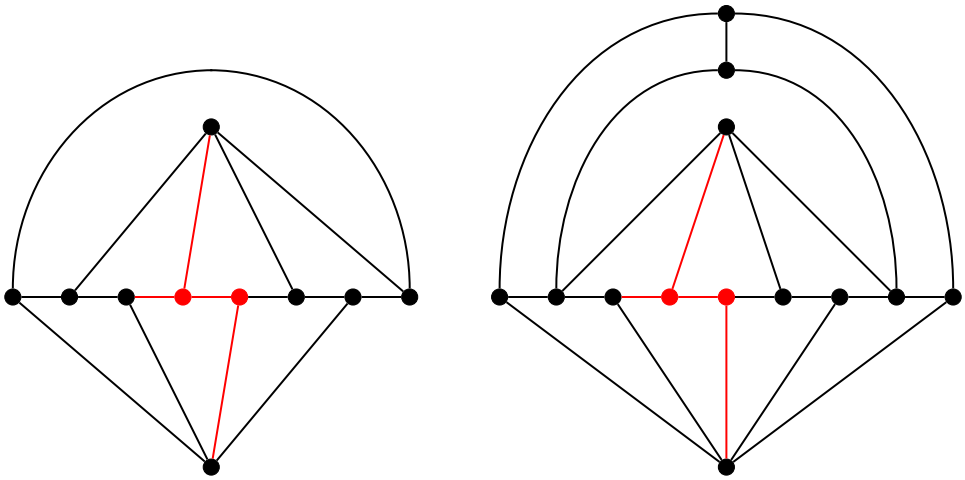

Figure 3.5: The graph Q_n^3 which minimizes the Wiener index among all 3-connected quadrangulations on $n = 2k$ (left) and $n = 2k + 1$ vertices (right).

Easy calculations show (see section 3.4) that for the 3-connected quadrangulation Q_n^3 on n vertices, which is shown in Figure 3.5,

$$W(Q_n^3) = \begin{cases} \frac{5n^2}{4} - 5n + 8 & \text{if } n = 2k \\ \frac{5n^2}{4} - 3n - \frac{49}{4} & \text{if } n = 2k + 1. \end{cases} \tag{3.1}$$

Lemma 7. *Let Q a 3-connected quadrangulation of order n, with partite classes A, B. Then*

$$W(Q) \geq 2n^2 - 2n - |A||B| - 2\sum_v \binom{d(v)}{2}. \tag{3.2}$$

Equality holds precisely when the diameter of Q is at most 4 and Q has no non-facial 4-cycles. In particular, equality holds for $Q = Q_n^3$.

Proof. Let Q be an arbitrary 3-connected quadrangulation on n vertices, with partite classes A, B. Let D_i denote the number of unordered pairs of vertices at distance i in Q. Clearly $W(Q) = \sum_i i \cdot D_i$. Observe that $D_1 = 2n - 4$, the number of edges; $D_2 \leq \sum_v \binom{d(v)}{2} - 2(n-2)$ by Lemma 6; $D_4 + D_6 + D_8 \cdots \geq \binom{|A|}{2} + \binom{|B|}{2} - D_2$, as pairs of vertices from the same colorclass must be at even distance; and finally, $D_3 + D_5 + D_7 + \cdots \geq |A| \cdot |B| - (2n-4)$, as pairs of vertices from different colorclasses must be at odd distance.

Combining all of this information with the identity $|A| + |B| = n$, we obtain that

$$\begin{aligned} W(Q) &\geq (2n-4) + 2D_2 + 3\Big[|A| \cdot |B| - (2n-4)\Big] + 4\left[\binom{|A|}{2} + \binom{|B|}{2} - D_2\right] \\ &= 2n^2 - 2n - |A||B| - 2(D_2 + 2n - 4) \\ &\geq 2n^2 - 2n - |A||B| - 2\sum_v \binom{d(v)}{2}. \end{aligned}$$

The first inequality in the displayed formula is an equality precisely when the diameter of Q is at most 4, and the second inequality is an equality when Q has no non-facial 4-cycles. □

Theorem 10. *Assume $n \geq 8$, even. The graph Q_n^3 in Figure 3.5 minimizes the Wiener index among all 3-connected quadrangulations on n vertices. Moreover, up to isomorphism, this minimizer is unique.*

(**Remark:** There are no 3-connected quadrangulations on 9 vertices, or on fewer than 8 vertices, and there is a unique one on 8 vertices.)

Proof. As Q_8^3 is is the unique 3-connected quadrangulation on 8 vertices, we can assume $n \geq 10$. Let Q be an arbitrary 3-connected quadrangulation on $n = 2k$ vertices, with partite classes A, B; $k \geq 5$. Our lower bound for $W(Q)$ in (3.2) is made smaller if we change $|A||B|$ to $n/2, n/2$, and (independently from $|A|, |B|$) maximize the sum of binomial coefficients. First note that $\sum_v \binom{d(v)}{2} = \frac{1}{2}\sum_v d(v)^2 - (2n-4)$, and also that $\sum_{v\in A} d(v) = 2n - 4 = \sum_{v\in B} d(v)$. Since Q is 3-connected, for all v, we have $d(v) \geq 3$, and by Lemma 5, $d(v) \leq \Delta(Q) \leq \min(|A| - 1, |B| - 1) \leq \frac{n}{2} - 1$.

Assume that $\sum_i x_i = a > 0$ is given, the x_i's are required to be integers from the interval $[b, c]$ with $0 < b$, and we have to maximize $\sum_i x_i^2$. As long as for some $x_i \geq x_j$ $(i \neq j)$ we can change x_i to $x_i + 1$ and x_j to $x_j - 1$ within the given interval, we increase the sum of squares. In our case $a = 4n - 8$, the x_i's are the degrees, and $3 \leq x_i \leq \frac{n}{2} - 1$.

The optimum is when two of the x_i are equal to $\frac{n}{2} - 1$, and the remaining entries are all 3's. Given the condition on the graph that the degree sums in A and in B,

the fact that $|A|, |B| \geq (k-1)+1 = k$ the corresponding degree sequences in both A and B are given by the sequence $\frac{n}{2} - 1, 3, 3, ..., 3$, if it is a realizable sequence. This coincides with the other requirement that $|A| = |B| = \frac{n}{2}$, so this indeed gives the optimum, if realizable by a graph. The degree sequence and the minimized lower bound for $W(Q)$ in (3.2) is fortunately realized by Q_n^3 on the left side of Figure 3.5.

This extremal quadrangulation is in fact unique. Let T be a 3-connected quadrangulation on $n = 2k$ vertices, with the same Wiener index (and consequently the same degree sequence) as Q_n^3. In particular, by our earlier work, each partition classes must have k vertices, and there is precisely one vertex of degree $k-1 \geq 4$ in both partition classes, and the remaining vertices have degree 3.

Let v be a vertex of maximum degree $k-1$, and construct the sunflower graph S_v around v. S_v has exactly $n-1$ vertices, T has one additional vertex v'. This vertex v' is in the same partite class as the u_i vertices, and differs from v. So either v' has degree $k-1$ and hence is adjacent to all w_i (in which case we have Q_n^3) or the degree of v' is 3 and one of the u_i (say u_1) has degree $k-1$ in T. Assume that the latter is the case. Then there is a unique $j \in \{3, \ldots, k-2\}$ such that u_1 is not adjacent to w_j. Let $\mathcal{R}$ be the region bounded by the cycle $u_1 w_{j-1} u_j w_j u_{j+1} w_{j+1} u_1$ that does not contain v. As the interior of $\mathcal{R}$ contains no vertices from $S_v = T - v'$, and all neighbors of w_j other than u_i and u_{i-1} must be on the interior of $\mathcal{R}$, v' is in the interior of $\mathcal{R}$ and is adjacent to w_j. Since v' must be adjacent to 3 vertices in $w_1, \ldots, w_{k-1}$, and those can only be on the boundary of $\mathcal{R}$, v' is also adjacent to w_{j-1} and w_{j+1}. But this means that both w_{j-1} and w_{j+1} have degree at least 4, a contradiction. □

Lemma 8. *Assume $n = 2k+1$, where $k \geq 9$, and let Q be a 3-connected quadrangulation on n vertices with partite sets A, B. If*

$$\sum_{v \in V(G)} \binom{d(v)}{2} < k^2 + 2k + 7, \tag{3.3}$$

then $W(Q) > W(Q_n^3)$. In particular, if $\Delta(Q) \leq k-2$, then $W(Q) > W(Q_n^3)$.

Proof. Assume $n = 2k + 1$ and $k \geq 9$.

First note that for Q_n^3 we have that

$$\sum_{x \in V(Q_n^3)} \binom{d(x)}{2} = \frac{(k-1)(k-2)}{2} + \frac{(k-2)(k-3)}{2} + 12 + 3(2k-3) = k^2 + 2k + 7,$$

and $|A||B| \leq k(k+1)$, so if (3.3) holds then $W(Q) < W(Q_n)$ by (3.2).

Assume now that $\Delta(Q) \leq k-2$. Let $x_1, \ldots, x_{2k+1}$ be a sequence of integers with $3 \leq x_i \leq k-2$ that maximizes $\sum \binom{x_i}{2}$. As $\sum x_i$ is constant, we maximize $\sum \binom{x_i}{2}$ as we maximized $\sum x_i^2$ in the proof of Theorem 10. The maximizing sequence is $k-2, k-2, 6, 3, 3, \ldots, 3$, and

$$\sum_{x \in V(G)} \binom{d(x)}{2} \leq \sum_{i=1}^{2k+1} \binom{x_i}{2} = (k-2)(k-3)+15+3(2k-2) = k^2+k+15 < k^2+2k+7.$$

□

Theorem 11. *Assume $n \geq 11$, odd, and let Q be a 3-connected quadrangulation of order n. If $W(Q)$ is minimum among the n vertex 3-connected quadrangulations, then $Q \cong Q_n^3$.*

Proof. As Q_n^3 is indeed the unique 3-connected quadrangulation on n vertices with minimum Wiener index for $11 \leq n \leq 28$ as our computer search found, we assume $n \geq 29$. Let $n = 2k + 1$ and $k \geq 14$. Let Q be a 3 connected quadrangulation on n vertices with minimum Wiener index, and A, B be the partite classes of Q with $|A| > |B|$. We have $|A| \geq k+1$ and $|B| \leq k$. Lemma 8 gives $\Delta(Q) \leq |B| - 1 \leq k-1$. Lemma 5 gives $\Delta(Q) = k-1$, which in turn shows $|B| = k$, $|A| = k+1$. In addition, if $d(v) = k-1$, Lemma 8 gives $v \in B$. As the sum of the k degrees in B is the number of edges $2n - 4 = 4k - 2$, and every degree is at least 3, only two degree sequences are possible for B: $(k-1, 5, 3, 3, \ldots, 3)$ or $(k-1, 4, 4, 3, \ldots, 3)$. We claim that $\Delta(A)$, the maximum degree of a vertex in A is $k-2$. Indeed, Lemma 8 showed $\Delta(A) \leq |B| - 2 = k-2$. On the other hand, if $\Delta(A) \leq k-3$, then, as

$\sum_{x\in A} d(x) = 4k-2$, we have

$$\sum_{x\in A}\binom{d(x)}{2} \leq \binom{k-3}{2} + \binom{4}{3} + (k-1)\binom{3}{2}$$

and

$$\begin{aligned}\sum_{x\in V(Q)}\binom{d(x)}{2} &\leq \binom{k-3}{2} + \binom{4}{3} + (k-1)\binom{3}{2} + \binom{k-1}{2} + \binom{5}{3} + (k-1)\binom{3}{2} \\ &= k^2+k+15 < k^2+2k+7,\end{aligned}$$

and Lemma 5 contradicts the minimality of the Wiener index. Hence the degree sequence of A is $(k-2,3,\ldots,3)$. Now we figure what is the degree sequence of B. Assume that B has degree sequence $(k-1,5,3,3,\ldots,3)$. Refering the the sunflower graph S_v at vertex v, where $d(v) = k-1$, we have $B = \{v, w_1, \ldots, w_{k-1}\}$ and $A = \{w_1, \ldots, w_{k-1}, x, y\}$. We can assume without loss of generality that $w_1 \in B$ has degree 5. Since w_1 is adjacent to 3 other vertices of A besides u_1 and u_2, it is adjacent to at least one (and at most three) vertices u_j, where $3 \leq j \leq k-1$. But then these vertices u_j have degree at least 4. As A has a single vertex with degree more than 3, we conclude that there is a unique $j : 3 \leq j \leq k-1$ that w_1 is adjacent to, $d(u_j) = k-1$ and $d(x) = d(y) = 3$, and w_1 is adjacent to x and y.

Let $\mathcal{R}_1$ and $\mathcal{R}_2$ be the regions that do not contain v and are bounded by the cycles $w_1u_2w_2u_3\ldots w_{j-1}u_jw_1$ and $w_1u_jw_ju_{j+1}\ldots w_{k-1}u_1w_1$, respectively. Recall from the sunflower construction that $\mathcal{R}_v$ is bounded by the cycle $u_1w_1u_2w_2\ldots\ u_dw_du_1$ that contains the vertex v does not contain any other vertices of Q. Now $\mathcal{R}_1, \mathcal{R}_2$ and $\mathcal{R}_v$ share only vertices on the boundary, and the common boundary of $\mathcal{R}_1$ and $\mathcal{R}_2$ is the path u_jw_1. $\mathcal{R}_1$ has $j-2 \geq 1$ vertices $w_2, \ldots w_{j-1}$ from B on its common boundary with $\mathcal{R}_v$ that can only have additional neighbors in the interior of $\mathcal{R}_1$, and $\mathcal{R}_2$ has $k-j \geq 1$ vertices $w_jw_{j+1}, \ldots, w_{k-1}$ from B on its common boundary with $\mathcal{R}_v$ that can only have additional neighbors on the interior of $\mathcal{R}_2$. This means that x is in the interior of $\mathcal{R}_1$ and x is adjacent to $w_2, \ldots w_{j-1}$ and w_1, y is in the interior of $\mathcal{R}_2$ and is adjacent to $w_jw_{j+1}, \ldots, w_{k-1}$ and w_1. (Exchange the names of x and y

if needed.) Since $d(x) = d(y) = 3$, this means $3 = j - 1 = k - j + 1$, so $j = 4$ and $k = 6$, which contradicts $k \geq 14$. Thus the integer sequence that maximizes the the sum of squares, and satisfies the conditions we have for the degree sequence of G in B is $k-1, 4, 4, 3, \ldots, 3$, the degree sequence of Q_n^3. Since Q_n^3 is a 3-connected quadrangulation with diameter at most 3, this shows that $W(Q_n^3)$ is minimal, and the degree sequence of Q is the same as the degree sequence of Q_n^3, and furthermore, the degree sequences of their respective partite classes are the same. Last, we need to show that $Q \cong Q_n^3$.

There are two vertices in B with degree 4, we assume without loss of generality that those are w_1 and w_j. As all but one vertices of $A = \{u_1, \ldots, u_{k-1}, x, y\}$ have degree 3, at most one of the vertices $u_1, \ldots, u_{k-1}$ can have edges incident upon it that are not edges of S_v.

Assume that u_i has an edge incident upon it that is not an edge of S_v, then $d(u_i) = k - 2$ and all other vertices in A have degree 3. This means that u_i is adjacent to precisely $k-5$ of the vertices in $\{w_s : s \notin \{i-1, i\}, 1 \leq s \leq k-1\}$, so it is adjacent to at least one w_ℓ such that $\ell \notin \{i-1, i\}$. If $i < \ell \leq k-1$ then $u_i v u_\ell w_\ell u_i$ is a non-facial 4-cycle (as $u_i w_i u_{i+1} \ldots w_{\ell-1} u_\ell$ lies in one of the regions bounded by this cycle while $w_\ell u_{\ell+1} w_{\ell+1} \ldots w_{i-1} u_i$ lies in the other region). If $1 \leq \ell < i - 1$ then $u_i v u_{\ell+1} w_\ell u_i$ is a non-facial 4-cycle. Since Q can not have non-facial 4 cycles by Lemma 7, this is a contradiction

Therefore none of the u_i has a neighbor outside of S_v. In this case w_1 and w_j must both be adjacent to x and y. Without loss of generality the region $\mathcal{R}$ bounded by the cycle $x w_1 u_2 w_2 \ldots u_j w_j x$ that does not contain v contains y. (Otherwise we exchange the name of x and y). If $j = k - 1$, then x is on the interior of the 4-cycle $y w_{k-1} u_1 w_1 y$ that does not contain v, and the degree of x can only be 2, which is a contradiction. If $j = 2$, then $\mathcal{R}$ is bounded by a 4-cycle and y can have only degree 2, a contradiction. So $3 \leq j \leq k-2$, the $k - 1 - j \geq 1$ vertices $w_{j+1}, w_{j+2}, \ldots, w_{k-1}$

must have x as their third neighbor, and the the $j-2 \geq 1$ vertices $w_2, w_3, \ldots, w_{j-1}$ must have y as their third neighbor. So $\{d(x), d(y)\} = \{k+1-j, j\} = \{3, k-2\}$, which gives $j \in \{3, k-2\}$. This is precisely the graph Q_n^3.

□

3.3 Minimum Wiener Index of 5-Connected Triangulations

Euler's formula shows that there are no 5-connected triangulations on fewer than 12 vertices, and computer search shows that there is only one on 12 vertices and there are none on 13 vertices.

We state some facts about triangulations of a simple n-gon not using additional vertices. Such triangulations can be viewed as planar graphs, where all but one of the faces are bounded by a 3-cycle (we will refer to such a faces as *triangles*).

Lemma 9. *Let $n \geq 4$. Any triangulation of a simple n-gon uses $n-3$ additional edges (i.e. edges which are not edges of the n-gon), and has at least 2 triangles with exactly two of their boundary edges on the n-gon.*

Proof. The fact that the triangulation has $n-3$ edges (and consequently $n-2$ triangles) is an easy induction on n. When $n \geq 4$, all these triangles have at most 2 boundary edges on the n-gon. As there are $n-2$ triangles inside and n edges on the n-gon itself, by the pigeonhole principle some two triangles must have two edges from edges of the n-gon. □

We need the following basic facts about 5-connected triangulations:

Lemma 10. *Let T be a 5-connected triangulation on n vertices. Then the following are true:*

(a) *Every 3-cycle is a boundary of a face and every 4-cycle is a boundary of a region whose interior does not contain vertices of the graph, and contains exactly one edge.*

(b) *Every edge lies on exactly on two triangles. If abc and bcd are triangles, then ad is not an edge.*

(c) *For every edge xy there is precisely one* 4*-cycle that goes through its vertices but does not use the edge; hence the number of* 4*-cycles in* T *is* $3(n-2)$*.*

(d) *If* x, y *are non-adjacent vertices in* T*, then there is at most one* 4*-cycle that contains them.*

(e) *Let* D_i *denote the number of unordered pairs of vertices at distance* i *in* T*. We have* $D_1 = 3(n-2)$ *and*

$$D_2 = \sum_{x \in V(T)} \binom{d(x)}{2} - 9(n-2).$$

(f)

$$W(T) \geq 3\binom{n}{2} - \sum_{x \in V(T)} \binom{d(x)}{2} + 3(n-2),$$

with equality if and only if T *has diameter at most* 3*.*

Proof. (a) follows from the fact that if C is a cycle that bounds two regions that both contain vertices in their interior, then the vertices of C form a cutset, therefore C has at least 5 vertices.

(b): An edge bounds two faces that are triangles, and if there is a third 3-cycle using the edge, the other two edges of two of these 3-cycles give a 4-cycle that has vertices in both of its regions. If *abc* and *bcd* are triangles such that *ad* is an edge, then one of *abd*, *acd* would be a non-facial triangle unless $n = 4$. Both of these contradict (a), and (b) follows.

(c): Since every 4-cycle $abcd$ bounds a region that has no vertices but has an edge (say bc), and if bc is an edge then ad can not be an edge by (b), for every 4-cycle there is a unique edge that is not part of the cycle and connects two of its vertices. So we can map 4-cycles to edges by assigning this edge to each cycle. If two different 4-cycles map to the same edge, one of the 4-cycles must be a separating 4-cycle, so this map is injective. As the number of triangles an edge is on is 2, every edge is assigned to precisely one of these 4-cycles, so the map is surjective as well. Thus, the number of 4-cycles is the same as the number of edges, which is $3(n-2)$ in any planar triangulation. (c) follows.

(d): If x, y are non-adjacent vertices that appear on two 4-cycles, then there are at least three paths of length 2 that connects x with y. Some two of these form a 4-cycle that bounds two regions that have vertices on their interiors, contradicting (a).

(e): Observe that D_1 is exactly the number of edges of T, $3(n-2)$. The formula $\sum \binom{d(x)}{2}$ counts the number of paths of length 2 between unordered pairs of vertices. If an unordered pair of vertices have more than 1 such path, it appears on a 4-cycle, and by (c) and (d) this 4-cycle is unique. As each 4-cycle contains exactly two such unordered pairs of vertices, the number of unordered pairs of vertices that have a path of length 2 between them is $\sum \binom{d(x)}{2} - 6(n-2)$ by (c). As every edge is contained in exactly one 4-cycle, this equals $D_1 + D_2$, proving (e).

(f): As $\sum_i D_i = \binom{n}{2}$ and we get

$$\begin{aligned} W(T) &= \sum_i iD_i \geq D_1 + 2D_2 + 3\left(\binom{n}{2} - D_1 - D_2\right) = 3\binom{n}{2} - 2D_1 - D_2 \\ &= 3\binom{n}{2} - \sum_{x \in V(T)} \binom{d(x)}{2} + 3(n-2), \end{aligned}$$

and equality holds if the diameter is at most 3. □

As in the previous section, we define a graph construction, which we will use extensively. Let T be a 5-connected triangulation, and let $v \in V(T)$ have degree d.

We define the *mosaic graph* M_v at vertex v, together with its planar drawing, in the following way. M_v contains the neighbors of v in G, $u_1, u_2, \ldots, u_d$, with the edges vu_i, such that vertices u_i are labeled according the clockwise cyclic order of the edges. We add the $u_iu_{i+1} \in E(T)$ for every $1 \leq i \leq d$ (indices are taken modulo d) according the drawing of T. We also add a vertex $w_i \neq v$, which is a common neighbor of u_i and u_{i+1}, together with edges joining them to u_i and u_{i+1} in T, following the drawing of T, for every i. We will show that M_v has $2d+1$ distinct vertices.

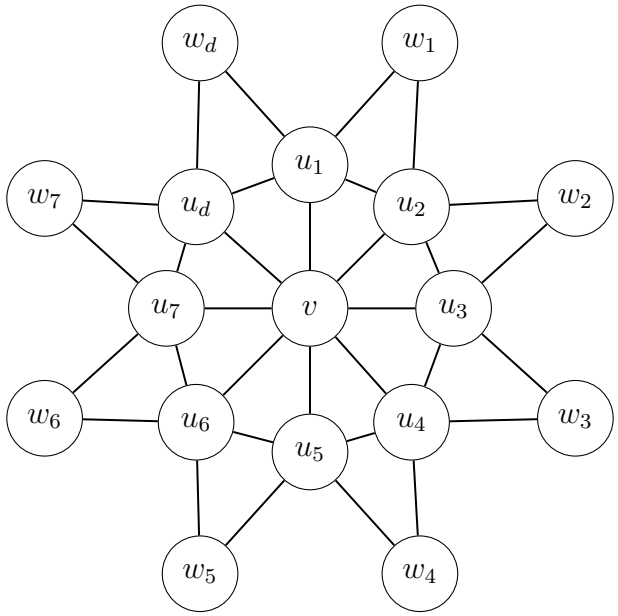

Figure 3.6: Mosaic graph around v, M_v.

Lemma 11. *If T is a drawing of a 5-connected triangulation and v is any vertex of it with degree $d(v) = d$, the mosaic graph M_v in T has $2d+1$ distinct vertices. Furthermore, the region $\mathcal{R}_v$ that is bounded by the cycle $u_1w_1u_2w_2 \ldots u_dw_d$ and contains the vertex v, contains edges and vertices from T only if they are in the mosaic graph M_v. In addition, T contains at least one vertex not in M_v, consequently $\Delta(T) \leq \lfloor \frac{n}{2} \rfloor - 1$.*

Proof. Since T is 5-connected, $\delta(T) \geq 5$. As before, label the neighbors of v by $u_1, \ldots, u_d$, in their planar clockwise cyclic order around v. We automatically get that u_iu_{i+1} is an edge, since we have a triangulation. For each pair of successive

neighbors u_i and u_{i+1} (indices taken modulo d) let $w_i \neq v$ be their common neighbor that completes the face that has u_iu_{i+1} on its boundary but not v. This means, in particular, that $\mathcal{R}_v$ will satisfy the required property, we just need to show that the vertices listed in M_v are all distinct.

If y is a neighbor of u_i and $y \notin \{v, w_{i-1}, w_i\}$ then y must lie between w_{i-1} and w_i in the planar cyclic order around w_i, In particular, as $d(u_i) \geq 5$, $w_{i-1} \neq w_i$. Also, $u_i \neq w_j$, as otherwise v, u_i, u_j would be a 3-element cutset.

Assume now that $w_i = w_j$ for some $j \neq i$. We already have that $j \notin \{i-1, i+1\}$ and the vertices $u_i, u_{i+1}, u_j, u_{j+1}$ are all distinct. We consider two regions of the planar drawing of T: $\mathcal{R}_1$ is bounded by the 4-cycle $u_{i+1}vu_jw_i$ and does not contain the vertex u_i, and $\mathcal{R}_2$ is bounded by the 4-cycle $u_ivu_{j+1}w_i$ and does not contain the vertex u_{i+1}.

The neighbors of u_i that differ from v, u_{i+1} and w_i, lie in $\mathcal{R}_2$ and the neighbors of u_j that differ from v, u_{j+1} and w_i, must lie in $\mathcal{R}_1$. Hence $\{v, u_{i+1}, u_{j+1}, w_i\}$ separates u_i from u_j (See Figure 3.7), contradicting that T is 5-connected.

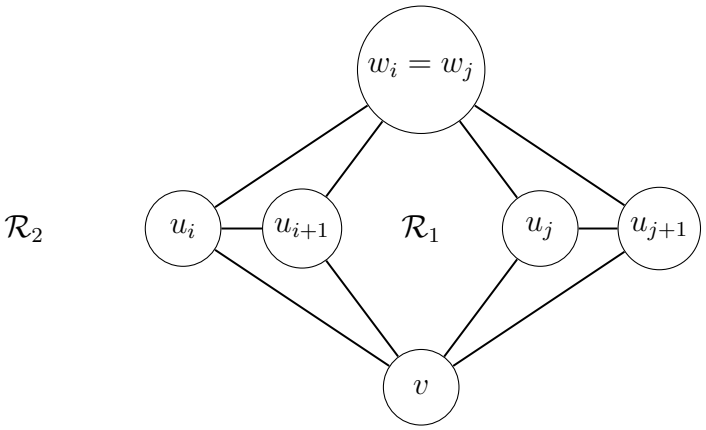

Figure 3.7: 4-element cutset in M_v when $w_i = w_j$.

Now let v be the vertex with maximum degree $\Delta = \Delta(T)$. The mosaic graph M_v around v contains $2\Delta + 1$ vertices. If T contains a vertex that is not in M_v, then $2\Delta(T) + 2 \leq n$, and the claim follows.

If T does not contain a vertex that is not in M_v, then set of edges E not in M_v form a triangulation of the 2Δ-cycle $u_1w_1u_2w_2\dots u_\Delta w_\Delta u_1$ on the region different from $\mathcal{R}_v$; consequently $|E| = 2\Delta - 3$. Note that for any $1 \le i < j \le \Delta$, we have $u_iu_j \notin E$, otherwise the 3-cycle u_iu_jv would contain the vertices w_i and w_{j+1}, and u_i, u_j, v would be a cutset of size 3, a contradiction. Also note that for any $j \notin \{i, i-1\}$, $u_iw_j \notin E$, otherwise the 4-cycle $u_iw_ju_jv$ has the vertices w_i and w_{i-1} on its different sides, giving a cutset of size 4. Therefore every edge in E connects two vertices of $W = \{w_1, \dots, w_\Delta\}$.

But then for every i, the edges $w_{i-1}u_i$ and u_iw_i lie on the boundary of the same face, giving $w_{i-1}w_i \in E$. Hence $w_1, \dots, w_\Delta$ determines a Δ-gon, which does not contain v, and this Δ-gon is triangulated by the remaining edges of E. Lemma 9 applies. Say, w_{i-1}, w_i, w_{i+1} is such a triangle. Then $d(w_i) = 4$, contradicting the fact that T is 5-connected. □

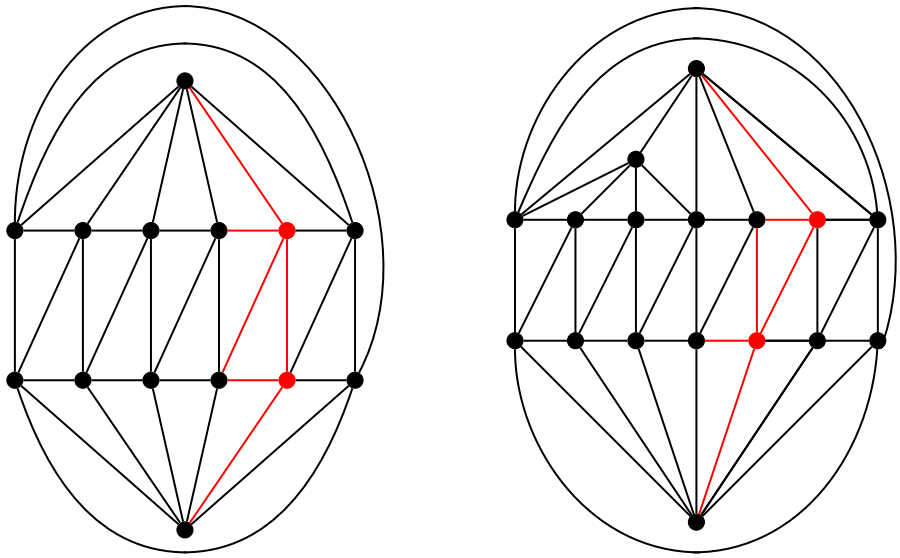

Figure 3.8: The graph T_n^5 which minimizes the Wiener index among all 5-connected triangulations on $n = 2k$ vertices (left) and on $n = 2k + 1$ vertices (right).

We will show that for every $n \ge 12$, $n \ne 13$ the 5-connected triangulations T_n^5 (see Figure 3.8) achieve the minimum Wiener index. It is easy to see that every T_n^5

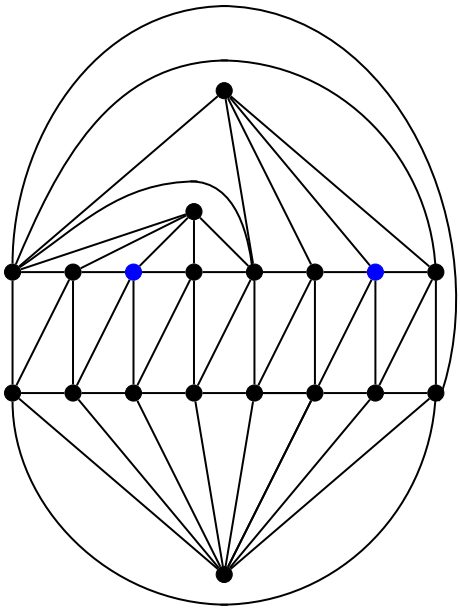

Figure 3.9: The graph X. The two blue vertices are at distance 4.

is 5-connected, and, as they have diameter 3, it can be easily shown that,

$$W(T_n^5) = \begin{cases} \frac{5n^2}{4} - 7n + 12 & \text{if } n = 2k \\ \frac{5n^2}{4} - 6n - \frac{9}{4} & \text{if } n = 2k+1. \end{cases} \tag{3.4}$$

The graph X, which is a 5-connected triangulation on 19 vertices (see Figure 3.9) has

$$W(X) = 335 = W(T_{19}^5). \tag{3.5}$$

We will also show that X the only 5-connected triangulation that is not isomorphic to any T_n^5 and achieves the minimum Wiener index for its order. Note that as X is of diameter 4, the lower bound in Lemma 10 (f) cannot be used to compute $W(X)$.

We define an extension of the mosaic graph M_v, the *extended mosaic graph* $M_v^\star$. Given a 5-connected triangulation G and a vertex v with mosaic graph M_v, we introduce the graph $M_v^\star$, on the vertex set of M_v, by setting

$$E(M_v^\star) = E(M_v) \cup \{w_i w_{i+1} : 1 \leq i \leq k-1, w_i w_{i+1} \in E(G)\}.$$

Note that $w_iw_{i+1} \in E(G)$ if an only if $d(u_i) = 5$. Let $\mathcal{R}_v^\star$ denote the extension of $\mathcal{R}_v$ by adding to it the faces bounded by the 3-cycles $u_iw_iw_{i+1}$ for all edges $w_iw_{i+1} \in E(G)$; let C_v denote the boundary cycle of $\mathcal{R}_v^\star$; and let $\mathcal{Q}_v^\star$ denote the other (outer) domain defined by the cycle C_v. Now all vertices of G that are not vertices of M_v and all edges of $E(G) \setminus E(M_v^\star)$ lie in the region $\mathcal{Q}_v^\star$ of the drawing of G.

We will use the following notation in the rest of the section. Given a 5-connected triangulation G and a vertex v, if a, b are vertices of C_v, then $P_v(a, b)$ denotes the path on the cycle C_v from a to b that follows the clockwise cyclic order. (So if $C_v = (w_1, w_2, \ldots, w_d)$ in clockwise cyclic order, then $P_v(w_1, w_2)$ is just the edge w_1w_2 with its endpoints, while $P_v(w_2, w_1)$ goes through all vertices of the cycle and misses only one edge).

Lemma 12. *Let G be a 5-connected triangulation on $n \geq 12$ vertices and let v be a vertex of G with $d(v) = d$. Consider the extended mosaic graph $M_v^\star$. The following are true:*

(a) Every vertex $z \in V(C_v)$ has an edge of $E(G) \setminus E(M_v^\star)$ incident upon it.

(b) If for some $z_1, z_2 \in V(C_v)$ we have $z_1z_2 \in E(G) \setminus E(M_v^\star)$, then z_1z_2 cuts $\mathcal{Q}_v^\star$ into two subregions, each containing a vertex of G in its interior.

(c) If n is even and $d(v) = \frac{n}{2} - 1$, then $G \cong T_n^5$

Proof. Set $W = \{w_1, \ldots, w_d\}$ and $U = \{u_1, \ldots, u_d\}$.

(a): Observe $W \subseteq V(C_v) \subseteq W \cup U$. Vertices in W have degree at most 4 in $M_v^\star$, and vertices of U have degree 5 in $M_v^\star$. If a vertex of U has degree 5 in G, then it is not a vertex of C_v. (a) follows.

(b): Assume $z_1, z_2 \in V(C_v)$ where $z_1z_2 \in E(G) \setminus E(M_v^\star)$, and z_1z_2 cuts $Q_v^\star$ into two subregions, one of which (say $\mathcal{R}$) contain no vertices in their interior. Then the edges lying in the interior of $\mathcal{R}$ are edges of $E(G) \setminus E(M_v^\star)$ that form a triangulation

of $\mathcal{R}$, and by Lemma 9 this triangulation contains two faces with two boundary edges on the boundary of $\mathcal{R}$. One of these faces, f, does not have z_1z_2 on its boundary; Let c be the common endvertex of the two edges of f on the boundary of $\mathcal{R}$. Then c can not have an edge from $E(G) \setminus E(M_v^\star)$ incident upon it, contradicting (a).

(c): Assume now that n is even and $d(v) = \frac{n}{2} - 1$. Lemma 11 gives $\Delta(G) = d(v)$. G has exactly one vertex, say x, not in M_v, and hence in the region $\mathcal{Q}_v^\star$. Then the previously proven (a) and (b) imply that $E(G) \setminus E(M_v^\star) = \{z_1x : z_1 \in V(C_v)\}$. As $W \subseteq C_v$, $d(x) \geq |W| = d(v) = \Delta(G)$, we get $d(x) = \frac{n}{2} - 1$ and $W = C_v$, and each edges of the form w_iw_{i+1} is an edge of $M_v^\star$. (c) follows. □

Theorem 12. *Assume that $n \geq 12$ and is even. The graph T_n^5 is the unique graph that minimizes the Wiener index among all 5-connected triangulations of order n.*

Proof. Let $n \geq 12$ even and assume T is a 5-connected triangulation on $n = 2k$ vertices ($k \geq 7$). The degree sum of T is $2(3n - 6) = 6n - 12$, and Lemma 11 gives $\Delta(T) \leq \frac{n}{2} - 1$. The integer sequence $y_1, \ldots, y_n$ that sums to $6n - 12$, satisfies $5 \leq y_i \leq \frac{n}{2} - 1$ and has the largest sum of squares is $\frac{n}{2} - 1, \frac{n}{2} - 1, 5, 5, \ldots, 5$, which is exactly the degree sequence of T_n^5. As T_n^5 has diameter 3, by Lemma 10 (f) T_n^5 indeed has the minimum Wiener index among all 5-connected n-vertex triangulations. We know that the degree sequence of T is the same as the degree sequence of T_n^5, $T \cong T_n^5$ follows from Lemma 12 (c). □

To handle odd order triangulations, we need a bit more information about their structure.

Lemma 13. *Let n be odd and G be a 5-connected triangulation on $n \geq 17$ vertices with $\Delta = \Delta(G) = \lfloor \frac{n}{2} \rfloor - 1$. Assume further that at most 4 vertices of G have degree greater than 5. Let v be a vertex with $d(v) = \Delta(G)$. Consider the extended mosaic graph $M_v^\star$, and the sets $W = \{w_1, \ldots, w_\Delta\}$ and $U = \{u_1, \ldots, u_\Delta\}$. Let x_1, x_2 denote the two vertices not in $M_v^\star$. The following are true:*

(a) *For* $i \in \{1,2\}$*, there are vertices* $a_i, b_i \in V(C_v)$ *such that* $N(x_i) \setminus \{x_{3-i}\} = V(P_v(a_i, b_i))$*. (We refer to these* a_i, b_i *vertices in the forthcoming cases.) Furthermore, if* $c \in V(P_v(a_1, b_1)) \cap V(P_v(a_2, b_2))$ *then* $c = a_1 = b_2$ *or* $c = a_2 = b_1$*.*

(b) *For* $i \in \{1,2\}$ *we have* $V(P_v(a_i, b_i)) \subseteq W$*. Consequently,* $d(x_i) \leq \Delta - 1$*, and if* $x_1x_2 \notin E(G)$ *then* $d(x_i) \leq \Delta - 2$*.*

(c) $x_1x_2 \in E(G)$ *implies the following:* $a_1 = b_2$*,* $b_1 = a_2$*,* $V(C_v) = W$*, all vertices in* $U \cup W \setminus \{a_1, a_2\}$ *have degree* 5*, and* $d(a_1) = d(a_2) = 6$*.*

(d) $x_1x_2 \notin E(G)$ *implies the following:* $a_1b_1, a_2b_2 \in E(G)$*. Moreover, for* $i \in \{1,2\}$*, if* $c_i \in V(P_v(b_i, a_{3-i}))$ *and* $z \in V(G) \setminus \{x_1, x_2\}$ *such that* $c_iz \in E(G) \setminus E(M_v^\star)$ *then* $z \in V(P_v(b_{3-i}, a_i))$*, the neighbors of* c_i *in* $P_v(b_{3-i}, a_i)$ *form a consecutive sequence of vertices on this path.*

(e) *If* $x_1x_2 \notin E(G)$*, and* $a_1 \neq b_2$*, and* $a_2 \neq b_1$*, then* G *has exactly* 4 *vertices of degree greater than* 5*, at least two of which are of degree* 7*.*

(f) *If* $x_1x_2 \notin E(G)$*,* $a_1 = b_2$ *and* $a_2 = b_1$*, then* $V(C_v) = W$*,* $d(a_1) = d(a_2) = 7$ *and for all* $c \in U$ *we have* $d(c) = 5$*.*

(g) *If* $x_1x_2 \notin E(G)$*,* $a_1 = b_2$ *and* $a_2 \neq b_1$*, then* $d(a_1) = 6 + |V(P_v(b_1, a_2))|$*. In addition, we have one of the following:*

[i] $V(C_v) = W$*,* $d(b_1) = d(a_2) = 6$ *and* $d(z) = 5$ *for all* $z \in (W \cup U) \setminus \{a_1, a_2, b_1\}$

[ii] There is a vertex $u \in U$ *such that* $P_v(b_1, a_2) = b_1ua_2$*,* $d(a_1) = 9$*,* $d(u) = 6$*, and for all* $c \in (W \cup U) \setminus \{a_1, u\}$*, we have* $d(c) = 5$*.*

Proof. Let G, v, x_1, x_2 be as in the conditions. Note $d(v) \geq 7$.

(a): Assume $i \in \{1,2\}$. As $d(x_i) \geq 5$, x_i has at least 4 neighbors on C_v, so there are vertices $a_i, b_i \in N(x_i) \cap V(C_v)$ such that all vertices in $N(x_i) \setminus \{x_{3-i}\}$ lie

on the path $P_v(a_i, b_i)$ and x_{3-i} does not lie in the interior of the subregion of $\mathcal{Q}_v^\star$ bounded by the cycle $x_i P_v(a_i, b_i)$. By Lemma 12 (b) no two vertices in $P_v(a_i, b_i)$ can be joined by an edge that is not in $M_v^\star$. As every vertex of C_v has at least one edge incident upon it from $E(G) \setminus E(M_v^\star)$, we must have $V(P_v(a_i, b_i)) \subseteq N(x_i)$, therefore $V(P_v(a_i, b_i)) = N(x_i) \setminus \{x_{3-i}\}$. As all edges incident upon x_1 or x_2 lie in the region $\mathcal{Q}_v^\star$ and do not cross, the two paths share at most their endvertices.

(b): For the contrary, assume $c \in U \cap V(P_v(a_i, b_i))$. As $d(x_i) \geq 5$, $P_v(a_i, b_i)$ has at least 4 vertices. Therefore, there exists an internal vertex $c^\star$ of the path $P_v(a_i, b_i)$, such that the edge $cc^\star$ is an edge of this path. This means $c^\star \in W$ (see Lemma 12(b)), $c^\star$ has at most 3 edges incident upon it in $E(M_v^\star)$, and the only edge in $E(G) \setminus E(M_v^\star)$ incident upon $c^\star$ is $c^\star x_i$, contradicting $d(c^\star) \geq 5$. Thus, $V(P_v(a_i, b_i)) \subseteq W$. By part (a), we have $|V(P_v(a_{3-i}, b_{3-i})) \setminus V(P_v(a_i, b_i))| \geq 2$. Hence $\Delta = |W| \geq |V(P_v(a_i, b_i))| + |V(P_v(a_{3-i}, b_{3-i})) \setminus V(P_v(a_i, b_i))|$. As $N(x_i) = V(P_v(a_i, b_i))$ or $V(P_v(a_i, b_i)) \cup \{x_{3-i}\}$, depending on whether $x_1 x_2$ is non-edge or edge, the claim (b) follows.

(c): Assume that $x_1 x_2 \in E(G)$ but $b_2 \neq a_1$. Then either $P_v(b_2, a_1)$ has an internal vertex c or $b_2 a_1 \in E(G)$. In the first case, as $c \in C_v$, there is at least one edge in $E(G) \setminus E(M_v^\star)$ incident upon c by Lemma 12 (a). All edges of $E(G) \setminus E(M_v^\star)$ incident upon c must lie in the subregion of $\mathcal{Q}_v^\star$ bounded by the cycle $P_v(b_2, a_1) x_1 x_2$. By Lemma 12 (b) these edges must be of the form cx_1 or cx_2. But none of these are edges of G, which is a contradiction. In the second case $b_2 a_1 \in E(G)$, and the subregion of $\mathcal{Q}_v^\star$ bounded by the 4-cycle $b_2 a_1 x_1 x_2$ has no vertices in its interior, so we must have either $x_1 b_2 \in E(G)$ or $x_2 a_1 \in E(G)$, a contradiction. So $a_1 = b_2$, and $b_2 = a_1$, which gives $C_v = W$ by (b). All $w \in W$ is incident to 4 edges of $M_v^\star$, but $a_1, a_2 \in W$ are incident to 2 more edges, and vertices of $W \setminus \{a_1, a_2\}$ are incident to one more, (c) follows.

For the remaining cases of this proof, it will be assumed that x_1 and x_2 are not adjacent.

(d): In this case the edges x_ia_i, x_ib_i lie on the boundary of the same face, so $a_ib_i \in E(G)$, and $a_ib_ix_i$ is a face. Let $c_i \in V(P_v(b_i, a_{3-i}))$ and $z \in V(G) \setminus \{x_1, x_2\}$ such that $c_iz \in E(G) \setminus E(M_v^\star)$. As c_i lies on the connected subregion $\mathcal{R}$ of $\mathcal{Q}_v^\star$ bounded by the cycle $P(b_1, a_2)P(b_2, a_1)$, Lemma 12 (b) gives that $z \in V(P(b_{3-i}, a_i))$, as claimed. Also, if $z_1, z_2 \in V(P(b_{3-i}, a_i))$ are different neighbors of c_i where z_1z_2 is not an edge of the path $P_v(b_{3-i}, a_i)$, then by Lemma 12(b) any vertex z_3 of the $z_1 - z_2$ subpath of $P(b_{3-i}, a_i)$ can only have the edge z_3c_i incident upon it from $E(G)\setminus E(M_v^\star)$. (d) follows.

(e): Assume that $a_1 \neq b_2$, $a_2 \neq b_1$. Let $\mathcal{R}$ be the connected subregion of $\mathcal{Q}_v^\star$ bounded by the cycle $P_v(b_2, a_1)P_v(a_2, b_1)$. The assumptions of the Lemma imply that $d(v) = \Delta \geq 6$ and no more than 4 vertices have degree greater than 5. Therefore, it is enough to find 4 vertices in $V(G) \setminus \{v\}$ with degree greater than 5. As $d(v) = \Delta \geq 6$, C_v contains at most 3 vertices with degree greater than 5. Case (d) gives that $a_1b_1, a_2b_2 \in E(G) \setminus E(M_v^\star)$. As by (b) $V(P(a_i, b_i)) \subseteq W$, each of a_1, a_2, b_1, b_2 has at least 3 edges incident upon them in $E(M_v^\star)$. (In fact, at least 4, unless they are connected to a vertex in U by an edge on C_v). Let z_1, z_2, c_1, c_2 be given such that b_jz_j, c_ia_i are edges of the path $P_v(b_j, a_i)$ for $i \neq j$. At this point, we only know $2 \leq |\{a_i, c_i, z_j, b_j\}|$. Since not all of a_1, b_1, a_2, b_2 has degree at least 6, we can assume without loss of generality that $d(a_1) \leq 5$.

By definition a_1 is joined to x_1, and by (d), to b_1. Then, by the paranthesized comment, we must have $c_1 \in U$ and hence $c_1 \neq b_2 \in W$. As we accounted for all 5 edges at a_1, no edge leaves a_1 between the edges a_1b_1 and a_1c_1 in $\mathcal{Q}_v^\star$, hence the edges a_1b_1 and a_1c_1 must lie on the boundary of a common face in $\mathcal{R}$, and as this face must be a triangle, we have $c_1b_1 \in E(G)$. As $c_1 \in U$, c_1 has 5 neighbors in M_v^*, and we

found its 6^{th} neighbor, b_1. $b_1 \in W$ by (b) has 3 incident edges in $M_v^\star$ as discussed above, and has 3 further neighbours, namely x_1, a_1, c_1. We conclude $d(b_1), d(c_1) \geq 6$.

As b_1, b_2, c_1, a_2 are 4 different vertices on C_v, and $d(b_1), d(c_1) \geq 6$, we have

$$d(a_2) \leq 5 \qquad \text{or} \qquad d(b_2) \leq 5. \tag{3.6}$$

If $d(a_2) \leq 5$, then we obtain by repeating the arguments of the paragraph above, changing i to $3 - i$ everywhere, that b_1, b_2, c_1, c_2 are 4 different vertices of degree greater than 5 in C_v, a contradiction. So we must have $d(a_2) \geq 6$, and by (3.6), $d(b_2) \leq 5$.

But then this gives as before that $z_2 \in U$, $z_2a_2 \in E(G)$, and the vertices b_1, c_1, a_2, z_2 all have degree greater than 5, so we must have $c_1 = z_2$. In addition as vertices of U have degree at least 6, and we already have 3 such vertices on C_v, $V(P_v(a_2, b_1)) \cap U = \emptyset$. This means that the 4 vertices $v, b_1, a_2, c_1 = z_2$ all have degree greater than 5. As c_1 is adjacent to a_2 and b_1, by (d) c_1 is adjacent to all vertices in $P_v(b_1, a_2)$. This shows $d(c_1) \geq 7$, but we cannot show here equality. However, $d(b_1) = d(a_2) = 7$, as both b_1 and a_2 have 4 edges in M_v^*, and we can account for all of their remanining neighbors: x_1, a_1, c_1 for b_1 and $x_2, b_2, z_2 = c_1$ for a_2. (e) follows.

(f): Assume $a_1 = b_2$ and $a_2 = b_1$. Then by (b) $V(C_v) = W$, and any edge from $E(G) \setminus E(M_v^\star)$ incident upon a vertex $w \in W \setminus \{a_1, a_2\}$ connects w to exactly one of x_1, x_2. As the 4-cylce $x_1a_1x_2a_2$ must have a diagonal edge inside, it must be $a_1a_2 \in E(G)$. Both a_i has 4 edges in M_v^*, and in addition, it joined to exactly 3 more vertices: x_1, x_2, a_{3-i}. (f) follows.

(g): Assume $a_1 = b_2$ and $a_2 \neq b_1$. We have $a_1x_1, a_1x_2 \in E(G)$ and by (d), all vertices of $P_v(b_1, a_2)$ are neighbors if a_1. All other neighbors of a_1 are one of the 4 neighbors of a_1 in $M_v^\star$. Consequently $d(a_1) = 6 + |V(P_v(b_1, a_2))|$, which finishes the main part of (g).

(g) [i]: Since $V(C_v) = W$, then we must have $d(a_2) = d(b_1) = 6$, as a_2, b_1 have each 4 neighbors in M_v^*, and both are joined to $a_1 = b_2$, and to a single x_i, and not

joined to anything else. As every $w \in W \setminus \{a_1, a_2, b_1\}$ has 4 neighbors in M_v^*, and is joined by an edge in $E(G) \setminus E(M_v^\star)$ to the vertex a_1, and nothing else, we have $d(w) = 5$. If $u \in U$, then only the 5 edges already in $E(M_v^\star)$ are incident upon u, so $d(u) = 5$. [i] follows.

(g) [ii]: Now we can assume $V(C_v) \neq W$. As both a_1 and v already have degree larger than 5, and every vertex in $V(C_v) \cap U$ has degree at least 6, C_v can contain at most two vertices from U. From (b) we know $V(C_v) \cap U \subseteq P_v(b_1, a_2)$. If $|V(P_v(b_1, a_2))| \geq 4$ and $c \in V(P_v(b_1, a_2)) \cap U$, then c has a neighbor z that is an internal vertex of $P_v(b_1, a_2)$ such that cz is an edge of this path, and consequently $z \in W$. Since the only edge in $E(G) \setminus E(M_v^\star)$ incident upon z is za_1, we have $d(z) \leq 4$, a contradiction. So $P(b_1, a_1) = b_1 u a_1$ for some vertex $u \in U$. The required degrees are easy to check. (g) follows. □

Lemma 14. *Let $n \geq 17$ be odd. If G is a 5-connected triangulation on n vertices, then it has at most one vertex of degree $\lfloor \frac{n}{2} \rfloor - 1$.*

Proof. Let $n = 2k + 1$, $k \geq 8$. Let G be a 5-connected triangulation on $n = 2k + 1$ vertices, with $\Delta(G) = \frac{n}{2} - 1 = k - 1$. Let v be a vertex of degree $k - 1$ and consider the extended mosaic graph $M_v^\star$. Let x_1, x_2 be the vertices of G not in M_v. Set $U = \{u_1, \ldots, u_{k-1}\}$ and $W = \{w_1, \ldots, w_{k-1}\}$. Let a_1, a_2, b_1, b_2 be the vertices described in Lemma 13 (a).

Assume that G has two vertices of degree $k - 1 \geq 7$. Then the degree sequence of G is $(k - 1, k - 1, 6, 5, \ldots 5)$. In particular, there are exactly 3 vertices G with degree bigger than 5, and exactly one of them has degree 6.

Therefore Lemma 13 (b), (c), (d) , (f), (e) and (g) (especially (g) [ii]) gives that we have $d(x_1), d(x_2) < k - 1$, $x_1 x_2 \notin E(G)$, $a_1 b_1, a_2 b_2 \in E(G)$, and without loss of generality $a_1 = b_2$ and $a_2 \neq b_1$, $d(a_1) = 9 = k - 1$, $n = 21$, $P_v(b_1, a_2) = b_1 u a_2$ for some $u \in U$, $d(u) = 6$, for all $z \in (U \cup W) \setminus \{u, a_1\}$ we have $d(z) = 5$, and consequently

$d(x_1) = d(x_2) = 5$. The resulting structure is the graph on Figure 3.10, which has a separating a 4-cycle, so it is not 5-connected.

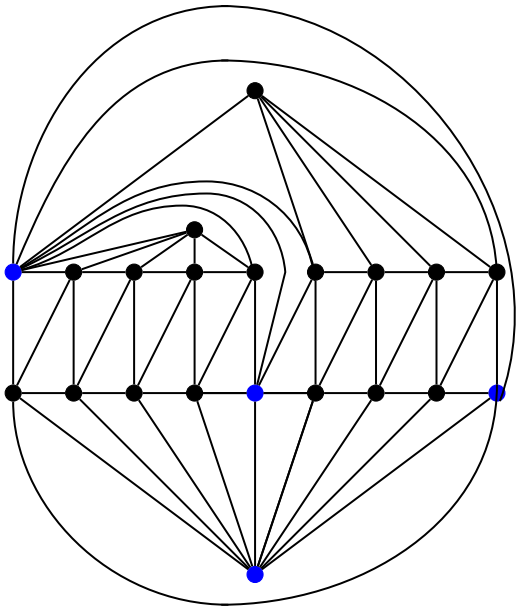

Figure 3.10: The planar triangulation on 21 vertices with degree sequence $(9, 9, 6, 5, 5, \ldots, 5)$. The blue vertices are on a 4-cycle, whose vertices form a cutset.

□

Lemma 15. *Let $n \geq 19$ be odd. If G is a 5-connected triangulation such that the three largest degrees are $\lfloor \frac{n}{2} \rfloor - 1, \lfloor \frac{n}{2} \rfloor - 2, d_3$ and $d_3 \geq 7$, then $n = 19$, $G \cong X$ and $W(G) = W(T_{19}^5)$*

Proof. Let $n = 2k + 1$, $k \geq 9$ and G be a 5-connected triangulation such that the three largest degrees are $d_1 = \lfloor \frac{n}{2} \rfloor - 1 > d_2 = \lfloor \frac{n}{2} \rfloor - 2 \geq d_3 \geq 7$. Then the degree sequence of G is $(k - 1, k - 2, 7, 5, 5, \ldots, 5)$.

Let v be a vertex of degree $k - 1$ and consider the extended mosaic graph $M_v^\star$. Let x_1, x_2 be the vertices of G not in M_v. Set $U = \{u_1, \ldots, u_{k-1}\}$ and $W = \{w_1, \ldots, w_{k-1}\}$. Let a_1, a_2, b_1, b_2 be the vertices described in Lemma 13 (a).

By Lemma 13 (b), (c), (d), (e), (f) and (g) we get that $x_1x_2 \notin E(G)$, $d(x_1) = d(x_2) < k-2$, $a_1b_1, a_2b_2 \in E(G)$, $a_2 = b_1$, $a_1 = b_2$, $d(a_1) = d(a_2) = 7$ and for all $c \in U \cup W \setminus \{a_1, a_2\}$ $d(c) = 5$. This implies that we must also have $d(x_1) = 5$, $d(x_2) = 5$ and $k - 2 = 7$, i.e. $n = 19$. This gives $G \cong X$ and $W(G) = W(T_{19}^5)$. □

Lemma 16. *Let $n \geq 19$ be odd. If G is a 5-connected triangulation with the same degree sequence as T_n^5, then either $G \cong T_n^5$ or $W(G) > W(T_n^5)$.*

Proof. Let $n = 2k + 1$, $k \geq 9$ and G be a 5-connected triangulation with degree sequence $(k-1, k-2, 6, 6, 5, 5, \ldots, 5)$. So G has 4 vertices with degree greater than 5, exactly two of which has degree 6.

Let v be a vertex of degree $k-1$ and consider the extended mosaic graph $M_v^\star$. Let x_1, x_2 be the vertices of G not in M_v. Set $U = \{u_1, \ldots, u_{k-1}\}$ and $W = \{w_1, \ldots, w_{k-1}\}$. Let a_1, a_2, b_1, b_2 be the vertices described in Lemma 13 (a).

By Lemma 13 (b), (c), (d), (e), (f), and (g) we have one of the following:

1. $x_1x_2 \in E(G)$, $a_1 = a_2$, $V(C_v) = W$, $d(a_1) = d(a_2) = 6$, and all vertices in $U \cup W \setminus \{a_1, a_2\}$ have degree 5; then without loss of generality $d(x_1) = k-2$, $d(x_2) = 5$. We have $G \cong T_n^5$.

2. $x_1x_2 \notin E(G)$, without loss of generality $a_1 = b_2$ and $a_2 \neq b_2$, $d(a_1) = 6 + |V(P_v(b_1, a_2))$, $V(C_v) = W$, $d(a_1) = d(a_2) = 6$ and $d(z) = 5$ for all $z \in W \cup U \setminus \{a_1, a_2, b_2\}$. This implies that $d(x_1) = d(x_2) = 5$, $|V(P_v(b_1, a_2))| = k-8 \geq 2$ (so $k \geq 10$ and $n \geq 21$) and $d(a_1) = k-2$. So we have exactly one graph for each $n \geq 21$, see Figure 3.11. These graphs have diameter at least 4, as indicated by the pair of vertices marked in Figure 3.11, and consequently by Lemma 10 (f) their Wiener index is larger than that of T_n^5, which is a graph with the same degree sequence.

□

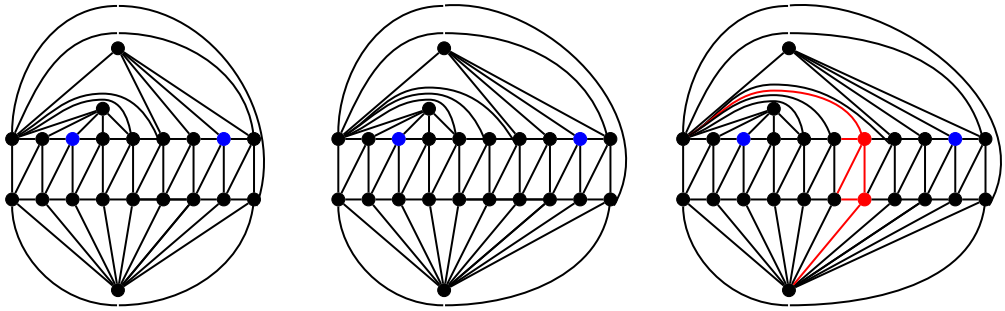

Figure 3.11: The other graphs that have the same degree sequence as T_n^5, on $21, 23$ and 25 vertices. The red vertices/edges on the 25 vertex graph indicate the pattern that needs to be repeated to get higher vertex count. The two blue vertices are at distance 4.

Theorem 13. *Let $n \geq 15$, odd. If $n \neq 19$, then the unique graph that among 5-connected triangulations on n vertices has minimum Wiener index is T_n^5. If $n = 19$, then there are precisely two such graphs, T_{19}^5 and X.*

Proof. Let $n = 2k + 1$. The theorem is true for $n \in \{15, 17\}$ by computer search, so assume $k \geq 9$. Let G be a 5-connected triangulation on n vertices with minimum Wiener index, and let $d_1 \geq d_2 \geq \ldots \geq d_n$ be the degree sequence of G. Then all d_i is at least 5 and by Lemmata 11 and 14 we have $d_1 \leq k - 1$ and $d_2 \leq k - 2$.

If $d_1 + d_2 < 2k - 3 = n - 4$, we set $(y_1, \ldots, y_k) = (d_1, \ldots, d_k)$ and we perform the following algorithm: If for $i \in \{1, 2\}$ $y_i < k - i$, pick $3 \leq j \leq n$ such that y_j is maximum among $\{y_\ell : 3 \leq \ell \leq n\}$, and set $y_i = y_i + 1$, $y_j = y_j - 1$. Every time we perform the operation, we do not change the sum of the numbers and we increase the sum of squares. Also, as the sum of the numbers is $6(n - 2)$, $\sum_{\ell \geq 3} y_i \geq 6(n-2)-(n-4) > 5(n-2)$, we have that the algorithm stops at a sequence $(y_1, \ldots, y_k)$ with $y_1 = k-1$, $y_2 = k-2$ and $y_i \geq 5$ for all i. Set $(x_1, \ldots, x_n)$ to be the permutation of the y_i sequence the algorithm stopped on given in decreasing order. Considering the fact that $\sum x_i = 6(n-2) = 12k-6$, $(x_1, \ldots, x_n)$ is one of $(k-1, k-2, 7, 5, \ldots, 5)$ and $(k - 1, k - 2, 6, 6, 5, \ldots, 5)$.

If $(x_1, \ldots, x_n) = (k-1, k-2, 7, 5, \ldots, 5)$, then, since any time we change the sequence in the algorithm, we change the largest of $\{y_\ell : 3 \leq \ell \leq n\}$, this implies that the starting sequence $(d_1, \ldots, d_n)$ satisfies $d_i = 5$ for all $4 \leq i \leq n$, so $d_3 = 12k - 6 - 5(2k-2) - (d_1 + d_2) = 2k + 4 - (d_1 + d_2) \geq 8$. Consider the sequence $(z_1, z_2 \ldots, z_n) = (k-1, k-3, 8, 5, \ldots, 5)$ that has at least as large sum of squares as $(d_1, \ldots, d_n)$. By Lemma 10 (f) and the fact that T_n^5 has diameter 3 we have

$$W(G) - W(T_n^5) \geq \sum_{v \in V(T_n^5)} \binom{d(v)}{2} - \sum_{i=1}^{n} \binom{z_i}{2} = \binom{k-2}{2} - \binom{k-3}{2} + 8 = k - 5 > 0,$$

contradicting the minimality of G. Therefore we must have $(x_1, \ldots, x_n)$ is the degree sequence of T_n^5, and we made at least one step with the algorithm. Lemma 10 (f) and the fact that T_n^5 has diameter 3 give

$$W(G) - W(T_n^5) \geq \left(\sum_{i=1}^{n} x_i^2\right) - \left(\sum_{i=1}^{n} d_i^2\right) > 0,$$

a contradiction. Therefore we must have $d_1 = k-1$, $d_2 = k-2$, and Lemma 15 yields G has the same degree sequence as T_n^5. Lemmata 15 and 16 then implies $G \cong T_n^5$ or $n = 19$ and $G \cong X$. □

3.4 Recursion Solutions

This section contains the worked out recursions for the equations of the Wiener index of the graphs in figures 3.1, 3.2, 3.8, 3.5. It should be noted that the variable k will be used to denote the number of layers of each structure. Adding another copy of the red pattern in the figures from the previous section would be equivalent to increasing k by 1. Let w_k denote the Wiener index on the given structure with k layers.

First, we consider 3 and 4-connected triangulations,

$$W(T_n^4) = n^2 - 4n + 6$$

Here, the layers represent the repeating 2-paths. When $n = k + 2$, we have,

$$w_{k+1} = w_k + 2k + 1 \text{ with } w_{10} = 102$$

$$\implies w_k = k^2 + 2 \implies w_n = n^2 - 4n + 6$$

Next, we consider quadrangulations,

$$W(K_{2,n-2}) = n^2 - 3n + 4$$

Here, the layers represent the repeating 2-paths. When $n = k + 2$, we have,

$$w_{k+1} = w_k + 2k + 2 \text{ with } w_{10} = 112$$

$$\implies w_k = k^2 + k + 2 \implies w_n = n^2 - 3n + 4$$

Next, we consider 3-connected quadrangulations,

$$W(Q_n^3) = \begin{cases} \frac{5n^2}{4} - 5n + 8 & \text{if } n = 2k + 2 \\ \frac{5n^2}{4} - 3n - \frac{49}{4} & \text{if } n = 2k + 3. \end{cases}$$

Here, the layers represent the repeating 3-paths. When $n = 2k + 2$, we have,

$$w_{k+1} = w_k + 10k + 5 \text{ with } w_6 = 183$$

$$\implies w_k = 5k^2 + 3 \implies w_n = \frac{5n^2}{4} - 5n + 8$$

If $n = 2k + 3$, we have,

$$w_{k+1} = w_k + 10k + 14 \text{ with } w_6 = 224$$

$$\implies w_k = 5k^2 + 9k - 10 \implies w_n = \frac{5n^2}{4} - 3n - \frac{49}{4}$$

Finally, we consider 5-connected triangulations,

$$W(T_n^5) = \begin{cases} \frac{5n^2}{4} - 7n + 12 & \text{if } n = 2k \\ \frac{5n^2}{4} - 6n - \frac{9}{4} & \text{if } n = 2k + 1. \end{cases}$$

Here, the layers represent the repeating 3-paths. When $n = 2k + 2$, we have,

$$w_{k+1} = w_k + 10k + 1 \text{ with } w_{10} = 463$$

$$\implies w_k = 5k^2 - 4k + 3 \implies w_n = \frac{5n^2}{4} - 7n + 12$$

If $n = 2k + 3$, we have,

$$w_{k+1} = w_k + 10k + 8 \text{ with } w_{10} = 521$$

$$\implies w_k = 5k^2 + 3k - 9 \implies w_n = \frac{5n^2}{4} - 6n - \frac{9}{4}$$

Chapter 4

Upper Bounds on Proximity

4.1 Theoretical Results

In this chapter we present bounds on the proximity of k-connected triangulations for $k \in \{3, 4, 5\}$ and k-connected quadrangulations for $k \in \{2, 3\}$. All bounds are sharp apart from an additive constant.

Our strategy is as follows: If G is a k-connected triangulation or quadrangulation, then we choose a central vertex v and derive certain properties of the sequence $n_0(v), n_1(v), n_2(v), \ldots, n_r(v)$. These properties will be used to obtain a bound on the average distance of v, which in turn is a bound on the proximity of G. The following lemmata were proved in [1] (see the proofs of Theorems 2.7, 2.9 and 2.10 there).

Lemma 17. *[1] Let G be a 3-connected plane graph of radius r with maximal face length ℓ. If v is a central vertex of G, then*
(i) $n_i(v) \geq 3$ *for* $i \in \{1, 2, \ldots, \lfloor \frac{\ell}{2} \rfloor\} \cup \{r - \lfloor \frac{\ell}{2} \rfloor, r - \lfloor \frac{\ell}{2} \rfloor + 1, \ldots, r - 1\}$,
(ii) $n_i(v) \geq 4$ *for* $i \in \{\lfloor \frac{\ell}{2} \rfloor + 1, \lfloor \frac{\ell}{2} \rfloor + 2, \ldots, \ell\} \cup \{r - \ell, r - \ell + 1, \ldots, r - \lfloor \frac{\ell}{2} \rfloor - 1\}$,
(iii) $n_i(v) \geq 6$ *for* $i \in \{\ell + 1, \ell + 2, \ldots, r - \ell - 1\}$.

Lemma 18. *[1] Let G be a 4-connected plane graph of radius r with maximal face length ℓ. If v is a central vertex of G, then*
(i) $n_i(v) \geq 4$ *for* $i \in \{1, 2, \ldots, \ell\} \cup \{r - \ell, r - \ell + 1, \ldots, r - 1\}$,
(ii) $n_i(v) \geq 6$ *for* $i \in \{\ell + 1, \ell + 2, \ldots, \lfloor \frac{3\ell}{2} \rfloor\} \cup \{r - \lfloor \frac{3\ell}{2} \rfloor, r - \lfloor \frac{3\ell}{2} \rfloor + 1, \ldots, r - \ell - 1\}$,
(iii) $n_i(v) \geq 8$ *for* $i \in \{\lfloor \frac{3\ell}{2} \rfloor + 1, \lfloor \frac{3\ell}{2} \rfloor + 2, \ldots, r - \lfloor \frac{3\ell}{2} \rfloor - 1\}$.

Lemma 19. *[1] Let G be a 5-connected plane graph of radius r with maximal face length ℓ. If v is a central vertex of G, then*

(i) $n_i(v) \geq 5$ *for* $i \in \{1, 2, \ldots, \ell\} \cup \{r - \ell, r - \ell + 1, \ldots, r - 1\}$,

(ii) $n_i(v) \geq 6$ *for* $i \in \{\ell + 1, \ell + 2, \ldots, \lfloor \frac{3\ell}{2} \rfloor\} \cup \{r - \lfloor \frac{3\ell}{2} \rfloor, r - \lfloor \frac{3\ell}{2} \rfloor + 1, \ldots, r - \ell - 1\}$,

(iii) $n_i(v) \geq 8$ *for* $i \in \{\lfloor \frac{3\ell}{2} \rfloor + 1, \lfloor \frac{3\ell}{2} \rfloor + 2, \ldots, 2\ell\} \cup \{r - 2\ell, r - 2\ell + 1, \ldots, r - \lfloor \frac{3\ell}{2} \rfloor - 1\}$.

(iii) $n_i(v) \geq 10$ *for* $i \in \{2\ell + 1, 2\ell + 2, \ldots, r - 2\ell - 1\}$.

We also need a corresponding result for 2-connected quadrangulations. The following two lemmata follow the approach in [1]. Given a fixed vertex v, recall that we say that a vertex in $N_i(v)$ is *active* if it has a neighbor in $N_{i+1}(v)$. The set of active vertices in $N_i(v)$ is denoted by $A_i(v)$.

Lemma 20. *Let G be a quadrangulation, v a vertex of G and $i \in \mathbb{N}$ with $1 \leq i \leq e(v) - 1$. For every active vertex $w \in N_i(v)$ there exists another active vertex $w' \in N_i(v)$ such that w and w' share a face of G.*

Proof. Let u be an arbitrary vertex in A_i. Since u is active, it has neighbors in N_{i-1} and in N_{i+1}. Number the neighbors of u as $x_0, x_1, \ldots, x_t$ such that the edges ux_j appear in clockwise order, x_0 is in N_{i-1} and, say, x_k is in N_{i+1}. Denote the face containing u, x_j, x_{j+1} and a fourth vertex by f_j for $j = 0, 1, \ldots, t$, where subscripts are taken modulo $t + 1$, and let y_j be the vertex on f_j not equal to u, x_j and x_{j+1}.

Consider the (x_0, x_k)-walk $W : x_0, y_0, x_1, y_1, \ldots, x_{k-1}, y_{k-1}x_k$. Since W joins a vertex in N_{i-1} to a vertex in N_{i+1}, it contains a vertex in N_i. Since G is bipartite, none of the neighbors of u is in N_i, so there exists a vertex y_j which is in N_i. We may assume that y_j is the last such vertex. Then y_j has a neighbor in N_{i+1} and is thus active. Since u and y_j share a face, Lemma 20 follows. □

Lemma 21. *Let G be a quadrangulation of order n and radius r. If v is a central vertex of G, then*

(i) $n_i \geq 2$ *for* $i \in \{1, 2, r-2, r-1\}$, *and*

(ii) $n_i \geq 4$ *for* $i \in \{3, 4, \ldots, r-3\}$.

Proof. Let v be a central vertex of the quadrangulation G and let $i \in \{1, 2, \ldots, r-1\}$, where r is the radius of G.

(i) Clearly, N_i contains an active vertex. It follows from Lemma 20 that every active vertex in N_i shares a face with some other active vertex in N_i. Hence N_i contains at least two active vertices, and so (i) follows.

(ii) Suppose to the contrary that (ii) does not hold. Then there exists $i \in \{3, 4, \ldots, r-3\}$ such that $n_i \leq 3$.

Denote the set of active vertices in N_i by A_i. Since by Lemma 20 every vertex in A_i shares a face with some other vertex in A_i, and since $|A_i| \leq |N_i| = 3$, there exist a vertex $z_i \in A_i$ that shares a face in G with every other vertices of A_i. Since all faces of G have length 4, it follows that $d_G(z_i, y_i) \leq 2$ for all $y_i \in A_i$. Let $z_3 \in N_3$ be a vertex on a shortest (v, z_i)-path in G, so that $d_G(z_3, z_i) = i - 3$.

We now bound $d(z_3, x)$ for all $x \in V(G)$. First let $x \in \bigcup_{j=0}^{r-4} N_i$. Then $d_G(z_3, x) \leq d_G(z_3, v) + d_G(v, x) \leq 3 + (r-4) = r - 1$. Now let $x \in \bigcup_{j=r-3}^{r} N_i$. Let $x_i \in N_i$ be a vertex on a shortest (v, x)-path in G. Then $x_i \in A_i$, and so $d_G(z_i, x_i) \leq 2$. Hence $d_G(z_3, x) \leq d_G(z_3, z_i) + d_G(z_i, x_i) + d_G(x_i, x) \leq (i-3) + 2 + (r-i) = r - 1$. So $d_G(z_3, x) \leq r - 1$ in all cases, a contradiction to r being the radius of G. Hence our assumption $n_i \leq 3$ is false, and (ii) follows. □

For the proofs below we define the function F which assigns to a finite sequence $X = (x_0, x_1, \ldots, x_k)$ of integers the value $F(X) = \sum_{i=0}^{k} i x_i$. So if v is a vertex of eccentricity r in a connected graph G, then $\sigma(v) = \sum_{i=0}^{r} i n_i(v) = F(n_0, n_1, \ldots, n_r)$.

Theorem 14. *Let G be a planar graph of order n and v a central vertex of G.*

(a) If G is a triangulation, then

$$\pi(G) \leq \frac{n + 19}{12} + \frac{25}{3(n-1)}.$$

(b) If G is a 4-connected triangulation, then

$$\pi(G) \leq \frac{n+35}{16} + \frac{91}{4(n-1)}.$$

(c) If G is a 5-connected triangulation, then

$$\pi(G) \leq \frac{n+57}{20} + \frac{393}{10(n-1)}.$$

(d) If G is a quadrangulation, then

$$\pi(G) \leq \frac{n+11}{8} + \frac{9}{2(n-1)}.$$

(e) If G is a 3-connected quadrangulation, then

$$\pi(G) \leq \frac{n+25}{12} + \frac{169}{12(n-1)}.$$

Proof. We only prove part (a) of the theorem; the proofs of (b)-(e) are analogous. Let G be a simple triangulation, let r be its radius and let v be a central vertex of G. It suffices to prove that

$$\sigma(v) \leq \frac{1}{12}(n^2 + 18n + 81).$$

Let $n_i := n_i(v)$ for $i = 0, 1, \ldots, r$. Then $\sigma(v) = \sum_{i=1}^{r} i n_i = F(n_0, n_1, \ldots, n_r)$. By Lemma 17, we have $n_1, n_{r-1} \geq 3$, $n_2, n_3, n_{r-3}, n_{r-2} \geq 4$, and $n_i \geq 6$ for $i = 4, 5, \ldots, r-4$. Moreover, we have $n_0 = 1$, $n_r \geq 1$, and $\sum_{i=0}^{r} n_i = n$.

Now assume that for given n the integers $r', n'_0, n'_1 \ldots, n'_{r'}$ are chosen to maximise $F(n'_0, n'_1, \ldots, n'_{r'})$ subject to the above conditions (with n_i and r replaced by n'_i and r', respectively). Then clearly $n'_0 = 1$, $n'_1 = 3$, $n'_2 = n'_3 = 4$, $n'_i = 6$ for $i = 4, 5, \ldots, r'-4$, and $n'_{r'-3} = n'_{r'-2} = 4$, $n'_{r'-1} = 3$.

Consider the sequence $X^* = (n'_0, n'_1, \ldots, n'_{r'-1}, 1)$. The sum of the entries of X^* is $n - n_{r'} + 1$. Simple calculations show that $r' = \frac{1}{6}(n - n'_{r'} + 19)$ and $F(X^*) = \frac{1}{2}r'(n - n'_{r'} + 1)$. Hence

$$F(n'_0, n'_1, \ldots, n'_{r'}) = F(X^*) + r'(n'_{r'} - 1) = \frac{1}{2}r'(n + n'_{r'} - 1).$$

Substituting $r' = \frac{1}{6}(n - n'_{r'} + 19)$ yields, after simplification, $F(n'_0, n'_1, \ldots, n'_{r'}) = \frac{1}{12}(n^2 + 18n - (n'_{r'})^2 + 20n'_{r'} - 19)$. Since the function $-x^2 + 20x$ attains its maximum 100 for $x = 10$, we have $-(n'_{r'})^2 + 20n'_{r'} \leq 100$, and so we get $F(n'_0, n'_1, \ldots, n'_{r'}) \leq \frac{1}{12}(n^2 + 18n + 81)$ and thus

$$\sigma(v) = F(n_0, n_1, \ldots, n_r) \leq F(n'_0, n'_1, \ldots, n'_{r'}) \leq \frac{1}{12}(n^2 + 18n + 81),$$

as desired. □

The bounds in Theorem 14 appear not to be sharp. The graphs constructed in the following section show that the bounds are sharp up to an additive constant.

4.2 Computational Results

This section contains numerous figures, functions and tables summarizing months of computer searches. Once again, I'd like to thank Plantri. For each category of problem (triangulations, 4-connected triangulations, 5-connected triangulations, quadrangulations and 3-connected quadrangulations) there is a function (see formulae (4.1), (4.2), (4.3), (4.4), and (4.5)), which states the minimum status of the structures, along a table, which summarizes the largest minimum status found for a given order in that category and a "Count", telling how many graphs attain the optimal value. Define the minimum status to be $(n-1)\pi(G)$. We use the minimum status rather than the proximity in the tables to remain in the domain of integers. If any Table entry displays a dash, there exists no graphs in that category on the given number of vertices. The number of isomorphism classes that our code searched matches the number of isomorphism classes in [6], [8], [7], [34], [42], verifying that the values that the search provides are in fact maximal. In each figure below, red edges represent the repeating pattern and the black node marks one of the vertices which minimizes the status in that graph and likely maximizes the proximity within the category defined by order and connectivity. Due to fact that few congruence classes of a large modulus fall

into the range, where we can do exhaustive calculations, finding a repeating pattern was difficult by brute force calculations. Once a pattern was established, extensive sampling was conducted in order to test if a better structure could be found, but no such structure ever arose. We conjecture that the bounds presented in this section are optimal.

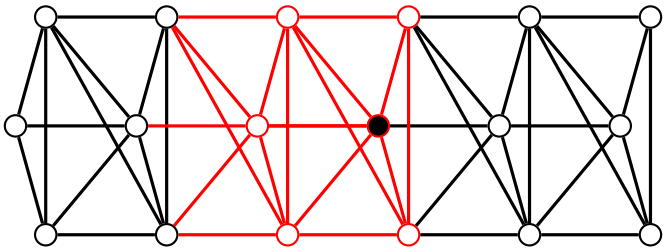

Figure 4.1: A triangulation T_n on $n = 6k$ vertices which is conjectured to maximize the proximity among triangulations of this order.

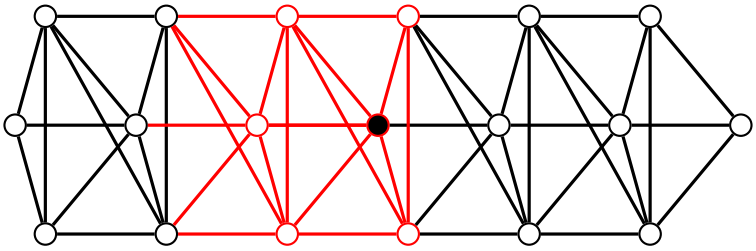

Figure 4.2: A triangulation T_n on $n = 6k + 1$ vertices which is conjectured to maximize the proximity among triangulations of this order.

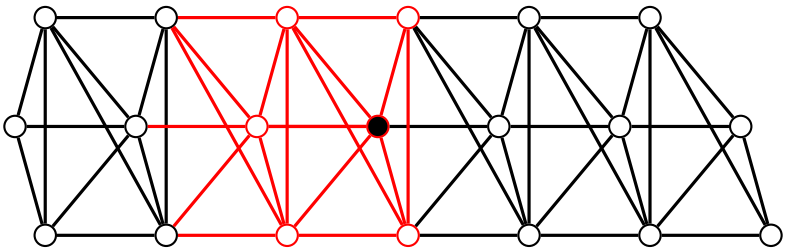

Figure 4.3: A triangulation T_n on $n = 6k + 2$ vertices which is conjectured to maximize the proximity among triangulations of this order.

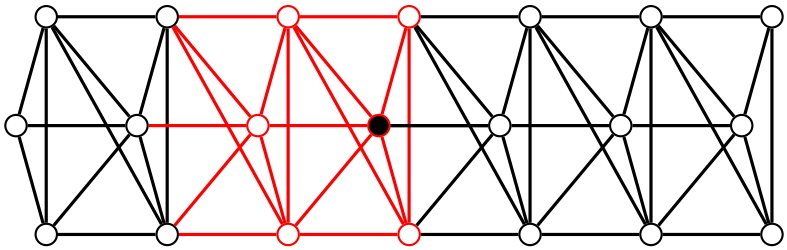

Figure 4.4: A triangulation T_n on $n = 6k + 3$ vertices which is conjectured to maximize the proximity among triangulations of this order.

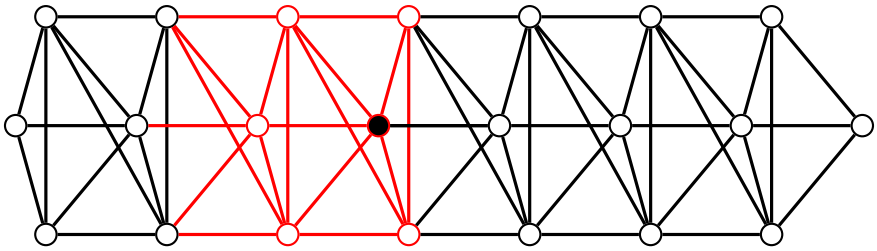

Figure 4.5: A triangulation T_n on $n = 6k + 4$ vertices which is conjectured to maximize the proximity among triangulations of this order.

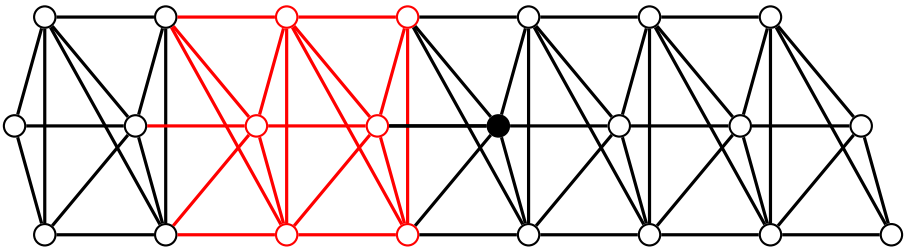

Figure 4.6: A triangulation T_n on $n = 6k + 5$ vertices which is conjectured to maximize the proximity among triangulations of this order.

$$\pi(T_n) = \begin{cases} \frac{n+5}{12} + \frac{5}{12(n-1)} & \text{if } n = 6k \\ \frac{n+5}{12} & \text{if } n = 6k+1 \\ \frac{n+5}{12} + \frac{5}{12(n-1)} & \text{if } n = 6k+2 \\ \frac{n+5}{12} + \frac{2}{3(n-1)} & \text{if } n = 6k+3 \\ \frac{n+5}{12} + \frac{3}{4(n-1)} & \text{if } n = 6k+4 \\ \frac{n+5}{12} + \frac{2}{3(n-1)} & \text{if } n = 6k+5 \end{cases} \tag{4.1}$$

Table 4.1: A summary of the largest minimum status among all triangulations.

Order	Min Status	Count
4	3	1
5	4	1
6	6	1
7	7	2
8	9	2
9	11	1
10	13	1
11	14	44
12	18	1
13	19	2
14	22	1
15	24	2
16	27	2
17	30	1
18	33	3

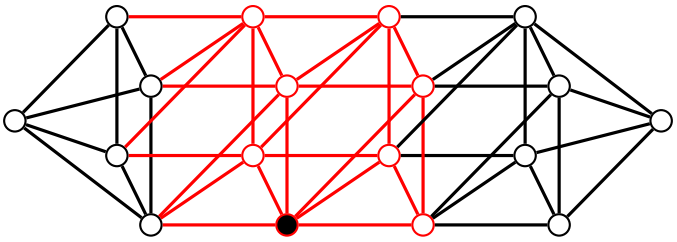

Figure 4.7: A 4-connected triangulation T_n^4 on $n = 8k + 2$ vertices which is conjectured to maximize the proximity among triangulations of this order and connectivity.

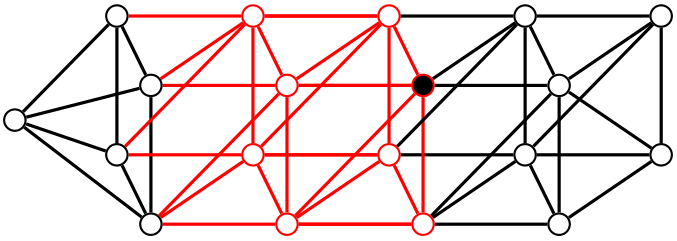

Figure 4.8: A 4-connected triangulation T_n^4 on $n = 8k + 3$ vertices which is conjectured to maximize the proximity among triangulations of this order and connectivity.

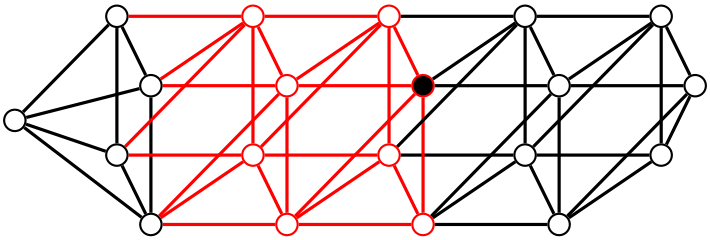

Figure 4.9: A 4-connected triangulation T_n^4 on $n = 8k + 4$ vertices which is conjectured to maximize the proximity among triangulations of this order and connectivity.

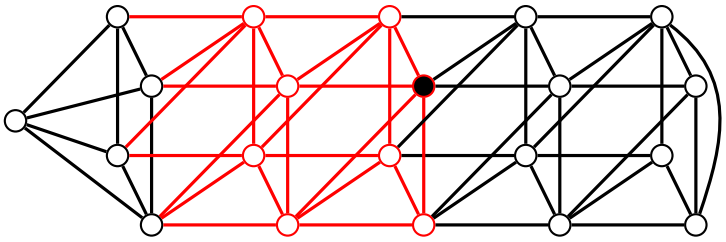

Figure 4.10: A 4-connected triangulation T_n^4 on $n = 8k + 5$ vertices which is conjectured to maximize the proximity among triangulations of this order and connectivity.

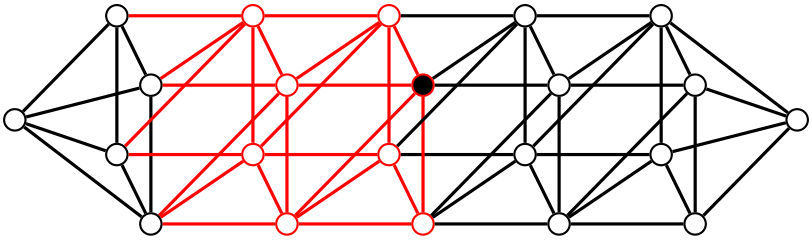

Figure 4.11: A 4-connected triangulation T_n^4 on $n = 8k + 6$ vertices which is conjectured to maximize the proximity among triangulations of this order and connectivity.

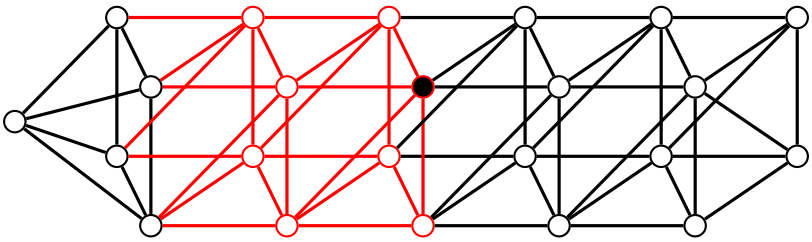

Figure 4.12: A 4-connected triangulation T_n^4 on $n = 8k + 7$ vertices which is conjectured to maximize the proximity among triangulations of this order and connectivity.

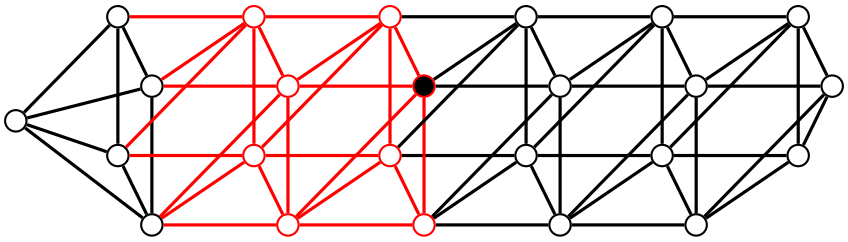

Figure 4.13: A 4-connected triangulation T_n^4 on $n = 8k$ vertices which is conjectured to maximize the proximity among triangulations of this order and connectivity.

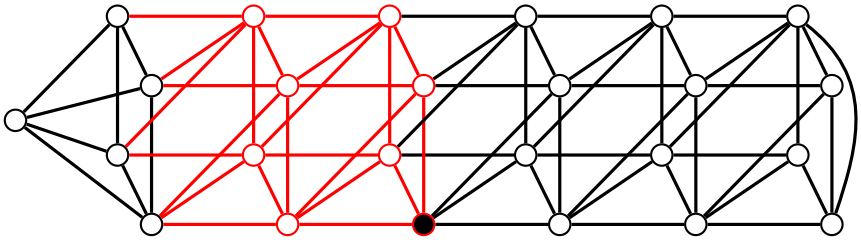

Figure 4.14: A 4-connected triangulation T_n^4 on $n = 8k + 1$ vertices which is conjectured to maximize the proximity among triangulations of this order and connectivity.

$$\pi(T_n^4) = \begin{cases} \frac{n+9}{16} + \frac{21}{16(n-1)} & \text{if } n = 8k+2 \\ \frac{n+9}{16} + \frac{3}{2(n-1)} & \text{if } n = 8k+3 \\ \frac{n+9}{16} + \frac{25}{16(n-1)} & \text{if } n = 8k+4 \\ \frac{n+9}{16} + \frac{3}{2(n-1)} & \text{if } n = 8k+5 \\ \frac{n+9}{16} + \frac{21}{16(n-1)} & \text{if } n = 8k+6 \\ \frac{n+9}{16} + \frac{1}{n-1} & \text{if } n = 8k+7 \\ \frac{n+9}{16} + \frac{25}{16(n-1)} & \text{if } n = 8k \\ \frac{n+9}{16} + \frac{1}{n-1} & \text{if } n = 8k+1 \end{cases} \tag{4.2}$$

Table 4.2: A summary of the largest minimum status among all 4-connected triangulations.

Order	Min Status	Count
6	6	1
7	7	1
8	9	1
9	11	1
10	13	1
11	14	10
12	18	1
13	19	1
14	21	13
15	24	1
16	27	1
17	29	5
18	32	2
19	34	28
20	37	13
21	40	6
22	44	5

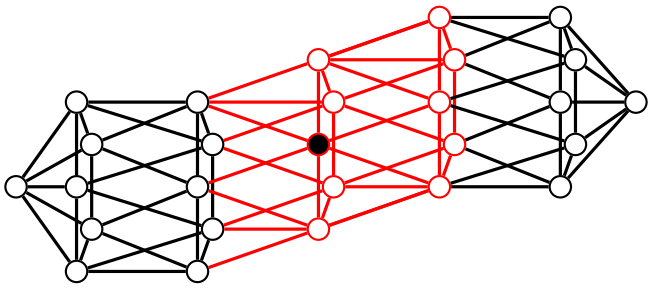

Figure 4.15: A 5-connected triangulation T_n^5 on $n = 10k + 7$ vertices which is conjectured to maximize the proximity among triangulations of this order and connectivity.

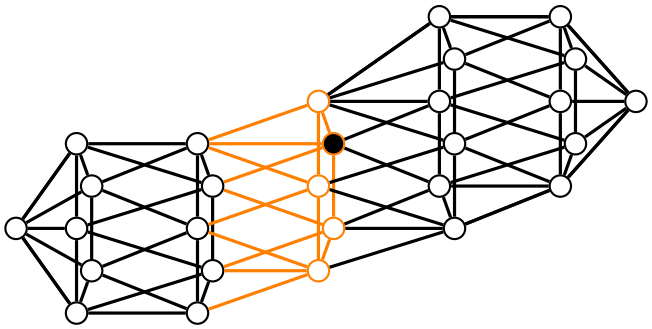

Figure 4.16: A 5-connected triangulation T_n^5 on $n = 10k + 8$ vertices which is conjectured to maximize the proximity among triangulations of this order and connectivity. Here, the orange pattern is to be repeated twice rather than once.

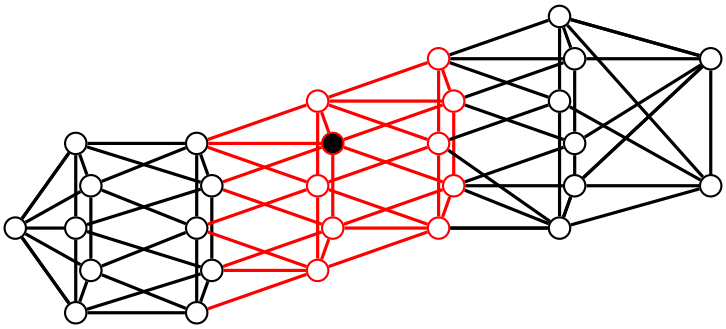

Figure 4.17: A 5-connected triangulation T_n^5 on $n = 10k + 9$ vertices which is conjectured to maximize the proximity among triangulations of this order and connectivity.

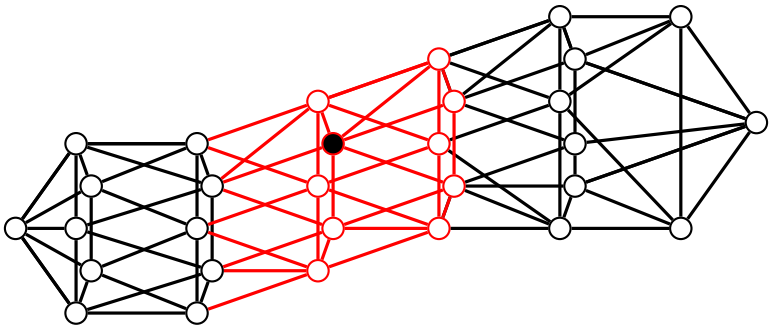

Figure 4.18: A 5-connected triangulation T_n^5 on $n = 10k$ vertices which is conjectured to maximize the proximity among triangulations of this order and connectivity.

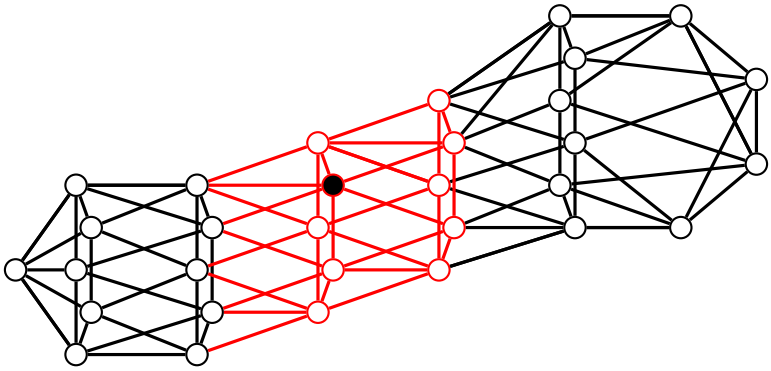

Figure 4.19: A 5-connected triangulation T_n^5 on $n = 10k + 1$ vertices which is conjectured to maximize the proximity among triangulations of this order and connectivity.

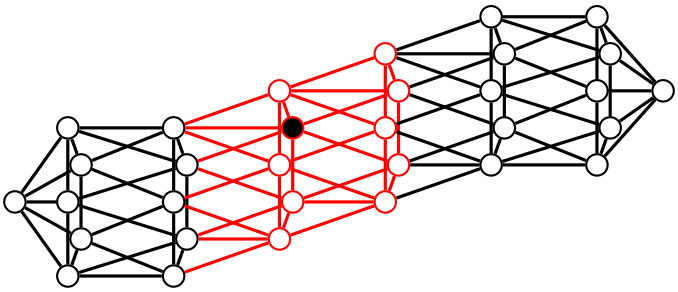

Figure 4.20: A 5-connected triangulation T_n^5 on $n = 10k + 2$ vertices which is conjectured to maximize the proximity among triangulations of this order and connectivity.

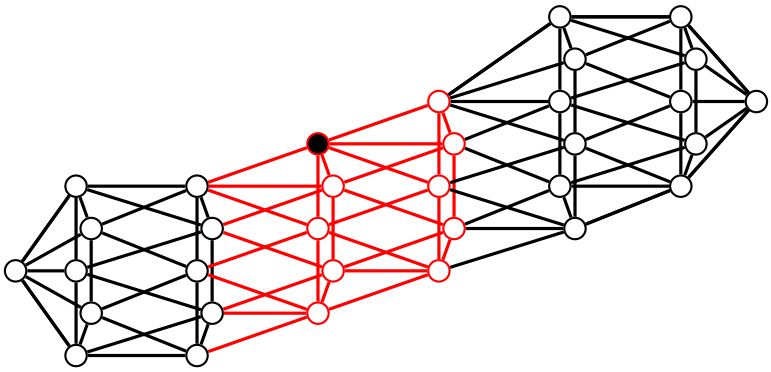

Figure 4.21: A 5-connected triangulation T_n^5 on $n = 10k + 3$ vertices which is conjectured to maximize the proximity among triangulations of this order and connectivity.

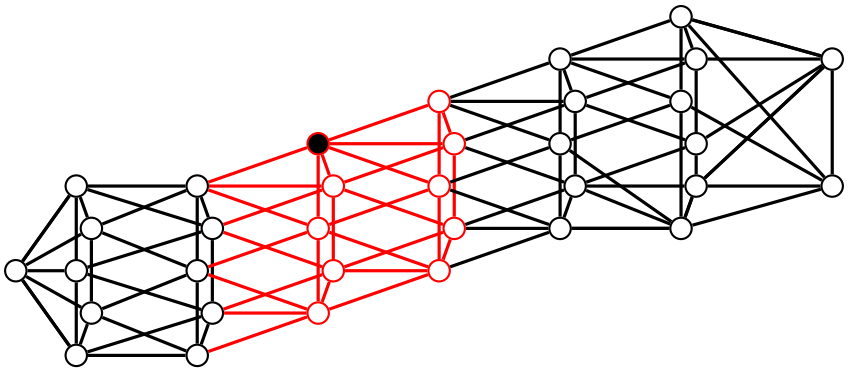

Figure 4.22: A 5-connected triangulation T_n^5 on $n = 10k + 4$ vertices which is conjectured to maximize the proximity among triangulations of this order and connectivity.

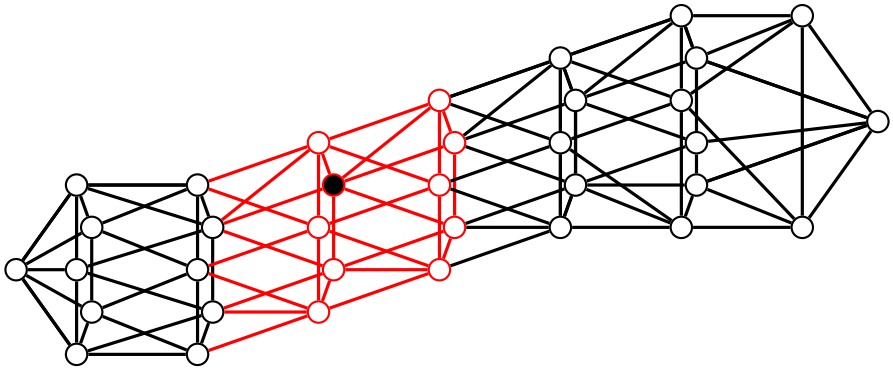

Figure 4.23: A 5-connected triangulation T_n^5 on $n = 10k + 5$ vertices which is conjectured to maximize the proximity among triangulations of this order and connectivity.

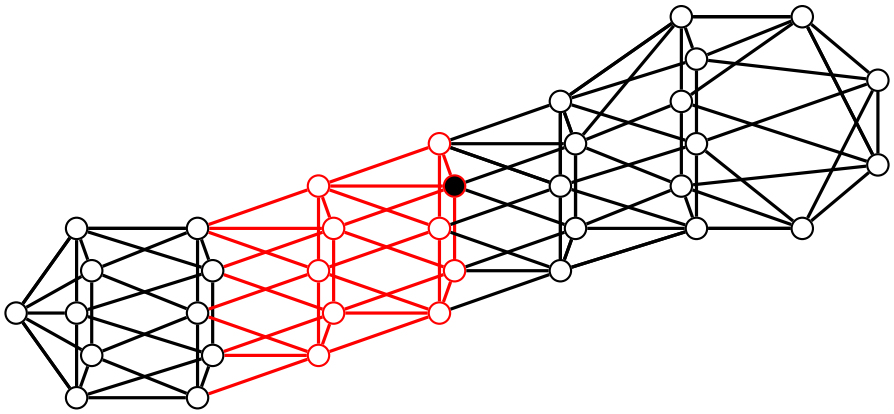

Figure 4.24: A 5-connected triangulation T_n^5 on $n = 10k + 6$ vertices which is conjectured to maximize the proximity among triangulations of this order and connectivity.

$$\pi(T_n^5) = \begin{cases} \frac{n+13}{20} & \text{if } n = 10k+7 \\ \frac{n+13}{20} - \frac{7}{20(n-1)} & \text{if } n = 10k+8 \\ \frac{n+13}{20} - \frac{4}{5(n-1)} & \text{if } n = 10k+9 \\ \frac{n+13}{20} - \frac{7}{20(n-1)} & \text{if } n = 10k \\ \frac{n+13}{20} - \frac{1}{n-1} & \text{if } n = 10k+1 \\ \frac{n+13}{20} - \frac{1}{4(n-1)} & \text{if } n = 10k+2 \\ \frac{n+13}{20} - \frac{8}{5(n-1)} & \text{if } n = 10k+3 \\ \frac{n+13}{20} - \frac{11}{20(n-1)} & \text{if } n = 10k+4 \\ \frac{n+13}{20} - \frac{3}{5(n-1)} & \text{if } n = 10k+5 \\ \frac{n+13}{20} - \frac{3}{4(n-1)} & \text{if } n = 10k+6 \end{cases} \tag{4.3}$$

Table 4.3: A summary of the largest minimum status among all 5-connected triangulations.

Order	Min Status	Count
12	18	1
13	—	0
14	21	1
15	24	1
16	27	1
17	29	1
18	32	1
19	34	4
20	37	6
21	40	3
22	44	2
23	46	5
24	49	19
25	52	18
26	56	3
27	60	3
28	63	3
29	66	2
30	69	59
31	73	2
32	80	1

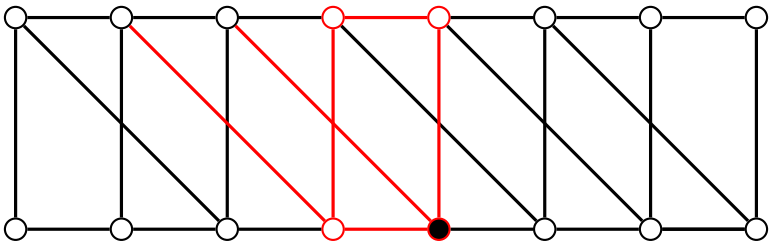

Figure 4.25: A quadrangulation Q_n on $n = 4k$ vertices which is conjectured to maximize the proximity among quadrangulations of this order.

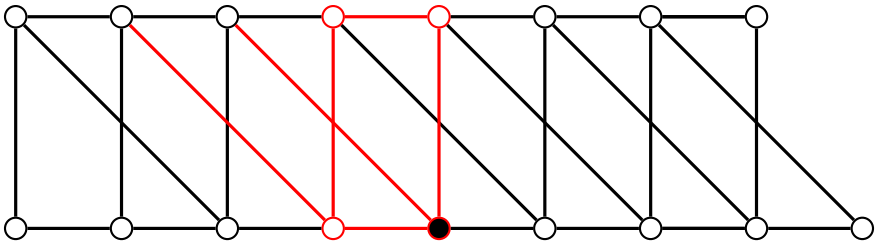

Figure 4.26: A quadrangulation Q_n on $n = 4k + 1$ vertices which is conjectured to maximize the proximity among quadrangulations of this order.

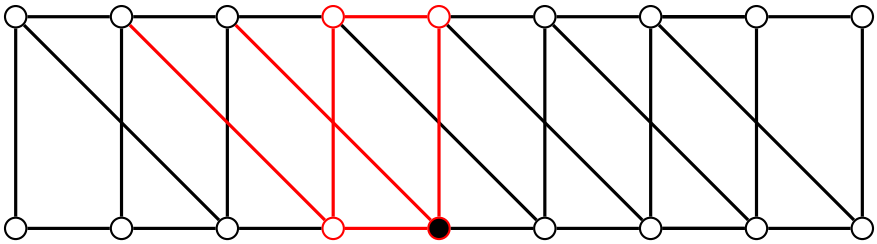

Figure 4.27: A quadrangulation Q_n on $n = 4k + 2$ vertices which is conjectured to maximize the proximity among quadrangulations of this order.

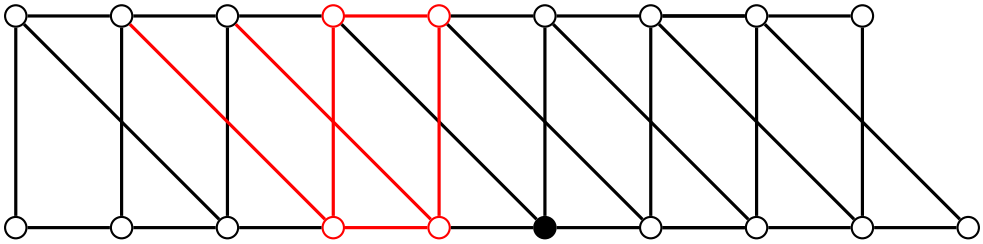

Figure 4.28: A quadrangulation Q_n on $n = 4k + 3$ vertices which is conjectured to maximize the proximity among quadrangulations of this order.

$$\pi(Q_n) = \begin{cases} \frac{n+1}{8} + \frac{17}{8(n-1)} & \text{if } n = 4k \\ \frac{n+1}{8} + \frac{2}{n-1} & \text{if } n = 4k+1 \\ \frac{n+1}{8} + \frac{21}{8(n-1)} & \text{if } n = 4k+2 \\ \frac{n+1}{8} + \frac{2}{n-1} & \text{if } n = 4k+3 \end{cases} \tag{4.4}$$

Table 4.4: A summary of the largest minimum status among all quadrangulations.

Order	Min Status	Count
4	4	1
5	5	1
6	7	1
7	8	2
8	12	1
9	13	1
10	16	1
11	18	1
12	20	19
13	23	1
14	28	1
15	30	1
16	34	2
17	38	1
18	43	1
19	47	1
20	52	2

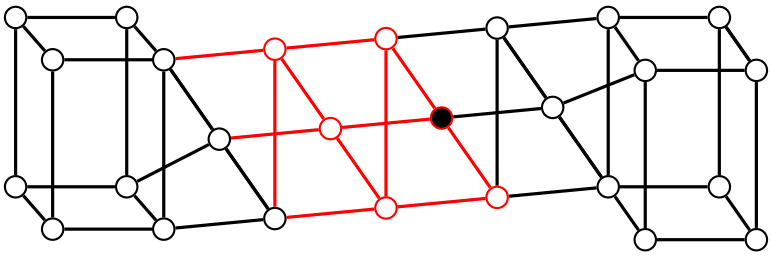

Figure 4.29: A 3-connected quadrangulation Q_n^3 on $n = 6k + 2$ vertices which is conjectured to maximize the proximity among quadrangulations of this order and connectivity.

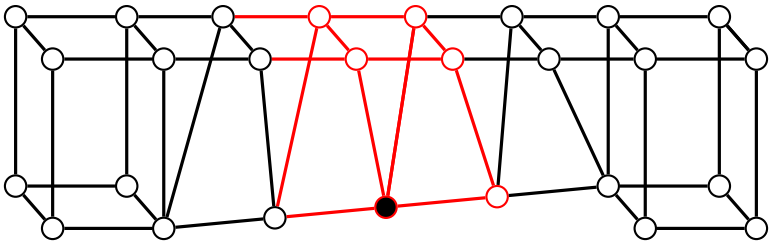

Figure 4.30: A 3-connected quadrangulation Q_n^3 on $n = 6k + 3$ vertices which is conjectured to maximize the proximity among quadrangulations of this order and connectivity.

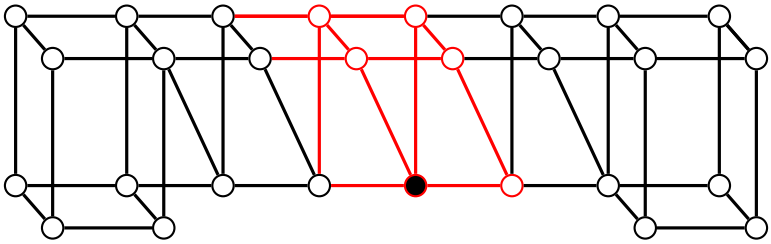

Figure 4.31: A 3-connected quadrangulation Q_n^3 on $n = 6k + 4$ vertices which is conjectured to maximize the proximity among quadrangulations of this order and connectivity.

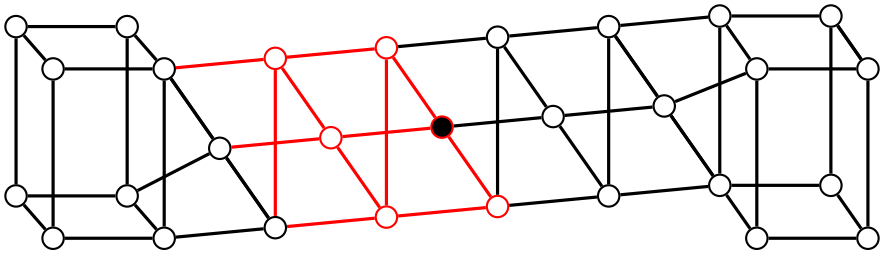

Figure 4.32: A 3-connected quadrangulation Q_n^3 on $n = 6k + 5$ vertices which is conjectured to maximize the proximity among quadrangulations of this order and connectivity.

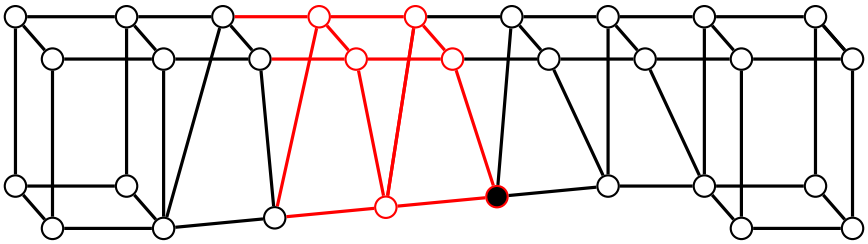

Figure 4.33: A 3-connected quadrangulation Q_n^3 on $n = 6k$ vertices which is conjectured to maximize the proximity.

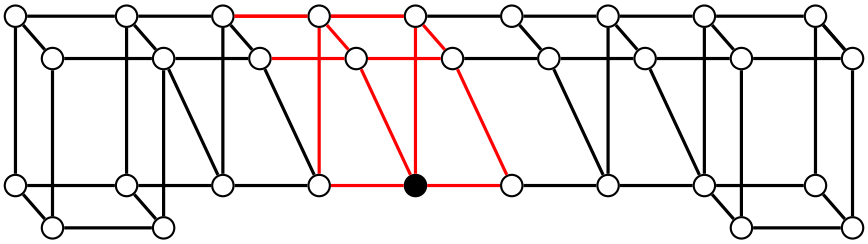

Figure 4.34: A 3-connected quadrangulation Q_n^3 on $n = 6k+1$ vertices which is conjectured to maximize the proximity among quadrangulations of this order and connectivity.

$$\pi(Q_n^3) = \begin{cases} \frac{n+9}{12} - \frac{23}{12(n-1)} & \text{if } n = 6k+2 \\ \frac{n+9}{12} - \frac{4}{n-1} & \text{if } n = 6k+3 \\ \frac{n+9}{12} - \frac{13}{4(n-1)} & \text{if } n = 6k+4 \\ \frac{n+9}{12} - \frac{8}{3(n-1)} & \text{if } n = 6k+5 \\ \frac{n+9}{12} - \frac{13}{4(n-1)} & \text{if } n = 6k \\ \frac{n+9}{12} - \frac{3}{n-1} & \text{if } n = 6k+1 \end{cases} \tag{4.5}$$

Table 4.5: A summary of the largest minimum status among all 3-connected quadrangulations.

Order	Min Status	Count
8	12	1
9	—	0
10	15	1
11	18	1
12	20	2
13	22	1
14	28	1
15	29	1
16	32	4
17	35	1
18	39	1
19	41	3
20	44	23
21	47	7
22	55	1
23	57	1
24	60	16
25	65	1
26	71	1
27	74	3
28	80	2

4.3 Recursion Solutions

This section contains the worked out recursions for the equations of the proximity of the graphs in figures 4.1 through 4.34. Rather than $\pi(G)$, we find the minimum status, $\pi^*(G) = (n-1)\pi(G)$. It should be noted that the variable k will be used to denote the number of layers of each structure. Adding another copy of the red pattern in the figures from the previous section would be equivalent to increasing k by 1. Let p_k denote the status on the given structure with k layers.

First, for triangulations, we have,

$$\pi^*(T_n) = \begin{cases} \frac{n^2}{12} + \frac{n}{3} & \text{if } n = 6k \\ \frac{n^2}{12} + \frac{n}{3} - \frac{5}{12} & \text{if } n = 6k+1 \\ \frac{n^2}{12} + \frac{n}{3} & \text{if } n = 6k+2 \\ \frac{n^2}{12} + \frac{n}{3} + \frac{1}{4} & \text{if } n = 6k+3 \\ \frac{n^2}{12} + \frac{n}{3} + \frac{1}{3} & \text{if } n = 6k+4 \\ \frac{n^2}{12} + \frac{n}{3} + \frac{1}{4} & \text{if } n = 6k+5 \end{cases}$$

Here, the layers represent the repeating consecutive 3-cycles with all edges joining them. If $n = 6k$, we have,

$$p_{k+1} = p_k + 6k - 1 \text{ with } p_2 = 16$$

$$\implies p_k = 3k^2 + 2k \implies p_n = \frac{n^2}{12} + \frac{n}{3}$$

If $n = 6k + 1$, we have,

$$p_{k+1} = p_k + 6k \text{ with } p_2 = 18$$

$$\implies p_k = 3k^2 + 2k \implies p_n = \frac{n^2}{12} + \frac{n}{3} - \frac{5}{12}$$

If $n = 6k + 2$, we have,

$$p_{k+1} = p_k + 6k + 1 \text{ with } p_2 = 21$$

$$\implies p_k = 3k^2 + 2k \implies p_n = \frac{n^2}{12} + \frac{n}{3}$$

If $n = 6k + 3$, we have,

$$p_{k+1} = p_k + 6k + 2 \text{ with } p_2 = 24$$

$$\implies p_k = 3k^2 + 2k \implies p_n = \frac{n^2}{12} + \frac{n}{3} + \frac{1}{4}$$

If $n = 6k + 4$, we have,

$$p_{k+1} = p_k + 6k + 3 \text{ with } p_2 = 27$$

$$\implies p_k = 3k^2 + 2k \implies p_n = \frac{n^2}{12} + \frac{n}{3} + \frac{1}{3}$$

If $n = 6k + 5$, we have,

$$p_{k+1} = p_k + 6k + 4 \text{ with } p_2 = 30$$

$$\implies p_k = 3k^2 + 2k \implies p_n = \frac{n^2}{12} + \frac{n}{3} + \frac{1}{4}$$

Next, we consider 4-connected triangulations,

$$\pi^*(T_n^4) = \begin{cases} \frac{n^2}{16} + \frac{n}{2} + \frac{3}{4} & \text{if } n = 8k + 2 \\ \frac{n^2}{16} + \frac{n}{2} + \frac{15}{16} & \text{if } n = 8k + 3 \\ \frac{n^2}{16} + \frac{n}{2} + 1 & \text{if } n = 8k + 4 \\ \frac{n^2}{16} + \frac{n}{2} + \frac{15}{16} & \text{if } n = 8k + 5 \\ \frac{n^2}{16} + \frac{n}{2} + \frac{3}{4} & \text{if } n = 8k + 6 \\ \frac{n^2}{16} + \frac{n}{2} + \frac{7}{16} & \text{if } n = 8k + 7 \\ \frac{n^2}{16} + \frac{n}{2} + 1 & \text{if } n = 8k + 8 \\ \frac{n^2}{16} + \frac{n}{2} + \frac{7}{16} & \text{if } n = 8k + 9 \end{cases}$$

Here, the layers represent the repeating consecutive 4-cycles with all edges joining them. If $n = 8k + 2$, we have,

$$p_k = p_{k-1} + 8k + 2 \text{ with } p_2 = 30$$

$$\implies p_k = 4k^2 + 6k + 2 \implies p_n = \frac{n^2}{16} + \frac{n}{2} + \frac{3}{4}$$

If $n = 8k + 3$, we have,

$$p_k = p_{k-1} + 8k + 3 \text{ with } p_2 = 33$$

$$\implies p_k = 4k^2 + 7k + 3 \implies p_n = \frac{n^2}{16} + \frac{n}{2} + \frac{15}{16}$$

If $n = 8k + 4$, we have,

$$p_k = p_{k-1} + 8k + 4 \text{ with } p_2 = 36$$

$$\implies p_k = 4k^2 + 8k + 4 \implies p_n = \frac{n^2}{16} + \frac{n}{2} + 1$$

If $n = 8k + 5$, we have,

$$p_k = p_{k-1} + 8k + 5 \text{ with } p_2 = 39$$

$$\implies p_k = 4k^2 + 9k + 5 \implies p_n = \frac{n^2}{16} + \frac{n}{2} + \frac{15}{16}$$

If $n = 8k + 6$, we have,

$$p_k = p_{k-1} + 8k + 6 \text{ with } p_2 = 42$$

$$\implies p_k = 4k^2 + 10k + 6 \implies p_n = \frac{n^2}{16} + \frac{n}{2} + \frac{3}{4}$$

If $n = 8k + 7$, we have,

$$p_k = p_{k-1} + 8k + 7 \text{ with } p_2 = 45$$

$$\implies p_k = 4k^2 + 11k + 7 \implies p_n = \frac{n^2}{16} + \frac{n}{2} + \frac{7}{16}$$

If $n = 8k + 8$, we have,

$$p_k = p_{k-1} + 8k + 8 \text{ with } p_2 = 49$$

$$\implies p_k = 4k^2 + 12k + 9 \implies p_n = \frac{n^2}{16} + \frac{n}{2} + 1$$

If $n = 8k + 9$, we have,

$$p_k = p_{k-1} + 8k + 9 \text{ with } p_2 = 52$$

$$\implies p_k = 4k^2 + 13k + 10 \implies p_n = \frac{n^2}{16} + \frac{n}{2} + \frac{7}{16}$$

Now, we consider 5-connected triangulations,

$$\pi^*(T_n^5) = \begin{cases} \frac{n^2}{20} + \frac{3n}{5} - \frac{13}{20} & \text{if } n = 10k+7 \\ \frac{n^2}{20} + \frac{3n}{5} - 1 & \text{if } n = 10k+8 \\ \frac{n^2}{20} + \frac{3n}{5} - \frac{29}{20} & \text{if } n = 10k+9 \\ \frac{n^2}{20} + \frac{3n}{5} - 1 & \text{if } n = 10k+10 \\ \frac{n^2}{20} + \frac{3n}{5} - \frac{33}{20} & \text{if } n = 10k+11 \\ \frac{n^2}{20} + \frac{3n}{5} - \frac{2}{5} & \text{if } n = 10k+12 \\ \frac{n^2}{20} + \frac{3n}{5} - \frac{9}{4} & \text{if } n = 10k+13 \\ \frac{n^2}{20} + \frac{3n}{5} - \frac{6}{5} & \text{if } n = 10k+14 \\ \frac{n^2}{20} + \frac{3n}{5} - \frac{5}{4} & \text{if } n = 10k+15 \\ \frac{n^2}{20} + \frac{3n}{5} - \frac{7}{5} & \text{if } n = 10k+16 \end{cases}$$

Here, the layers represent the repeating consecutive 5-cycles with all edges joining them. If $n = 10k+7$, we have,

$$p_k = p_{k-1} + 10k + 8 \text{ with } p_3 = 90$$

$$\implies p_k = 5k^2 + 13k + 6 \implies p_n = \frac{n^2}{20} + \frac{3n}{5} - \frac{13}{20}$$

If $n = 10k+8$, we have,

$$p_k = p_{k-1} + 10k + 9 \text{ with } p_3 = 94$$

$$\implies p_k = 5k^2 + 14k + 7 \implies p_n = \frac{n^2}{20} + \frac{3n}{5} - 1$$

If $n = 10k+9$, we have,

$$p_k = p_{k-1} + 10k + 10 \text{ with } p_3 = 98$$

$$\implies p_k = 5k^2 + 15k + 8 \implies p_n = \frac{n^2}{20} + \frac{3n}{5} - \frac{29}{20}$$

If $n = 10k + 10$, we have,

$$p_k = p_{k-1} + 10k + 11 \text{ with } p_3 = 103$$

$$\implies p_k = 5k^2 + 16k + 10 \implies p_n = \frac{n^2}{20} + \frac{3n}{5} - 1$$

If $n = 10k + 11$, we have,

$$p_k = p_{k-1} + 10k + 12 \text{ with } p_3 = 107$$

$$\implies p_k = 5k^2 + 17k + 11 \implies p_n = \frac{n^2}{20} + \frac{3n}{5} - \frac{33}{20}$$

If $n = 10k + 12$, we have,

$$p_k = p_{k-1} + 10k + 13 \text{ with } p_3 = 113$$

$$\implies p_k = 5k^2 + 18k + 14 \implies p_n = \frac{n^2}{20} + \frac{3n}{5} - \frac{2}{5}$$

If $n = 10k + 13$, we have,

$$p_k = p_{k-1} + 10k + 14 \text{ with } p_3 = 116$$

$$\implies p_k = 5k^2 + 19k + 14 \implies p_n = \frac{n^2}{20} + \frac{3n}{5} - \frac{9}{4}$$

If $n = 10k + 14$, we have,

$$p_k = p_{k-1} + 10k + 15 \text{ with } p_3 = 122$$

$$\implies p_k = 5k^2 + 20k + 17 \implies p_n = \frac{n^2}{20} + \frac{3n}{5} - \frac{6}{5}$$

If $n = 10k + 15$, we have,

$$p_k = p_{k-1} + 10k + 16 \text{ with } p_3 = 127$$

$$\implies p_k = 5k^2 + 21k + 19 \implies p_n = \frac{n^2}{20} + \frac{3n}{5} - \frac{5}{4}$$

If $n = 10k + 16$, we have,

$$p_k = p_{k-1} + 10k + 17 \text{ with } p_3 = 132$$

$$\implies p_k = 5k^2 + 22k + 21 \implies p_n = \frac{n^2}{20} + \frac{3n}{5} - \frac{7}{5}$$

Now, we consider equation general quadrangulations,

$$\pi^*(Q_n) = \begin{cases} \frac{n^2}{8} + 2 & \text{if } n = 4k \\ \frac{n^2}{8} + \frac{15}{8} & \text{if } n = 4k+1 \\ \frac{n^2}{8} + \frac{5}{2} & \text{if } n = 4k+2 \\ \frac{n^2}{8} + \frac{15}{8} & \text{if } n = 4k+3 \end{cases}$$

Here, the layers represent the repeating consecutive 1-paths with all edges joining them. If $n = 4k$, we have,

$$p_k = p_{k-1} + 4k - 2 \text{ with } p_3 = 20$$
$$\implies p_k = 2k^2 + 2 \implies p_n = \frac{n^2}{8} + 2$$

If $n = 4k + 1$, we have,

$$p_k = p_{k-1} + 4k - 2 \text{ with } p_3 = 20$$
$$\implies p_k = 2k^2 + 2 \implies p_n = \frac{n^2}{8} + \frac{15}{8}$$

If $n = 4k + 2$, we have,

$$p_k = p_{k-1} + 4k - 2 \text{ with } p_3 = 20$$
$$\implies p_k = 2k^2 + 2 \implies p_n = \frac{n^2}{8} + \frac{5}{2}$$

If $n = 4k + 3$, we have,

$$p_k = p_{k-1} + 4k - 2 \text{ with } p_3 = 20$$
$$\implies p_k = 2k^2 + 2 \implies p_n = \frac{n^2}{8} + \frac{15}{8}$$

Finally, we consider equation 3-connected quadrangulations,

$$\pi^*(Q_n^3) = \begin{cases} \frac{n^2}{12} + \frac{2n}{3} - \frac{8}{3} & \text{if } n = 6k+14 \\ \frac{n^2}{12} + \frac{2n}{3} - \frac{19}{4} & \text{if } n = 6k+15 \\ \frac{n^2}{12} + \frac{2n}{3} - 4 & \text{if } n = 6k+16 \\ \frac{n^2}{12} + \frac{2n}{3} - \frac{41}{12} & \text{if } n = 6k+17 \\ \frac{n^2}{12} + \frac{2n}{3} - 4 & \text{if } n = 6k+18 \\ \frac{n^2}{12} + \frac{2n}{3} - \frac{15}{4} & \text{if } n = 6k+19 \end{cases}$$

Here, the layers represent the repeating consecutive 2-paths with all edges joining them. If $n = 6k + 14$, we have,

$$p_k = p_{k-1} + 6k + 15 \text{ with } p_1 = 44$$

$$\implies p_k = 3k^2 + 18k + 23 \implies p_n = \frac{n^2}{12} + \frac{2n}{3} - \frac{8}{3}$$

If $n = 6k + 15$, we have,

$$p_k = p_{k-1} + 6k + 16 \text{ with } p_1 = 46$$

$$\implies p_k = 3k^2 + 19k + 24 \implies p_n = \frac{n^2}{12} + \frac{2n}{3} - \frac{19}{4}$$

If $n = 6k + 16$, we have,

$$p_k = p_{k-1} + 6k + 17 \text{ with } p_1 = 51$$

$$\implies p_k = 3k^2 + 20k + 28 \implies p_n = \frac{n^2}{12} + \frac{2n}{3} - 4$$

If $n = 6k + 17$, we have,

$$p_k = p_{k-1} + 6k + 18 \text{ with } p_1 = 56$$

$$\implies p_k = 3k^2 + 21k + 32 \implies p_n = \frac{n^2}{12} + \frac{2n}{3} - \frac{41}{12}$$

If $n = 6k + 18$, we have,

$$p_k = p_{k-1} + 6k + 19 \text{ with } p_1 = 60$$

$$\implies p_k = 3k^2 + 22k + 35 \implies p_n = \frac{n^2}{12} + \frac{2n}{3} - 4$$

If $n = 6k + 19$, we have,

$$p_k = p_{k-1} + 6k + 20 \text{ with } p_1 = 65$$

$$\implies p_k = 3k^2 + 23k + 39 \implies p_n = \frac{n^2}{12} + \frac{2n}{3} - \frac{15}{4}$$

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Appendix A

SageMath Triangulations Graph Database

```
#general triangulations which maximize the WI
t4maxwi = Graph([(1, 2), (1, 3), (1, 4), (2, 3), (2, 4), (3, 4)])
t5maxwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 4), (2, 5), (3, 4), (4,
     5)])
t6_1maxwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 3), (2, 5), (2, 6),
    (3, 4), (3, 5), (4, 5), (5, 6)])
t6_2maxwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (3, 4),
    (3, 6), (4, 5), (4, 6), (5, 6)])
t7_1maxwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (1, 7), (2, 3), (2, 6),
    (2, 7), (3, 4), (3, 5), (3, 6), (4, 5), (5, 6), (6, 7)])
t7_2maxwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (1, 7), (2, 3), (2, 4),
    (2, 7), (3, 4), (4, 5), (4, 6), (4, 7), (5, 6), (6, 7)])
t7_3maxwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (1, 7), (2, 3), (2, 4),
    (2, 6), (2, 7), (3, 4), (4, 5), (4, 6), (5, 6), (6, 7)])
t7_4maxwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 3), (2, 6), (2, 7),
    (3, 4), (3, 7), (4, 5), (4, 7), (5, 6), (5, 7), (6, 7)])
t7_5maxwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (2, 7),
    (3, 4), (3, 7), (4, 5), (4, 7), (5, 6), (5, 7), (6, 7)])
t8_1maxwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (1, 7), (2, 3), (2, 6),
    (2, 7), (2, 8), (3, 4), (3, 5), (3, 6), (4, 5), (5, 6), (6, 7), (6, 8), (7, 8)])
t8_2maxwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 3), (2, 6), (2, 7),
    (2, 8), (3, 4), (3, 5), (3, 8), (4, 5), (5, 6), (5, 8), (6, 7), (6, 8), (7, 8)])
t9maxwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (1, 7), (2, 3), (2, 6), (2,
     7), (2, 8), (2, 9), (3, 4), (3, 5), (3, 6), (4, 5), (5, 6), (6, 7), (6, 9), (7,
     8), (7, 9), (8, 9)])
t10maxwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (1, 7), (2, 3), (2, 6),
    (2, 7), (2, 8), (2, 9), (3, 4), (3, 5), (3, 6), (4, 5), (5, 6), (6, 7), (6, 9),
    (7, 8), (7, 9), (7, 10), (8, 9), (8, 10), (9, 10)])
t11maxwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (1, 7), (2, 3), (2, 6),
    (2, 7), (2, 8), (2, 9), (3, 4), (3, 5), (3, 6), (4, 5), (5, 6), (6, 7), (6, 9),
    (7, 8), (7, 9), (7, 10), (8, 9), (8, 10), (8, 11), (9, 10), (9, 11), (10, 11)])
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t12maxwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (1, 7), (2, 3), (2, 6),
    (2, 7), (2, 8), (2, 9), (3, 4), (3, 5), (3, 6), (4, 5), (5, 6), (6, 7), (6, 9),
    (7, 8), (7, 9), (7, 10), (8, 9), (8, 10), (8, 11), (8, 12), (9, 10), (9, 12),
    (10, 11), (10, 12), (11, 12)])
t13maxwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (1, 7), (2, 3), (2, 6),
    (2, 7), (2, 8), (2, 9), (3, 4), (3, 5), (3, 6), (4, 5), (5, 6), (6, 7), (6, 9),
    (7, 8), (7, 9), (7, 10), (8, 9), (8, 10), (8, 11), (8, 12), (9, 10), (9, 12),
    (10, 11), (10, 12), (10, 13), (11, 12), (11, 13), (12, 13)])
t14maxwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (1, 7), (2, 3), (2, 6),
    (2, 7), (2, 8), (2, 9), (3, 4), (3, 5), (3, 6), (4, 5), (5, 6), (6, 7), (6, 9),
    (7, 8), (7, 9), (7, 10), (8, 9), (8, 10), (8, 11), (8, 12), (9, 10), (9, 12),
    (10, 11), (10, 12), (10, 13), (11, 12), (11, 13), (11, 14), (12, 13), (12, 14),
    (13, 14)])
t15maxwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (1, 7), (2, 3), (2, 6),
    (2, 7), (2, 8), (2, 9), (3, 4), (3, 5), (3, 6), (4, 5), (5, 6), (6, 7), (6, 9),
    (7, 8), (7, 9), (7, 10), (8, 9), (8, 10), (8, 11), (8, 12), (9, 10), (9, 12),
    (10, 11), (10, 12), (10, 13), (11, 12), (11, 13), (11, 14), (11, 15), (12, 13),
    (12, 15), (13, 14), (13, 15), (14, 15)])
t16maxwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (1, 7), (2, 3), (2, 6),
    (2, 7), (2, 8), (2, 9), (3, 4), (3, 5), (3, 6), (4, 5), (5, 6), (6, 7), (6, 9),
    (7, 8), (7, 9), (7, 10), (8, 9), (8, 10), (8, 11), (8, 12), (9, 10), (9, 12),
    (10, 11), (10, 12), (10, 13), (11, 12), (11, 13), (11, 14), (11, 15), (12, 13),
    (12, 15), (13, 14), (13, 15), (13, 16), (14, 15), (14, 16), (15, 16)])
t17maxwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (1, 7), (2, 3), (2, 6),
    (2, 7), (2, 8), (2, 9), (3, 4), (3, 5), (3, 6), (4, 5), (5, 6), (6, 7), (6, 9),
    (7, 8), (7, 9), (7, 10), (8, 9), (8, 10), (8, 11), (8, 12), (9, 10), (9, 12),
    (10, 11), (10, 12), (10, 13), (11, 12), (11, 13), (11, 14), (11, 15), (12, 13),
    (12, 15), (13, 14), (13, 15), (13, 16), (14, 15), (14, 16), (14, 17), (15, 16),
    (15, 17), (16, 17)])
t18maxwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (1, 7), (2, 3), (2, 6),
    (2, 7), (2, 8), (2, 9), (3, 4), (3, 5), (3, 6), (4, 5), (5, 6), (6, 7), (6, 9),
    (7, 8), (7, 9), (7, 10), (8, 9), (8, 10), (8, 11), (8, 12), (9, 10), (9, 12),
    (10, 11), (10, 12), (10, 13), (11, 12), (11, 13), (11, 14), (11, 15), (12, 13),
    (12, 15), (13, 14), (13, 15), (13, 16), (14, 15), (14, 16), (14, 17), (14, 18),
    (15, 16), (15, 18), (16, 17), (16, 18), (17, 18)])

#4-connected triangulations which maximize the WI
t4c6maxwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (3, 4),
    (3, 6), (4, 5), (4, 6), (5, 6)])
t4c7maxwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (3, 4),
    (3, 6), (3, 7), (4, 5), (4, 7), (5, 6), (5, 7), (6, 7)])
```

```
t4c8_1maxwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (2, 7),
     (3, 4), (3, 7), (3, 8), (4, 5), (4, 8), (5, 6), (5, 8), (6, 7), (6, 8), (7, 8)
    ])
t4c8_2maxwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (3, 4),
     (3, 6), (3, 7), (3, 8), (4, 5), (4, 8), (5, 6), (5, 7), (5, 8), (6, 7), (7, 8)
    ])
t4c9_1maxwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 3), (2, 6), (2, 7),
     (2, 8), (3, 4), (3, 8), (3, 9), (4, 5), (4, 9), (5, 6), (5, 7), (5, 9), (6, 7),
     (7, 8), (7, 9), (8, 9)])
t4c9_2maxwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (2, 7),
     (3, 4), (3, 7), (3, 8), (3, 9), (4, 5), (4, 9), (5, 6), (5, 9), (6, 7), (6, 8),
     (6, 9), (7, 8), (8, 9)])
t4c9_3maxwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (3, 4),
     (3, 6), (3, 7), (3, 8), (4, 5), (4, 8), (5, 6), (5, 8), (5, 9), (6, 7), (6, 9),
     (7, 8), (7, 9), (8, 9)])
t4c9_4maxwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (3, 4),
     (3, 6), (3, 7), (3, 8), (3, 9), (4, 5), (4, 9), (5, 6), (5, 7), (5, 8), (5, 9),
     (6, 7), (7, 8), (8, 9)])
t4c10maxwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (2, 7),
    (3, 4), (3, 7), (3, 8), (3, 9), (4, 5), (4, 6), (4, 9), (5, 6), (6, 7), (6, 9),
    (6, 10), (7, 8), (7, 10), (8, 9), (8, 10), (9, 10)])
t4c11maxwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (2, 7),
    (2, 8), (3, 4), (3, 8), (3, 9), (3, 10), (4, 5), (4, 6), (4, 7), (4, 10), (5, 6)
    , (6, 7), (7, 8), (7, 10), (7, 11), (8, 9), (8, 11), (9, 10), (9, 11), (10, 11)
    ])
t4c12maxwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 3), (2, 6), (2, 7),
    (2, 8), (2, 9), (3, 4), (3, 9), (3, 10), (3, 11), (4, 5), (4, 7), (4, 8), (4,
    11), (5, 6), (5, 7), (6, 7), (7, 8), (8, 9), (8, 11), (8, 12), (9, 10), (9, 12),
     (10, 11), (10, 12), (11, 12)])
t4c13maxwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (1, 7), (2, 3), (2, 7),
    (2, 8), (2, 9), (2, 10), (3, 4), (3, 10), (3, 11), (3, 12), (4, 5), (4, 8), (4,
    9), (4, 12), (5, 6), (5, 8), (6, 7), (6, 8), (7, 8), (8, 9), (9, 10), (9, 12),
    (9, 13), (10, 11), (10, 13), (11, 12), (11, 13), (12, 13)])
t4c14maxwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (1, 7), (2, 3), (2, 7),
    (2, 8), (2, 9), (2, 10), (3, 4), (3, 10), (3, 11), (3, 12), (4, 5), (4, 8), (4,
    9), (4, 12), (5, 6), (5, 8), (5, 13), (6, 7), (6, 13), (7, 8), (7, 13), (8, 9),
    (8, 13), (9, 10), (9, 12), (9, 14), (10, 11), (10, 14), (11, 12), (11, 14), (12,
     14)])
t4c15maxwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (1, 7), (2, 3), (2, 7),
    (2, 8), (3, 4), (3, 8), (3, 9), (3, 10), (4, 5), (4, 10), (4, 11), (4, 12), (5,
    6), (5, 12), (5, 13), (5, 14), (6, 7), (6, 10), (6, 11), (6, 14), (7, 8), (7, 9)
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    , (7, 10), (8, 9), (9, 10), (10, 11), (11, 12), (11, 14), (11, 15), (12, 13),
    (12, 15), (13, 14), (13, 15), (14, 15)])
t4c16maxwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (1, 7), (2, 3), (2, 7),
    (2, 8), (2, 9), (3, 4), (3, 9), (3, 10), (3, 11), (4, 5), (4, 11), (4, 12), (4,
    13), (5, 6), (5, 13), (5, 14), (5, 15), (6, 7), (6, 11), (6, 12), (6, 15), (7,
    8), (7, 10), (7, 11), (8, 9), (8, 10), (9, 10), (10, 11), (11, 12), (12, 13),
    (12, 15), (12, 16), (13, 14), (13, 16), (14, 15), (14, 16), (15, 16)])
t4c17maxwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (1, 7), (2, 3), (2, 7),
    (2, 8), (2, 9), (2, 10), (3, 4), (3, 10), (3, 11), (3, 12), (4, 5), (4, 12), (4,
     13), (4, 14), (5, 6), (5, 14), (5, 15), (5, 16), (6, 7), (6, 12), (6, 13), (6,
    16), (7, 8), (7, 11), (7, 12), (8, 9), (8, 11), (9, 10), (9, 11), (10, 11), (11,
     12), (12, 13), (13, 14), (13, 16), (13, 17), (14, 15), (14, 17), (15, 16), (15,
     17), (16, 17)])
t4c18maxwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (1, 7), (2, 3), (2, 7),
    (2, 8), (2, 9), (2, 10), (3, 4), (3, 10), (3, 11), (3, 12), (4, 5), (4, 12), (4,
     13), (4, 14), (5, 6), (5, 14), (5, 15), (5, 16), (6, 7), (6, 12), (6, 13), (6,
    16), (7, 8), (7, 11), (7, 12), (8, 9), (8, 11), (8, 17), (9, 10), (9, 17), (10,
    11), (10, 17), (11, 12), (11, 17), (12, 13), (13, 14), (13, 16), (13, 18), (14,
    15), (14, 18), (15, 16), (15, 18), (16, 18)])
t4c19maxwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (1, 7), (2, 3), (2, 7),
    (2, 8), (2, 9), (2, 10), (3, 4), (3, 10), (3, 11), (3, 12), (4, 5), (4, 12), (4,
     13), (4, 14), (5, 6), (5, 14), (5, 15), (5, 16), (6, 7), (6, 12), (6, 13), (6,
    16), (7, 8), (7, 11), (7, 12), (8, 9), (8, 11), (8, 17), (8, 18), (9, 10), (9,
    18), (10, 11), (10, 17), (10, 18), (11, 12), (11, 17), (12, 13), (13, 14), (13,
    16), (13, 19), (14, 15), (14, 19), (15, 16), (15, 19), (16, 19), (17, 18)])
t4c20maxwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (1, 7), (2, 3), (2, 7),
    (2, 8), (2, 9), (2, 10), (3, 4), (3, 10), (3, 11), (3, 12), (4, 5), (4, 12), (4,
     13), (4, 14), (5, 6), (5, 14), (5, 15), (5, 16), (6, 7), (6, 12), (6, 13), (6,
    16), (7, 8), (7, 11), (7, 12), (8, 9), (8, 11), (8, 17), (8, 18), (9, 10), (9,
    18), (9, 19), (10, 11), (10, 17), (10, 19), (11, 12), (11, 17), (12, 13), (13,
    14), (13, 16), (13, 20), (14, 15), (14, 20), (15, 16), (15, 20), (16, 20), (17,
    18), (17, 19), (18, 19)])
t4c21maxwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (1, 7), (2, 3), (2, 7),
    (2, 8), (2, 9), (2, 10), (3, 4), (3, 10), (3, 11), (3, 12), (4, 5), (4, 12), (4,
     13), (4, 14), (5, 6), (5, 14), (5, 15), (5, 16), (6, 7), (6, 12), (6, 13), (6,
    16), (7, 8), (7, 11), (7, 12), (8, 9), (8, 11), (8, 17), (8, 18), (9, 10), (9,
    18), (9, 19), (9, 20), (10, 11), (10, 17), (10, 20), (11, 12), (11, 17), (12,
    13), (13, 14), (13, 16), (13, 21), (14, 15), (14, 21), (15, 16), (15, 21), (16,
    21), (17, 18), (17, 19), (17, 20), (18, 19), (19, 20)])
t4c22maxwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (1, 7), (2, 3), (2, 7),
    (2, 8), (2, 9), (2, 10), (3, 4), (3, 10), (3, 11), (3, 12), (4, 5), (4, 12), (4,
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     13), (4, 14), (5, 6), (5, 14), (5, 15), (5, 16), (6, 7), (6, 12), (6, 13), (6,
    16), (7, 8), (7, 11), (7, 12), (8, 9), (8, 11), (8, 17), (8, 18), (9, 10), (9,
    18), (9, 19), (9, 20), (10, 11), (10, 17), (10, 20), (11, 12), (11, 17), (12,
    13), (13, 14), (13, 16), (13, 21), (14, 15), (14, 21), (15, 16), (15, 21), (16,
    21), (17, 18), (17, 20), (17, 22), (18, 19), (18, 22), (19, 20), (19, 22), (20,
    22)])

#4-connected triangulations which minimize the WI
t4c6minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (3, 4),
    (3, 6), (4, 5), (4, 6), (5, 6)])
t4c7minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (3, 4),
    (3, 6), (3, 7), (4, 5), (4, 7), (5, 6), (5, 7), (6, 7)])
t4c8_1minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (2, 7),
     (3, 4), (3, 7), (3, 8), (4, 5), (4, 8), (5, 6), (5, 8), (6, 7), (6, 8), (7, 8)
    ])
t4c8_2minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (3, 4),
     (3, 6), (3, 7), (3, 8), (4, 5), (4, 8), (5, 6), (5, 7), (5, 8), (6, 7), (7, 8)
    ])
t4c9_1minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 3), (2, 6), (2, 7),
     (2, 8), (3, 4), (3, 8), (3, 9), (4, 5), (4, 9), (5, 6), (5, 7), (5, 9), (6, 7),
     (7, 8), (7, 9), (8, 9)])
t4c9_2minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (2, 7),
     (3, 4), (3, 7), (3, 8), (3, 9), (4, 5), (4, 9), (5, 6), (5, 9), (6, 7), (6, 8),
     (6, 9), (7, 8), (8, 9)])
t4c9_3minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (3, 4),
     (3, 6), (3, 7), (3, 8), (4, 5), (4, 8), (5, 6), (5, 8), (5, 9), (6, 7), (6, 9),
     (7, 8), (7, 9), (8, 9)])
t4c9_4minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (3, 4),
     (3, 6), (3, 7), (3, 8), (3, 9), (4, 5), (4, 9), (5, 6), (5, 7), (5, 8), (5, 9),
     (6, 7), (7, 8), (8, 9)])
t4c10_1minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 3), (2, 6), (2, 7)
    , (2, 8), (3, 4), (3, 8), (4, 5), (4, 8), (4, 9), (4, 10), (5, 6), (5, 10), (6,
    7), (6, 10), (7, 8), (7, 9), (7, 10), (8, 9), (9, 10)])
t4c10_2minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (2, 7)
    , (2, 8), (2, 9), (3, 4), (3, 9), (4, 5), (4, 9), (4, 10), (5, 6), (5, 10), (6,
    7), (6, 10), (7, 8), (7, 10), (8, 9), (8, 10), (9, 10)])
t4c10_3minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (2, 7)
    , (2, 8), (3, 4), (3, 8), (4, 5), (4, 8), (4, 9), (4, 10), (5, 6), (5, 10), (6,
    7), (6, 9), (6, 10), (7, 8), (7, 9), (8, 9), (9, 10)])
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t4c10_4minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (3, 4)
    , (3, 6), (3, 7), (3, 8), (3, 9), (4, 5), (4, 9), (5, 6), (5, 8), (5, 9), (5,
    10), (6, 7), (6, 10), (7, 8), (7, 10), (8, 9), (8, 10)])
t4c10_5minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (3, 4)
    , (3, 6), (3, 7), (3, 8), (3, 9), (3, 10), (4, 5), (4, 10), (5, 6), (5, 7), (5,
    8), (5, 9), (5, 10), (6, 7), (7, 8), (8, 9), (9, 10)])
t4c10_6minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 3), (2, 6), (2, 7)
    , (2, 8), (2, 9), (3, 4), (3, 9), (4, 5), (4, 9), (5, 6), (5, 7), (5, 9), (5,
    10), (6, 7), (7, 8), (7, 10), (8, 9), (8, 10), (9, 10)])
t4c11_1minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 3), (2, 6), (2, 7)
    , (2, 8), (2, 9), (2, 10), (3, 4), (3, 10), (4, 5), (4, 10), (4, 11), (5, 6),
    (5, 7), (5, 11), (6, 7), (7, 8), (7, 11), (8, 9), (8, 11), (9, 10), (9, 11),
    (10, 11)])
t4c11_2minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (2, 7)
    , (2, 8), (2, 9), (3, 4), (3, 9), (4, 5), (4, 9), (4, 10), (4, 11), (5, 6), (5,
    11), (6, 7), (6, 10), (6, 11), (7, 8), (7, 10), (8, 9), (8, 10), (9, 10), (10,
    11)])
t4c11_3minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (2, 7)
    , (2, 8), (2, 9), (2, 10), (3, 4), (3, 10), (4, 5), (4, 10), (4, 11), (5, 6),
    (5, 11), (6, 7), (6, 11), (7, 8), (7, 11), (8, 9), (8, 11), (9, 10), (9, 11),
    (10, 11)])
t4c11_4minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (2, 7)
    , (3, 4), (3, 7), (3, 8), (3, 9), (3, 10), (3, 11), (4, 5), (4, 11), (5, 6), (5,
     10), (5, 11), (6, 7), (6, 8), (6, 9), (6, 10), (7, 8), (8, 9), (9, 10), (10,
    11)])
t4c11_5minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (3, 4)
    , (3, 6), (3, 7), (3, 8), (3, 9), (4, 5), (4, 9), (5, 6), (5, 9), (5, 10), (5,
    11), (6, 7), (6, 11), (7, 8), (7, 10), (7, 11), (8, 9), (8, 10), (9, 10), (10,
    11)])
t4c11_6minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (2, 7)
    , (2, 8), (2, 9), (3, 4), (3, 9), (4, 5), (4, 7), (4, 9), (4, 10), (4, 11), (5,
    6), (5, 11), (6, 7), (6, 11), (7, 8), (7, 10), (7, 11), (8, 9), (8, 10), (9, 10)
    ])
t4c11_7minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (2, 7)
    , (2, 8), (2, 9), (2, 10), (3, 4), (3, 10), (4, 5), (4, 6), (4, 7), (4, 8), (4,
    10), (4, 11), (5, 6), (6, 7), (7, 8), (8, 9), (8, 11), (9, 10), (9, 11), (10,
    11)])
t4c11_8minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (2, 7)
    , (2, 8), (2, 9), (2, 10), (2, 11), (3, 4), (3, 11), (4, 5), (4, 6), (4, 7), (4,
     8), (4, 9), (4, 10), (4, 11), (5, 6), (6, 7), (7, 8), (8, 9), (9, 10), (10, 11)
    ])
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t4c11_9minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (1, 7), (2, 3), (2, 7)
    , (2, 8), (3, 4), (3, 8), (3, 9), (3, 10), (4, 5), (4, 10), (5, 6), (5, 10), (6,
     7), (6, 8), (6, 10), (6, 11), (7, 8), (8, 9), (8, 11), (9, 10), (9, 11), (10,
    11)])
t4c11_10minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 3), (2, 6), (2,
    7), (3, 4), (3, 7), (3, 8), (3, 9), (4, 5), (4, 9), (4, 10), (5, 6), (5, 10),
    (6, 7), (6, 10), (6, 11), (7, 8), (7, 11), (8, 9), (8, 10), (8, 11), (9, 10),
    (10, 11)])
t4c12_1minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 3), (2, 6), (2, 7)
    , (2, 8), (2, 9), (2, 10), (2, 11), (3, 4), (3, 11), (4, 5), (4, 11), (4, 12),
    (5, 6), (5, 7), (5, 12), (6, 7), (7, 8), (7, 12), (8, 9), (8, 12), (9, 10), (9,
    12), (10, 11), (10, 12), (11, 12)])
t4c12_2minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (2, 7)
    , (2, 8), (2, 9), (2, 10), (3, 4), (3, 10), (4, 5), (4, 10), (4, 11), (4, 12),
    (5, 6), (5, 12), (6, 7), (6, 11), (6, 12), (7, 8), (7, 11), (8, 9), (8, 11), (9,
     10), (9, 11), (10, 11), (11, 12)])
t4c12_3minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (2, 7)
    , (2, 8), (2, 9), (2, 10), (2, 11), (3, 4), (3, 11), (4, 5), (4, 11), (4, 12),
    (5, 6), (5, 12), (6, 7), (6, 12), (7, 8), (7, 12), (8, 9), (8, 12), (9, 10), (9,
     12), (10, 11), (10, 12), (11, 12)])
t4c12_4minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (2, 7)
    , (3, 4), (3, 7), (3, 8), (3, 9), (3, 10), (3, 11), (4, 5), (4, 11), (5, 6), (5,
     11), (5, 12), (6, 7), (6, 8), (6, 9), (6, 12), (7, 8), (8, 9), (9, 10), (9, 12)
    , (10, 11), (10, 12), (11, 12)])
t4c12_5minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (2, 7)
    , (3, 4), (3, 7), (3, 8), (3, 9), (3, 10), (3, 11), (3, 12), (4, 5), (4, 12),
    (5, 6), (5, 10), (5, 11), (5, 12), (6, 7), (6, 8), (6, 9), (6, 10), (7, 8), (8,
    9), (9, 10), (10, 11), (11, 12)])
t4c12_6minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (2, 7)
    , (2, 8), (2, 9), (3, 4), (3, 9), (4, 5), (4, 9), (4, 10), (4, 11), (4, 12), (5,
     6), (5, 12), (6, 7), (6, 11), (6, 12), (7, 8), (7, 11), (8, 9), (8, 10), (8,
    11), (9, 10), (10, 11), (11, 12)])
t4c12_7minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (3, 4)
    , (3, 6), (3, 7), (3, 8), (3, 9), (3, 10), (4, 5), (4, 10), (5, 6), (5, 9), (5,
    10), (5, 11), (5, 12), (6, 7), (6, 12), (7, 8), (7, 11), (7, 12), (8, 9), (8,
    11), (9, 10), (9, 11), (11, 12)])
t4c12_8minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (3, 4)
    , (3, 6), (3, 7), (3, 8), (3, 9), (3, 10), (4, 5), (4, 10), (5, 6), (5, 8), (5,
    10), (5, 11), (5, 12), (6, 7), (6, 12), (7, 8), (7, 12), (8, 9), (8, 11), (8,
    12), (9, 10), (9, 11), (10, 11)])
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t4c12_9minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (2, 7)
    , (2, 8), (2, 9), (2, 10), (2, 11), (3, 4), (3, 11), (4, 5), (4, 6), (4, 7), (4,
     8), (4, 9), (4, 11), (4, 12), (5, 6), (6, 7), (7, 8), (8, 9), (9, 10), (9, 12),
     (10, 11), (10, 12), (11, 12)])
t4c12_10minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (2,
    7), (2, 8), (2, 9), (2, 10), (2, 11), (2, 12), (3, 4), (3, 12), (4, 5), (4, 6),
    (4, 7), (4, 8), (4, 9), (4, 10), (4, 11), (4, 12), (5, 6), (6, 7), (7, 8), (8,
    9), (9, 10), (10, 11), (11, 12)])
t4c13_1minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (2, 7)
    , (2, 8), (2, 9), (2, 10), (3, 4), (3, 10), (4, 5), (4, 10), (4, 11), (4, 12),
    (4, 13), (5, 6), (5, 13), (6, 7), (6, 12), (6, 13), (7, 8), (7, 12), (8, 9), (8,
     12), (9, 10), (9, 11), (9, 12), (10, 11), (11, 12), (12, 13)])
t4c13_2minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 3), (2, 6), (2, 7)
    , (2, 8), (2, 9), (2, 10), (2, 11), (2, 12), (3, 4), (3, 12), (4, 5), (4, 12),
    (4, 13), (5, 6), (5, 7), (5, 13), (6, 7), (7, 8), (7, 13), (8, 9), (8, 13), (9,
    10), (9, 13), (10, 11), (10, 13), (11, 12), (11, 13), (12, 13)])
t4c13_3minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (2, 7)
    , (2, 8), (2, 9), (2, 10), (2, 11), (3, 4), (3, 11), (4, 5), (4, 11), (4, 12),
    (4, 13), (5, 6), (5, 13), (6, 7), (6, 12), (6, 13), (7, 8), (7, 12), (8, 9), (8,
     12), (9, 10), (9, 12), (10, 11), (10, 12), (11, 12), (12, 13)])
t4c13_4minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (2, 7)
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     10), (9, 13), (10, 11), (10, 13), (11, 12), (11, 13), (12, 13)])
t4c13_5minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (2, 7)
    , (3, 4), (3, 7), (3, 8), (3, 9), (3, 10), (3, 11), (3, 12), (4, 5), (4, 12),
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    10), (9, 13), (10, 11), (10, 13), (11, 12), (11, 13), (12, 13)])
t4c13_6minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (2, 7)
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    (4, 12), (4, 13), (5, 6), (5, 13), (6, 7), (6, 13), (7, 8), (7, 13), (8, 9), (8,
     12), (8, 13), (9, 10), (9, 12), (10, 11), (10, 12), (12, 13)])
t4c13_7minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (2, 7)
    , (2, 8), (2, 9), (2, 10), (2, 11), (3, 4), (3, 11), (4, 5), (4, 10), (4, 11),
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     13), (9, 10), (9, 12), (9, 13), (10, 11), (10, 12), (12, 13)])
t4c13_8minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (2, 7)
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    (7, 13), (8, 9), (8, 13), (9, 10), (9, 13), (10, 11), (11, 12)])
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t4c13_9minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (2, 7)
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    , (9, 11), (9, 12), (9, 13), (10, 11), (11, 12), (12, 13)])
t4c13_10minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (2,
    7), (2, 8), (2, 9), (2, 10), (3, 4), (3, 10), (4, 5), (4, 6), (4, 10), (4, 11),
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     12), (9, 10), (9, 11), (9, 12), (10, 11), (11, 12), (12, 13)])
t4c13_11minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (2,
    7), (2, 8), (2, 9), (2, 10), (2, 11), (3, 4), (3, 11), (4, 5), (4, 6), (4, 7),
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    10), (9, 12), (9, 13), (10, 11), (10, 12), (11, 12), (12, 13)])
t4c13_12minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (2,
    7), (2, 8), (2, 9), (2, 10), (2, 11), (3, 4), (3, 11), (4, 5), (4, 6), (4, 7),
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    13), (9, 10), (9, 12), (9, 13), (10, 11), (10, 12), (11, 12)])
t4c13_13minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (2,
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    9), (9, 10), (10, 11), (10, 13), (11, 12), (11, 13), (12, 13)])
t4c13_14minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (2,
    7), (2, 8), (2, 9), (2, 10), (2, 11), (2, 12), (2, 13), (3, 4), (3, 13), (4, 5),
     (4, 6), (4, 7), (4, 8), (4, 9), (4, 10), (4, 11), (4, 12), (4, 13), (5, 6), (6,
     7), (7, 8), (8, 9), (9, 10), (10, 11), (11, 12), (12, 13)])
t4c14_1minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (2, 7)
    , (2, 8), (2, 9), (2, 10), (2, 11), (3, 4), (3, 11), (4, 5), (4, 11), (4, 12),
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    (8, 9), (8, 13), (9, 10), (9, 13), (10, 11), (10, 12), (10, 13), (11, 12), (12,
    13), (13, 14)])
t4c14_2minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 3), (2, 6), (2, 7)
    , (2, 8), (2, 9), (2, 10), (2, 11), (2, 12), (2, 13), (3, 4), (3, 13), (4, 5),
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    14), (9, 10), (9, 14), (10, 11), (10, 14), (11, 12), (11, 14), (12, 13), (12,
    14), (13, 14)])
t4c14_3minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (2, 7)
    , (2, 8), (2, 9), (2, 10), (2, 11), (2, 12), (3, 4), (3, 12), (4, 5), (4, 12),
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    (8, 9), (8, 13), (9, 10), (9, 13), (10, 11), (10, 13), (11, 12), (11, 13), (12,
    13), (13, 14)])
t4c14_4minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (2, 7)
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    14), (13, 14)])
t4c14_5minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (2, 7)
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     13), (8, 14), (9, 10), (9, 13), (10, 11), (10, 13), (11, 12), (11, 13), (12,
    13), (13, 14)])
t4c14_6minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (2, 7)
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     14), (7, 8), (8, 9), (9, 10), (9, 14), (10, 11), (10, 14), (11, 12), (11, 14),
    (12, 13)])
t4c14_7minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (2, 7)
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     10), (6, 14), (7, 8), (8, 9), (9, 10), (10, 11), (10, 14), (11, 12), (11, 14),
    (12, 13)])
t4c14_8minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (2, 7)
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    (6, 8), (6, 9), (6, 10), (7, 8), (8, 9), (9, 10), (10, 11), (11, 12), (12, 13),
    (13, 14)])
t4c14_9minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (2, 7)
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    11), (4, 12), (4, 13), (4, 14), (5, 6), (6, 7), (7, 8), (7, 14), (8, 9), (8, 13)
    , (8, 14), (9, 10), (9, 11), (9, 12), (9, 13), (10, 11), (11, 12), (12, 13),
    (13, 14)])
t4c14_10minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (2,
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    14), (9, 10), (9, 11), (9, 12), (9, 13), (9, 14), (10, 11), (11, 12), (12, 13),
    (13, 14)])
t4c14_11minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (3,
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    (8, 9), (8, 12), (8, 13), (8, 14), (9, 10), (9, 12), (10, 11), (10, 12), (12,
    13), (13, 14)])
t4c14_12minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (3,
    4), (3, 6), (3, 7), (3, 8), (3, 9), (3, 10), (3, 11), (3, 12), (4, 5), (4, 12),
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     (13, 14)])
t4c14_13minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (3,
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     (11, 12)])
t4c14_14minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (2,
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     7), (7, 8), (8, 9), (9, 10), (10, 11), (11, 12), (11, 14), (12, 13), (12, 14),
    (13, 14)])
t4c14_15minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (2,
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    (4, 14), (5, 6), (6, 7), (7, 8), (8, 9), (9, 10), (10, 11), (11, 12), (12, 13),
    (13, 14)])
t4c15_1minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (2, 7)
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    (7, 14), (8, 9), (8, 14), (9, 10), (9, 14), (10, 11), (10, 14), (11, 12), (11,
    13), (11, 14), (12, 13), (13, 14), (14, 15)])
t4c15_2minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 3), (2, 6), (2, 7)
    , (2, 8), (2, 9), (2, 10), (2, 11), (2, 12), (2, 13), (2, 14), (3, 4), (3, 14),
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     (12, 15), (13, 14), (13, 15), (14, 15)])
t4c15_3minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (2, 7)
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    14), (12, 13), (12, 14), (13, 14), (14, 15)])
t4c15_4minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (2, 7)
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    , (12, 15), (13, 14), (13, 15), (14, 15)])
t4c15_5minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (2, 7)
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t4c15_6minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (2, 7)
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    (11, 12), (12, 13), (13, 14), (14, 15)])
t4c15_7minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 3), (2, 6), (2, 7)
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    , (11, 12), (11, 14), (12, 13), (14, 15)])
t4c15_8minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (2, 7)
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    (11, 12), (11, 15), (12, 13), (13, 14)])
t4c15_9minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (2, 7)
    , (3, 4), (3, 7), (3, 8), (3, 9), (3, 10), (3, 11), (3, 12), (3, 13), (3, 14),
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    (11, 12), (11, 15), (12, 13), (13, 14)])
t4c15_10minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (2,
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    (11, 12), (12, 13), (13, 14), (14, 15)])
t4c15_11minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (2,
    7), (2, 8), (2, 9), (2, 10), (2, 11), (2, 12), (2, 13), (3, 4), (3, 13), (4, 5),
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    , (11, 12), (11, 14), (12, 13), (14, 15)])
t4c15_12minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (2,
    7), (2, 8), (2, 9), (2, 10), (2, 11), (2, 12), (2, 13), (3, 4), (3, 13), (4, 5),
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    12), (11, 14), (12, 13), (12, 14), (14, 15)])
t4c15_13minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (2,
    7), (2, 8), (2, 9), (2, 10), (2, 11), (2, 12), (2, 13), (2, 14), (3, 4), (3, 14)
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t4c15_14minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (2,
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     (11, 12), (12, 13), (13, 14), (14, 15)])
t4c15_15minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (2,
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    (11, 14), (12, 13), (13, 14), (14, 15)])
t4c15_16minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (2,
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     7), (7, 8), (8, 9), (9, 10), (10, 11), (10, 15), (11, 12), (11, 14), (11, 15),
    (12, 13), (12, 14), (13, 14), (14, 15)])
t4c15_17minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (2,
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    (11, 15), (12, 13), (12, 14), (13, 14)])
t4c15_18minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (2,
    7), (2, 8), (2, 9), (2, 10), (2, 11), (2, 12), (2, 13), (2, 14), (3, 4), (3, 14)
    , (4, 5), (4, 6), (4, 7), (4, 8), (4, 9), (4, 10), (4, 11), (4, 12), (4, 14),
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    (12, 15), (13, 14), (13, 15), (14, 15)])
t4c15_19minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (2,
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    (11, 12), (12, 13), (13, 14), (14, 15)])
t4c16_1minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (2, 7)
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t4c16_2minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 3), (2, 6), (2, 7)
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    (12, 13), (12, 16), (13, 14), (13, 16), (14, 15), (14, 16), (15, 16)])
t4c16_3minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (2, 7)
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    , (11, 15), (12, 13), (12, 15), (13, 14), (13, 15), (14, 15), (15, 16)])
t4c16_4minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (2, 7)
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t4c16_5minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (2, 7)
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    (11, 15), (12, 13), (12, 15), (13, 14), (13, 15), (14, 15), (15, 16)])
t4c16_6minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (2, 7)
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t4c16_7minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (3, 4)
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     11), (10, 16), (11, 12), (11, 16), (12, 13), (13, 14), (14, 15)])
t4c16_10minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (2,
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t4c16_11minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (2,
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t4c16_12minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (2,
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t4c16_13minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (2,
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    , (11, 15), (11, 16), (12, 13), (12, 15), (13, 14), (13, 15), (15, 16)])
t4c16_14minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (2,
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t4c16_15minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (2,
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t4c16_16minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (2,
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t4c16_17minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (2,
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t4c16_18minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (2,
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t4c16_19minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (2,
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    (11, 16), (12, 13), (12, 15), (12, 16), (13, 14), (13, 15), (14, 15)])
t4c16_20minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (2,
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t4c16_21minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (2,
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     10), (10, 11), (11, 12), (12, 13), (13, 14), (14, 15), (15, 16)])
t4c17_1minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (2, 7)
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t4c17_2minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 3), (2, 6), (2, 7)
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    (11, 17), (12, 13), (12, 17), (13, 14), (13, 17), (14, 15), (14, 17), (15, 16),
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t4c17_3minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (2, 7)
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t4c17_4minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (2, 7)
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    , (15, 17), (16, 17)])
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t4c17_5minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (2, 7)
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t4c17_6minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (2, 7)
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    16), (16, 17)])
t4c17_7minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (2, 7)
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    16), (16, 17)])
t4c17_8minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (3, 4)
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    , (16, 17)])
t4c17_9minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 3), (2, 6), (2, 7)
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     15), (16, 17)])
t4c17_10minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (2,
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    15), (15, 16)])
t4c17_11minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (2,
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15), (15, 16)])
t4c17_12minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (2,
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(7, 8), (8, 9), (9, 10), (10, 11), (11, 12), (12, 13), (13, 14), (14, 15), (15,
16), (16, 17)])
t4c17_13minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (2,
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(10, 11), (10, 16), (11, 12), (11, 15), (11, 16), (12, 13), (12, 15), (13, 14),
(15, 16), (16, 17)])
t4c17_14minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (2,
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(10, 17), (11, 12), (11, 17), (12, 13), (12, 16), (12, 17), (13, 14), (13, 16),
(14, 15), (16, 17)])
t4c17_15minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (2,
7), (2, 8), (2, 9), (2, 10), (2, 11), (2, 12), (2, 13), (2, 14), (2, 15), (3, 4)
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 (11, 12), (11, 17), (12, 13), (12, 16), (12, 17), (13, 14), (13, 16), (14, 15),
 (14, 16), (16, 17)])
t4c17_16minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (2,
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11), (10, 17), (11, 12), (11, 17), (12, 13), (12, 17), (13, 14), (13, 17), (14,
15), (14, 17), (15, 16)])
t4c17_17minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (2,
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(10, 11), (10, 16), (10, 17), (11, 12), (11, 16), (12, 13), (12, 16), (13, 14),
(14, 15), (16, 17)])
t4c17_18minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (2,
7), (2, 8), (2, 9), (2, 10), (2, 11), (2, 12), (2, 13), (2, 14), (2, 15), (3, 4)
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(10, 11), (10, 17), (11, 12), (11, 16), (11, 17), (12, 13), (12, 16), (13, 14),
(14, 15), (16, 17)])
t4c17_19minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (2,
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, (9, 10), (9, 17), (10, 11), (10, 17), (11, 12), (11, 17), (12, 13), (13, 14),
(14, 15), (15, 16)])
t4c17_20minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (2,
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(15, 16), (16, 17)])
t4c17_21minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (2,
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(15, 16), (16, 17)])
t4c17_22minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (3,
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(8, 9), (8, 16), (9, 10), (9, 16), (10, 11), (11, 12), (12, 13), (13, 14), (14,
15), (16, 17)])
t4c17_23minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (3,
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(7, 17), (8, 9), (8, 17), (9, 10), (10, 11), (11, 12), (12, 13), (13, 14), (14,
15), (15, 16)])
t4c17_25minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (3,
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    , (16, 17)])
t4c18_1minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (2, 7)
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    (10, 17), (11, 12), (11, 17), (12, 13), (12, 17), (13, 14), (13, 17), (14, 15),
    (14, 16), (14, 17), (15, 16), (16, 17), (17, 18)])
t4c18_2minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 3), (2, 6), (2, 7)
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    (6, 7), (7, 8), (7, 18), (8, 9), (8, 18), (9, 10), (9, 18), (10, 11), (10, 18),
    (11, 12), (11, 18), (12, 13), (12, 18), (13, 14), (13, 18), (14, 15), (14, 18),
    (15, 16), (15, 18), (16, 17), (16, 18), (17, 18)])
t4c18_3minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (2, 7)
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    (14, 17), (15, 16), (15, 17), (16, 17), (17, 18)])
t4c18_4minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (2, 7)
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    (15, 16), (15, 18), (16, 17), (16, 18), (17, 18)])
t4c18_5minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (2, 7)
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    (10, 17), (11, 12), (11, 17), (12, 13), (12, 17), (13, 14), (13, 17), (14, 15),
    (14, 17), (15, 16), (15, 17), (16, 17), (17, 18)])
t4c18_6minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (2, 7)
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t4c18_7minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (2, 7)
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     (14, 15), (15, 16), (16, 17), (17, 18)])
t4c18_8minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (3, 4)
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    (14, 15), (15, 16), (16, 17), (17, 18)])
t4c18_9minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (3, 4)
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t4c18_10minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 3), (2, 6), (2,
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t4c18_11minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (2,
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    13), (13, 14), (14, 15), (15, 16), (16, 17)])
t4c18_12minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (2,
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    13), (13, 14), (14, 15), (15, 16), (16, 17)])
t4c18_13minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (2,
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t4c18_14minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (2,
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t4c18_15minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (2,
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    , (13, 18), (14, 15), (14, 17), (15, 16), (17, 18)])
t4c18_16minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (2,
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t4c18_19minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (2,
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t4c18_23minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (2,
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    (14, 16), (14, 17), (15, 16), (16, 17), (17, 18)])
t4c18_24minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (2,
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     18), (15, 16), (15, 17), (16, 17), (17, 18)])
t4c18_25minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (2,
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t4c18_27minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (2,
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t4c19_10minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (3,
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t4c19_14minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (2,
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t4c19_15minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (2,
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t4c19_16minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (2,
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    16), (3, 4), (3, 16), (4, 5), (4, 14), (4, 15), (4, 16), (4, 17), (4, 18), (4,
    19), (5, 6), (5, 19), (6, 7), (6, 18), (6, 19), (7, 8), (7, 18), (8, 9), (8, 18)
    , (9, 10), (9, 18), (10, 11), (10, 18), (11, 12), (11, 18), (12, 13), (12, 18),
    (13, 14), (13, 17), (13, 18), (14, 15), (14, 17), (15, 16), (17, 18), (18, 19)])
t4c19_17minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (2,
    7), (2, 8), (2, 9), (2, 10), (2, 11), (2, 12), (2, 13), (2, 14), (2, 15), (2,
    16), (2, 17), (3, 4), (3, 17), (4, 5), (4, 15), (4, 16), (4, 17), (4, 18), (4,
    19), (5, 6), (5, 19), (6, 7), (6, 19), (7, 8), (7, 19), (8, 9), (8, 19), (9, 10)
    , (9, 19), (10, 11), (10, 19), (11, 12), (11, 19), (12, 13), (12, 19), (13, 14),
     (13, 19), (14, 15), (14, 18), (14, 19), (15, 16), (15, 18), (16, 17), (18, 19)
    ])
t4c19_18minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (2,
    7), (2, 8), (2, 9), (2, 10), (2, 11), (2, 12), (2, 13), (2, 14), (2, 15), (2,
    16), (2, 17), (3, 4), (3, 17), (4, 5), (4, 16), (4, 17), (4, 18), (4, 19), (5,
    6), (5, 19), (6, 7), (6, 19), (7, 8), (7, 19), (8, 9), (8, 19), (9, 10), (9, 19)
    , (10, 11), (10, 19), (11, 12), (11, 19), (12, 13), (12, 19), (13, 14), (13, 19)
    , (14, 15), (14, 18), (14, 19), (15, 16), (15, 18), (16, 17), (16, 18), (18, 19)
    ])
t4c19_19minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (2,
    7), (2, 8), (2, 9), (2, 10), (2, 11), (2, 12), (2, 13), (2, 14), (2, 15), (2,
    16), (2, 17), (2, 18), (3, 4), (3, 18), (4, 5), (4, 16), (4, 17), (4, 18), (4,
    19), (5, 6), (5, 19), (6, 7), (6, 19), (7, 8), (7, 19), (8, 9), (8, 19), (9, 10)
    , (9, 19), (10, 11), (10, 19), (11, 12), (11, 19), (12, 13), (12, 19), (13, 14),
     (13, 19), (14, 15), (14, 19), (15, 16), (15, 19), (16, 17), (16, 19), (17, 18)
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t4c19_20minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 3), (2, 6), (2,
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    16), (2, 17), (3, 4), (3, 17), (4, 5), (4, 13), (4, 14), (4, 15), (4, 16), (4,
    17), (4, 18), (4, 19), (5, 6), (5, 19), (6, 7), (6, 19), (7, 8), (7, 19), (8, 9)
    , (8, 19), (9, 10), (9, 19), (10, 11), (10, 19), (11, 12), (11, 19), (12, 13),
    (12, 18), (12, 19), (13, 14), (13, 18), (14, 15), (15, 16), (16, 17), (18, 19)])
t4c19_21minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (2,
    7), (3, 4), (3, 7), (3, 8), (3, 9), (3, 10), (3, 11), (3, 12), (3, 13), (3, 14),
     (3, 15), (3, 16), (3, 17), (3, 18), (4, 5), (4, 18), (5, 6), (5, 13), (5, 14),
    (5, 15), (5, 16), (5, 17), (5, 18), (5, 19), (6, 7), (6, 8), (6, 9), (6, 10),
    (6, 11), (6, 19), (7, 8), (8, 9), (9, 10), (10, 11), (11, 12), (11, 19), (12,
    13), (12, 19), (13, 14), (13, 19), (14, 15), (15, 16), (16, 17), (17, 18)])
t4c19_22minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (2,
    7), (3, 4), (3, 7), (3, 8), (3, 9), (3, 10), (3, 11), (3, 12), (3, 13), (3, 14),
     (3, 15), (3, 16), (3, 17), (3, 18), (4, 5), (4, 18), (5, 6), (5, 13), (5, 14),
    (5, 15), (5, 16), (5, 17), (5, 18), (5, 19), (6, 7), (6, 8), (6, 9), (6, 10),
    (6, 11), (6, 12), (6, 19), (7, 8), (8, 9), (9, 10), (10, 11), (11, 12), (12, 13)
    , (12, 19), (13, 14), (13, 19), (14, 15), (15, 16), (16, 17), (17, 18)])
t4c19_23minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (2,
    7), (3, 4), (3, 7), (3, 8), (3, 9), (3, 10), (3, 11), (3, 12), (3, 13), (3, 14),
     (3, 15), (3, 16), (3, 17), (3, 18), (3, 19), (4, 5), (4, 19), (5, 6), (5, 12),
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    (6, 9), (6, 10), (6, 11), (6, 12), (7, 8), (8, 9), (9, 10), (10, 11), (11, 12),
    (12, 13), (13, 14), (14, 15), (15, 16), (16, 17), (17, 18), (18, 19)])
t4c19_24minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (2,
    7), (2, 8), (2, 9), (2, 10), (2, 11), (2, 12), (2, 13), (2, 14), (2, 15), (2,
    16), (2, 17), (3, 4), (3, 17), (4, 5), (4, 13), (4, 14), (4, 15), (4, 16), (4,
    17), (4, 18), (4, 19), (5, 6), (5, 19), (6, 7), (6, 19), (7, 8), (7, 19), (8, 9)
    , (8, 19), (9, 10), (9, 19), (10, 11), (10, 19), (11, 12), (11, 19), (12, 13),
    (12, 18), (12, 19), (13, 14), (13, 18), (14, 15), (15, 16), (16, 17), (18, 19)])
t4c19_25minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (2,
    7), (2, 8), (2, 9), (2, 10), (2, 11), (2, 12), (2, 13), (2, 14), (2, 15), (2,
    16), (2, 17), (3, 4), (3, 17), (4, 5), (4, 14), (4, 15), (4, 16), (4, 17), (4,
    18), (4, 19), (5, 6), (5, 19), (6, 7), (6, 19), (7, 8), (7, 19), (8, 9), (8, 19)
    , (9, 10), (9, 19), (10, 11), (10, 19), (11, 12), (11, 19), (12, 13), (12, 18),
    (12, 19), (13, 14), (13, 18), (14, 15), (14, 18), (15, 16), (16, 17), (18, 19)])
t4c19_26minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (2,
    7), (2, 8), (2, 9), (2, 10), (2, 11), (2, 12), (2, 13), (2, 14), (2, 15), (2,
    16), (2, 17), (2, 18), (3, 4), (3, 18), (4, 5), (4, 14), (4, 15), (4, 16), (4,
    17), (4, 18), (4, 19), (5, 6), (5, 19), (6, 7), (6, 19), (7, 8), (7, 19), (8, 9)
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    (12, 19), (13, 14), (13, 19), (14, 15), (14, 19), (15, 16), (16, 17), (17, 18)])
t4c19_27minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (2,
    7), (2, 8), (2, 9), (2, 10), (2, 11), (2, 12), (3, 4), (3, 12), (3, 13), (4, 5),
     (4, 6), (4, 7), (4, 8), (4, 9), (4, 10), (4, 11), (4, 13), (4, 14), (4, 15),
    (4, 16), (4, 17), (4, 18), (4, 19), (5, 6), (6, 7), (7, 8), (8, 9), (9, 10),
    (10, 11), (11, 12), (11, 19), (12, 13), (12, 14), (12, 15), (12, 16), (12, 17),
    (12, 18), (12, 19), (13, 14), (14, 15), (15, 16), (16, 17), (17, 18), (18, 19)])
t4c19_28minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (2,
    7), (2, 8), (2, 9), (2, 10), (2, 11), (2, 12), (2, 13), (2, 14), (2, 15), (2,
    16), (3, 4), (3, 16), (4, 5), (4, 6), (4, 7), (4, 8), (4, 9), (4, 10), (4, 11),
    (4, 12), (4, 16), (4, 17), (4, 18), (4, 19), (5, 6), (6, 7), (7, 8), (8, 9), (9,
     10), (10, 11), (11, 12), (12, 13), (12, 19), (13, 14), (13, 18), (13, 19), (14,
     15), (14, 18), (15, 16), (15, 17), (15, 18), (16, 17), (17, 18), (18, 19)])
t4c19_29minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (3,
    4), (3, 6), (3, 7), (3, 8), (3, 9), (3, 10), (3, 11), (3, 12), (3, 13), (3, 14),
     (3, 15), (3, 16), (3, 17), (4, 5), (4, 17), (5, 6), (5, 9), (5, 10), (5, 11),
    (5, 12), (5, 13), (5, 14), (5, 15), (5, 16), (5, 17), (5, 18), (5, 19), (6, 7),
    (6, 19), (7, 8), (7, 18), (7, 19), (8, 9), (8, 18), (9, 10), (9, 18), (10, 11),
    (11, 12), (12, 13), (13, 14), (14, 15), (15, 16), (16, 17), (18, 19)])
t4c19_30minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (3,
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     (3, 15), (3, 16), (3, 17), (4, 5), (4, 17), (5, 6), (5, 8), (5, 10), (5, 11),
    (5, 12), (5, 13), (5, 14), (5, 15), (5, 16), (5, 17), (5, 18), (5, 19), (6, 7),
    (6, 19), (7, 8), (7, 19), (8, 9), (8, 18), (8, 19), (9, 10), (9, 18), (10, 11),
    (10, 18), (11, 12), (12, 13), (13, 14), (14, 15), (15, 16), (16, 17)])
t4c19_31minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (3,
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     (3, 15), (3, 16), (3, 17), (3, 18), (4, 5), (4, 18), (5, 6), (5, 8), (5, 9),
    (5, 10), (5, 11), (5, 12), (5, 13), (5, 14), (5, 15), (5, 16), (5, 17), (5, 18),
     (5, 19), (6, 7), (6, 19), (7, 8), (7, 19), (8, 9), (8, 19), (9, 10), (10, 11),
    (11, 12), (12, 13), (13, 14), (14, 15), (15, 16), (16, 17), (17, 18)])
t4c19_32minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (3,
    4), (3, 6), (3, 7), (3, 8), (3, 9), (3, 10), (3, 11), (3, 12), (3, 13), (3, 14),
     (3, 15), (3, 16), (3, 17), (3, 18), (3, 19), (4, 5), (4, 19), (5, 6), (5, 7),
    (5, 8), (5, 9), (5, 10), (5, 11), (5, 12), (5, 13), (5, 14), (5, 15), (5, 16),
    (5, 17), (5, 18), (5, 19), (6, 7), (7, 8), (8, 9), (9, 10), (10, 11), (11, 12),
    (12, 13), (13, 14), (14, 15), (15, 16), (16, 17), (17, 18), (18, 19)])
t4c20_1minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (2, 7)
    , (2, 8), (2, 9), (2, 10), (2, 11), (2, 12), (2, 13), (2, 14), (2, 15), (2, 16),
     (2, 17), (3, 4), (3, 17), (4, 5), (4, 17), (4, 18), (4, 19), (4, 20), (5, 6),
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    (9, 19), (10, 11), (10, 19), (11, 12), (11, 19), (12, 13), (12, 19), (13, 14),
    (13, 19), (14, 15), (14, 19), (15, 16), (15, 19), (16, 17), (16, 18), (16, 19),
    (17, 18), (18, 19), (19, 20)])
t4c20_2minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 3), (2, 6), (2, 7)
    , (2, 8), (2, 9), (2, 10), (2, 11), (2, 12), (2, 13), (2, 14), (2, 15), (2, 16),
     (2, 17), (2, 18), (2, 19), (3, 4), (3, 19), (4, 5), (4, 19), (4, 20), (5, 6),
    (5, 7), (5, 20), (6, 7), (7, 8), (7, 20), (8, 9), (8, 20), (9, 10), (9, 20),
    (10, 11), (10, 20), (11, 12), (11, 20), (12, 13), (12, 20), (13, 14), (13, 20),
    (14, 15), (14, 20), (15, 16), (15, 20), (16, 17), (16, 20), (17, 18), (17, 20),
    (18, 19), (18, 20), (19, 20)])
t4c20_3minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (2, 7)
    , (2, 8), (2, 9), (2, 10), (2, 11), (2, 12), (2, 13), (2, 14), (2, 15), (2, 16),
     (2, 17), (2, 18), (3, 4), (3, 18), (4, 5), (4, 18), (4, 19), (4, 20), (5, 6),
    (5, 20), (6, 7), (6, 19), (6, 20), (7, 8), (7, 19), (8, 9), (8, 19), (9, 10),
    (9, 19), (10, 11), (10, 19), (11, 12), (11, 19), (12, 13), (12, 19), (13, 14),
    (13, 19), (14, 15), (14, 19), (15, 16), (15, 19), (16, 17), (16, 19), (17, 18),
    (17, 19), (18, 19), (19, 20)])
t4c20_4minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (2, 7)
    , (2, 8), (2, 9), (2, 10), (2, 11), (2, 12), (2, 13), (2, 14), (2, 15), (2, 16),
     (2, 17), (2, 18), (2, 19), (3, 4), (3, 19), (4, 5), (4, 19), (4, 20), (5, 6),
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    (14, 15), (14, 20), (15, 16), (15, 20), (16, 17), (16, 20), (17, 18), (17, 20),
    (18, 19), (18, 20), (19, 20)])
t4c20_5minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (2, 7)
    , (2, 8), (2, 9), (2, 10), (2, 11), (2, 12), (2, 13), (2, 14), (2, 15), (2, 16),
     (2, 17), (2, 18), (3, 4), (3, 18), (4, 5), (4, 18), (4, 19), (4, 20), (5, 6),
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    (13, 19), (14, 15), (14, 19), (15, 16), (15, 19), (16, 17), (16, 19), (17, 18),
    (17, 19), (18, 19), (19, 20)])
t4c20_6minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (2, 7)
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     (2, 17), (2, 18), (3, 4), (3, 18), (4, 5), (4, 18), (4, 19), (4, 20), (5, 6),
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    (10, 11), (10, 20), (11, 12), (11, 20), (12, 13), (12, 20), (13, 14), (13, 20),
    (14, 15), (14, 19), (14, 20), (15, 16), (15, 19), (16, 17), (16, 19), (17, 18),
    (17, 19), (18, 19), (19, 20)])
t4c20_7minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (2, 7)
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(10, 11), (10, 19), (10, 20), (11, 12), (11, 19), (12, 13), (12, 19), (13, 14),
(13, 19), (14, 15), (14, 19), (15, 16), (15, 19), (16, 17), (16, 19), (17, 18),
(17, 19), (18, 19), (19, 20)])
t4c20_8minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (2, 7)
, (2, 8), (3, 4), (3, 8), (3, 9), (4, 5), (4, 6), (4, 9), (4, 10), (4, 11), (4,
12), (4, 13), (4, 14), (4, 15), (4, 16), (4, 17), (4, 18), (4, 19), (4, 20), (5,
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 11), (11, 12), (12, 13), (13, 14), (14, 15), (15, 16), (16, 17), (17, 18), (18,
 19), (19, 20)])
t4c20_9minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (2, 7)
, (2, 8), (2, 9), (2, 10), (2, 11), (2, 12), (2, 13), (2, 14), (2, 15), (2, 16),
 (2, 17), (2, 18), (3, 4), (3, 18), (4, 5), (4, 6), (4, 7), (4, 8), (4, 9), (4,
10), (4, 18), (4, 19), (4, 20), (5, 6), (6, 7), (7, 8), (8, 9), (9, 10), (10,
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15), (14, 19), (14, 20), (15, 16), (15, 19), (16, 17), (16, 19), (17, 18), (17,
19), (18, 19), (19, 20)])
t4c20_10minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (3,
4), (3, 6), (3, 7), (3, 8), (3, 9), (3, 10), (3, 11), (3, 12), (3, 13), (3, 14),
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(14, 15), (14, 19), (14, 20), (15, 16), (15, 19), (16, 17), (16, 19), (17, 18),
(17, 19), (18, 19), (19, 20)])
t4c20_11minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (2,
7), (2, 8), (2, 9), (2, 10), (2, 11), (3, 4), (3, 11), (4, 5), (4, 6), (4, 9),
(4, 10), (4, 11), (4, 12), (4, 13), (4, 14), (4, 15), (4, 16), (4, 17), (4, 18),
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(7, 20), (8, 9), (8, 12), (8, 13), (8, 14), (8, 15), (8, 16), (8, 17), (9, 10),
(9, 12), (10, 11), (12, 13), (13, 14), (14, 15), (15, 16), (16, 17), (17, 18),
(18, 19), (19, 20)])
t4c20_12minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (2,
7), (2, 8), (2, 9), (2, 10), (2, 11), (2, 12), (2, 13), (2, 14), (2, 15), (2,
16), (2, 17), (2, 18), (3, 4), (3, 18), (4, 5), (4, 6), (4, 7), (4, 18), (4, 19)
, (4, 20), (5, 6), (6, 7), (7, 8), (7, 20), (8, 9), (8, 20), (9, 10), (9, 20),
(10, 11), (10, 20), (11, 12), (11, 20), (12, 13), (12, 19), (12, 20), (13, 14),
(13, 19), (14, 15), (14, 19), (15, 16), (15, 19), (16, 17), (16, 19), (17, 18),
(17, 19), (18, 19), (19, 20)])
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t4c20_13minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 3), (2, 6), (2,
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    16), (2, 17), (2, 18), (3, 4), (3, 18), (4, 5), (4, 11), (4, 12), (4, 13), (4,
    14), (4, 15), (4, 16), (4, 17), (4, 18), (4, 19), (4, 20), (5, 6), (5, 20), (6,
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    19), (10, 20), (11, 12), (11, 19), (12, 13), (13, 14), (14, 15), (15, 16), (16,
    17), (17, 18), (19, 20)])
t4c20_14minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (2,
    7), (3, 4), (3, 7), (3, 8), (3, 9), (3, 10), (3, 11), (3, 12), (3, 13), (3, 14),
     (3, 15), (3, 16), (3, 17), (3, 18), (3, 19), (4, 5), (4, 19), (5, 6), (5, 11),
    (5, 12), (5, 13), (5, 14), (5, 15), (5, 16), (5, 17), (5, 18), (5, 19), (5, 20),
     (6, 7), (6, 8), (6, 9), (6, 20), (7, 8), (8, 9), (9, 10), (9, 20), (10, 11),
    (10, 20), (11, 12), (11, 20), (12, 13), (13, 14), (14, 15), (15, 16), (16, 17),
    (17, 18), (18, 19)])
t4c20_15minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (2,
    7), (3, 4), (3, 7), (3, 8), (3, 9), (3, 10), (3, 11), (3, 12), (3, 13), (3, 14),
     (3, 15), (3, 16), (3, 17), (3, 18), (3, 19), (4, 5), (4, 19), (5, 6), (5, 11),
    (5, 12), (5, 13), (5, 14), (5, 15), (5, 16), (5, 17), (5, 18), (5, 19), (5, 20),
     (6, 7), (6, 8), (6, 9), (6, 10), (6, 20), (7, 8), (8, 9), (9, 10), (10, 11),
    (10, 20), (11, 12), (11, 20), (12, 13), (13, 14), (14, 15), (15, 16), (16, 17),
    (17, 18), (18, 19)])
t4c20_16minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (2,
    7), (3, 4), (3, 7), (3, 8), (3, 9), (3, 10), (3, 11), (3, 12), (3, 13), (3, 14),
     (3, 15), (3, 16), (3, 17), (3, 18), (3, 19), (3, 20), (4, 5), (4, 20), (5, 6),
    (5, 10), (5, 11), (5, 12), (5, 13), (5, 14), (5, 15), (5, 16), (5, 17), (5, 18),
     (5, 19), (5, 20), (6, 7), (6, 8), (6, 9), (6, 10), (7, 8), (8, 9), (9, 10),
    (10, 11), (11, 12), (12, 13), (13, 14), (14, 15), (15, 16), (16, 17), (17, 18),
    (18, 19), (19, 20)])
t4c20_17minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (2,
    7), (2, 8), (2, 9), (2, 10), (2, 11), (2, 12), (2, 13), (2, 14), (2, 15), (2,
    16), (2, 17), (3, 4), (3, 17), (4, 5), (4, 15), (4, 16), (4, 17), (4, 18), (4,
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    , (8, 19), (9, 10), (9, 19), (10, 11), (10, 19), (11, 12), (11, 19), (12, 13),
    (12, 19), (13, 14), (13, 19), (14, 15), (14, 18), (14, 19), (15, 16), (15, 18),
    (16, 17), (18, 19), (19, 20)])
t4c20_18minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (2,
    7), (2, 8), (2, 9), (2, 10), (2, 11), (2, 12), (2, 13), (2, 14), (2, 15), (2,
    16), (2, 17), (2, 18), (3, 4), (3, 18), (4, 5), (4, 16), (4, 17), (4, 18), (4,
    19), (4, 20), (5, 6), (5, 20), (6, 7), (6, 20), (7, 8), (7, 20), (8, 9), (8, 20)
    , (9, 10), (9, 20), (10, 11), (10, 20), (11, 12), (11, 20), (12, 13), (12, 20),
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    (16, 19), (17, 18), (19, 20)])
t4c20_19minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (2,
    7), (2, 8), (2, 9), (2, 10), (2, 11), (2, 12), (2, 13), (2, 14), (2, 15), (2,
    16), (2, 17), (2, 18), (3, 4), (3, 18), (4, 5), (4, 17), (4, 18), (4, 19), (4,
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    , (9, 20), (10, 11), (10, 20), (11, 12), (11, 20), (12, 13), (12, 20), (13, 14),
     (13, 20), (14, 15), (14, 20), (15, 16), (15, 19), (15, 20), (16, 17), (16, 19),
     (17, 18), (17, 19), (19, 20)])
t4c20_20minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (2,
    7), (2, 8), (2, 9), (2, 10), (2, 11), (2, 12), (2, 13), (2, 14), (2, 15), (2,
    16), (2, 17), (2, 18), (2, 19), (3, 4), (3, 19), (4, 5), (4, 17), (4, 18), (4,
    19), (4, 20), (5, 6), (5, 20), (6, 7), (6, 20), (7, 8), (7, 20), (8, 9), (8, 20)
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    (13, 14), (13, 20), (14, 15), (14, 20), (15, 16), (15, 20), (16, 17), (16, 20),
    (17, 18), (17, 20), (18, 19)])
t4c20_21minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 3), (2, 6), (2,
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    16), (2, 17), (2, 18), (3, 4), (3, 18), (4, 5), (4, 13), (4, 14), (4, 15), (4,
    16), (4, 17), (4, 18), (4, 19), (4, 20), (5, 6), (5, 20), (6, 7), (6, 20), (7,
    8), (7, 20), (8, 9), (8, 20), (9, 10), (9, 20), (10, 11), (10, 20), (11, 12),
    (11, 20), (12, 13), (12, 19), (12, 20), (13, 14), (13, 19), (14, 15), (15, 16),
    (16, 17), (17, 18), (19, 20)])
t4c20_22minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (2,
    7), (3, 4), (3, 7), (3, 8), (3, 9), (3, 10), (3, 11), (3, 12), (3, 13), (3, 14),
     (3, 15), (3, 16), (3, 17), (3, 18), (3, 19), (4, 5), (4, 19), (5, 6), (5, 13),
    (5, 14), (5, 15), (5, 16), (5, 17), (5, 18), (5, 19), (5, 20), (6, 7), (6, 8),
    (6, 9), (6, 10), (6, 11), (6, 20), (7, 8), (8, 9), (9, 10), (10, 11), (11, 12),
    (11, 20), (12, 13), (12, 20), (13, 14), (13, 20), (14, 15), (15, 16), (16, 17),
    (17, 18), (18, 19)])
t4c20_23minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (2,
    7), (3, 4), (3, 7), (3, 8), (3, 9), (3, 10), (3, 11), (3, 12), (3, 13), (3, 14),
     (3, 15), (3, 16), (3, 17), (3, 18), (3, 19), (4, 5), (4, 19), (5, 6), (5, 13),
    (5, 14), (5, 15), (5, 16), (5, 17), (5, 18), (5, 19), (5, 20), (6, 7), (6, 8),
    (6, 9), (6, 10), (6, 11), (6, 12), (6, 20), (7, 8), (8, 9), (9, 10), (10, 11),
    (11, 12), (12, 13), (12, 20), (13, 14), (13, 20), (14, 15), (15, 16), (16, 17),
    (17, 18), (18, 19)])
t4c20_24minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (2,
    7), (3, 4), (3, 7), (3, 8), (3, 9), (3, 10), (3, 11), (3, 12), (3, 13), (3, 14),
     (3, 15), (3, 16), (3, 17), (3, 18), (3, 19), (3, 20), (4, 5), (4, 20), (5, 6),
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    (18, 19), (19, 20)])
t4c20_25minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (2,
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    (12, 13), (12, 20), (13, 14), (13, 19), (13, 20), (14, 15), (14, 19), (15, 16),
    (16, 17), (17, 18), (19, 20)])
t4c20_26minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (2,
    7), (2, 8), (2, 9), (2, 10), (2, 11), (2, 12), (2, 13), (2, 14), (2, 15), (2,
    16), (2, 17), (2, 18), (3, 4), (3, 18), (4, 5), (4, 15), (4, 16), (4, 17), (4,
    18), (4, 19), (4, 20), (5, 6), (5, 20), (6, 7), (6, 20), (7, 8), (7, 20), (8, 9)
    , (8, 20), (9, 10), (9, 20), (10, 11), (10, 20), (11, 12), (11, 20), (12, 13),
    (12, 20), (13, 14), (13, 19), (13, 20), (14, 15), (14, 19), (15, 16), (15, 19),
    (16, 17), (17, 18), (19, 20)])
t4c20_27minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (2,
    7), (2, 8), (2, 9), (2, 10), (2, 11), (2, 12), (2, 13), (2, 14), (2, 15), (2,
    16), (2, 17), (2, 18), (2, 19), (3, 4), (3, 19), (4, 5), (4, 15), (4, 16), (4,
    17), (4, 18), (4, 19), (4, 20), (5, 6), (5, 20), (6, 7), (6, 20), (7, 8), (7,
    20), (8, 9), (8, 20), (9, 10), (9, 20), (10, 11), (10, 20), (11, 12), (11, 20),
    (12, 13), (12, 20), (13, 14), (13, 20), (14, 15), (14, 20), (15, 16), (15, 20),
    (16, 17), (17, 18), (18, 19)])
t4c20_28minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (2,
    7), (2, 8), (2, 9), (2, 10), (2, 11), (2, 12), (3, 4), (3, 12), (3, 13), (4, 5),
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    (4, 17), (4, 18), (4, 19), (4, 20), (5, 6), (6, 7), (7, 8), (8, 9), (9, 10),
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    (12, 16), (12, 17), (12, 18), (12, 19), (13, 14), (14, 15), (15, 16), (16, 17),
    (17, 18), (18, 19), (19, 20)])
t4c20_29minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (2,
    7), (2, 8), (2, 9), (2, 10), (2, 11), (2, 12), (2, 13), (3, 4), (3, 13), (3, 14)
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    (13, 16), (13, 17), (13, 18), (13, 19), (13, 20), (14, 15), (15, 16), (16, 17),
    (17, 18), (18, 19), (19, 20)])
t4c20_30minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (2,
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    15), (14, 19), (14, 20), (15, 16), (15, 19), (16, 17), (16, 18), (16, 19), (17,
    18), (18, 19), (19, 20)])
t4c20_31minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (2,
    7), (2, 8), (2, 9), (2, 10), (2, 11), (2, 12), (2, 13), (2, 14), (2, 15), (2,
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     (5, 6), (6, 7), (7, 8), (8, 9), (9, 10), (10, 11), (11, 12), (12, 13), (13, 14)
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    , (18, 19), (19, 20)])
t4c20_32minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (2,
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    , (17, 19), (18, 19)])
t4c20_33minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (2,
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    16), (2, 17), (2, 18), (2, 19), (3, 4), (3, 19), (4, 5), (4, 6), (4, 7), (4, 8),
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     (4, 19), (4, 20), (5, 6), (6, 7), (7, 8), (8, 9), (9, 10), (10, 11), (11, 12),
    (12, 13), (13, 14), (14, 15), (15, 16), (16, 17), (17, 18), (17, 20), (18, 19),
    (18, 20), (19, 20)])
t4c20_34minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (2,
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    (10, 11), (11, 12), (12, 13), (13, 14), (14, 15), (15, 16), (16, 17), (17, 18),
    (18, 19), (19, 20)])
t4c21_1minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (2, 7)
    , (2, 8), (2, 9), (2, 10), (2, 11), (2, 12), (2, 13), (2, 14), (2, 15), (2, 16),
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    14), (13, 20), (14, 15), (14, 20), (15, 16), (15, 20), (16, 17), (16, 20), (17,
    18), (17, 19), (17, 20), (18, 19), (19, 20), (20, 21)])
t4c21_2minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 3), (2, 6), (2, 7)
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    21), (18, 19), (18, 21), (19, 20), (19, 21), (20, 21)])
t4c21_3minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (2, 7)
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    18), (17, 20), (18, 19), (18, 20), (19, 20), (20, 21)])
t4c21_4minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (2, 7)
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    (5, 6), (5, 21), (6, 7), (6, 21), (7, 8), (7, 21), (8, 9), (8, 21), (9, 10), (9,
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    21), (14, 15), (14, 21), (15, 16), (15, 21), (16, 17), (16, 21), (17, 18), (17,
    21), (18, 19), (18, 21), (19, 20), (19, 21), (20, 21)])
t4c21_5minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (2, 7)
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    (2, 17), (2, 18), (2, 19), (3, 4), (3, 19), (4, 5), (4, 19), (4, 20), (4, 21),
    (5, 6), (5, 21), (6, 7), (6, 21), (7, 8), (7, 21), (8, 9), (8, 20), (8, 21), (9,
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    14), (13, 20), (14, 15), (14, 20), (15, 16), (15, 20), (16, 17), (16, 20), (17,
    18), (17, 20), (18, 19), (18, 20), (19, 20), (20, 21)])
t4c21_6minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (2, 7)
    , (2, 8), (2, 9), (2, 10), (2, 11), (2, 12), (2, 13), (2, 14), (2, 15), (2, 16),
    (2, 17), (2, 18), (2, 19), (3, 4), (3, 19), (4, 5), (4, 19), (4, 20), (4, 21),
    (5, 6), (5, 21), (6, 7), (6, 21), (7, 8), (7, 21), (8, 9), (8, 21), (9, 10), (9,
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    18), (17, 20), (18, 19), (18, 20), (19, 20), (20, 21)])
t4c21_7minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (2, 7)
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    (2, 17), (2, 18), (2, 19), (3, 4), (3, 19), (4, 5), (4, 19), (4, 20), (4, 21),
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    21), (10, 11), (10, 20), (10, 21), (11, 12), (11, 20), (12, 13), (12, 20), (13,
    14), (13, 20), (14, 15), (14, 20), (15, 16), (15, 20), (16, 17), (16, 20), (17,
    18), (17, 20), (18, 19), (18, 20), (19, 20), (20, 21)])
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t4c21_8minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (2, 7)
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    12), (4, 13), (4, 14), (4, 15), (4, 16), (4, 17), (4, 18), (4, 19), (4, 20), (4,
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    16), (9, 10), (10, 11), (11, 12), (12, 13), (13, 14), (14, 15), (15, 16), (16,
    17), (17, 18), (18, 19), (19, 20), (20, 21)])
t4c21_9minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (2, 7)
    , (2, 8), (3, 4), (3, 8), (3, 9), (4, 5), (4, 6), (4, 9), (4, 10), (4, 11), (4,
    12), (4, 13), (4, 14), (4, 15), (4, 16), (4, 17), (4, 18), (4, 19), (4, 20), (4,
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    19), (7, 20), (7, 21), (8, 9), (8, 10), (8, 11), (8, 12), (8, 13), (8, 14), (8,
    15), (9, 10), (10, 11), (11, 12), (12, 13), (13, 14), (14, 15), (15, 16), (16,
    17), (17, 18), (18, 19), (19, 20), (20, 21)])
t4c21_10minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (2,
    7), (2, 8), (2, 9), (2, 10), (2, 11), (2, 12), (2, 13), (2, 14), (2, 15), (2,
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    (8, 9), (9, 10), (10, 11), (11, 12), (11, 21), (12, 13), (12, 21), (13, 14),
    (13, 21), (14, 15), (14, 21), (15, 16), (15, 20), (15, 21), (16, 17), (16, 20),
    (17, 18), (17, 20), (18, 19), (18, 20), (19, 20), (20, 21)])
t4c21_11minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (2,
    7), (2, 8), (2, 9), (2, 10), (2, 11), (2, 12), (2, 13), (2, 14), (2, 15), (2,
    16), (2, 17), (2, 18), (2, 19), (3, 4), (3, 19), (4, 5), (4, 6), (4, 7), (4, 19)
    , (4, 20), (4, 21), (5, 6), (6, 7), (7, 8), (7, 21), (8, 9), (8, 21), (9, 10),
    (9, 21), (10, 11), (10, 21), (11, 12), (11, 21), (12, 13), (12, 21), (13, 14),
    (13, 21), (14, 15), (14, 21), (15, 16), (15, 20), (15, 21), (16, 17), (16, 20),
    (17, 18), (17, 20), (18, 19), (18, 20), (19, 20), (20, 21)])
t4c21_12minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (2,
    7), (2, 8), (2, 9), (2, 10), (2, 11), (2, 12), (2, 13), (2, 14), (2, 15), (2,
    16), (2, 17), (2, 18), (2, 19), (3, 4), (3, 19), (4, 5), (4, 6), (4, 7), (4, 8),
     (4, 9), (4, 19), (4, 20), (4, 21), (5, 6), (6, 7), (7, 8), (8, 9), (9, 10), (9,
     21), (10, 11), (10, 21), (11, 12), (11, 21), (12, 13), (12, 21), (13, 14), (13,
     21), (14, 15), (14, 21), (15, 16), (15, 20), (15, 21), (16, 17), (16, 20), (17,
     18), (17, 20), (18, 19), (18, 20), (19, 20), (20, 21)])
t4c21_13minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (2,
    7), (2, 8), (2, 9), (2, 10), (2, 11), (2, 12), (2, 13), (2, 14), (2, 15), (2,
    16), (2, 17), (2, 18), (2, 19), (3, 4), (3, 19), (4, 5), (4, 6), (4, 7), (4, 19)
    , (4, 20), (4, 21), (5, 6), (6, 7), (7, 8), (7, 21), (8, 9), (8, 21), (9, 10),
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    (17, 18), (17, 20), (18, 19), (18, 20), (19, 20), (20, 21)])
t4c21_14minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (2,
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    (16, 17), (17, 18), (18, 19), (19, 20), (20, 21)])
t4c21_15minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 3), (2, 6), (2,
    7), (2, 8), (2, 9), (2, 10), (2, 11), (2, 12), (2, 13), (2, 14), (2, 15), (2,
    16), (2, 17), (2, 18), (2, 19), (3, 4), (3, 19), (4, 5), (4, 11), (4, 12), (4,
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    15), (15, 16), (16, 17), (17, 18), (18, 19), (20, 21)])
t4c21_16minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (2,
    7), (3, 4), (3, 7), (3, 8), (3, 9), (3, 10), (3, 11), (3, 12), (3, 13), (3, 14),
     (3, 15), (3, 16), (3, 17), (3, 18), (3, 19), (3, 20), (4, 5), (4, 20), (5, 6),
    (5, 11), (5, 12), (5, 13), (5, 14), (5, 15), (5, 16), (5, 17), (5, 18), (5, 19),
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     16), (16, 17), (17, 18), (18, 19), (19, 20)])
t4c21_17minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (2,
    7), (3, 4), (3, 7), (3, 8), (3, 9), (3, 10), (3, 11), (3, 12), (3, 13), (3, 14),
     (3, 15), (3, 16), (3, 17), (3, 18), (3, 19), (3, 20), (4, 5), (4, 20), (5, 6),
    (5, 11), (5, 12), (5, 13), (5, 14), (5, 15), (5, 16), (5, 17), (5, 18), (5, 19),
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     10), (10, 11), (10, 21), (11, 12), (11, 21), (12, 13), (13, 14), (14, 15), (15,
     16), (16, 17), (17, 18), (18, 19), (19, 20)])
t4c21_18minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (2,
    7), (3, 4), (3, 7), (3, 8), (3, 9), (3, 10), (3, 11), (3, 12), (3, 13), (3, 14),
     (3, 15), (3, 16), (3, 17), (3, 18), (3, 19), (3, 20), (3, 21), (4, 5), (4, 21),
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    (8, 9), (9, 10), (10, 11), (11, 12), (12, 13), (13, 14), (14, 15), (15, 16),
    (16, 17), (17, 18), (18, 19), (19, 20), (20, 21)])
t4c21_19minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (2,
    7), (2, 8), (2, 9), (2, 10), (2, 11), (2, 12), (2, 13), (2, 14), (2, 15), (2,
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    (15, 20), (16, 17), (16, 19), (17, 18), (19, 20), (20, 21)])
t4c21_20minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (2,
    7), (2, 8), (2, 9), (2, 10), (2, 11), (2, 12), (2, 13), (2, 14), (2, 15), (2,
    16), (2, 17), (2, 18), (2, 19), (3, 4), (3, 19), (4, 5), (4, 17), (4, 18), (4,
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    (12, 21), (13, 14), (13, 21), (14, 15), (14, 21), (15, 16), (15, 21), (16, 17),
    (16, 20), (16, 21), (17, 18), (17, 20), (18, 19), (20, 21)])
t4c21_21minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (2,
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    , (9, 10), (9, 21), (10, 11), (10, 21), (11, 12), (11, 21), (12, 13), (12, 21),
    (13, 14), (13, 21), (14, 15), (14, 21), (15, 16), (15, 21), (16, 17), (16, 20),
    (16, 21), (17, 18), (17, 20), (18, 19), (18, 20), (20, 21)])
t4c21_22minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (2,
    7), (2, 8), (2, 9), (2, 10), (2, 11), (2, 12), (2, 13), (2, 14), (2, 15), (2,
    16), (2, 17), (2, 18), (2, 19), (2, 20), (3, 4), (3, 20), (4, 5), (4, 18), (4,
    19), (4, 20), (4, 21), (5, 6), (5, 21), (6, 7), (6, 21), (7, 8), (7, 21), (8, 9)
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    (12, 21), (13, 14), (13, 21), (14, 15), (14, 21), (15, 16), (15, 21), (16, 17),
    (16, 21), (17, 18), (17, 21), (18, 19), (18, 21), (19, 20)])
t4c21_23minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 3), (2, 6), (2,
    7), (2, 8), (2, 9), (2, 10), (2, 11), (2, 12), (2, 13), (2, 14), (2, 15), (2,
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    15), (4, 16), (4, 17), (4, 18), (4, 19), (4, 20), (4, 21), (5, 6), (5, 21), (6,
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    21), (11, 12), (11, 21), (12, 13), (12, 20), (12, 21), (13, 14), (13, 20), (14,
    15), (15, 16), (16, 17), (17, 18), (18, 19), (20, 21)])
t4c21_24minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (2,
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    (15, 16), (16, 17), (17, 18), (18, 19), (19, 20)])
t4c21_25minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (2,
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     16), (16, 17), (17, 18), (18, 19), (19, 20)])
t4c21_26minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (2,
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    (16, 17), (17, 18), (18, 19), (19, 20), (20, 21)])
t4c21_27minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (2,
    7), (2, 8), (2, 9), (2, 10), (2, 11), (2, 12), (2, 13), (2, 14), (2, 15), (2,
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    (14, 19), (15, 16), (16, 17), (17, 18), (19, 20), (20, 21)])
t4c21_28minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (2,
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    (15, 16), (15, 20), (16, 17), (17, 18), (18, 19), (20, 21)])
t4c21_29minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (2,
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    (15, 20), (16, 17), (16, 20), (17, 18), (18, 19), (20, 21)])
t4c21_30minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (2,
    7), (2, 8), (2, 9), (2, 10), (2, 11), (2, 12), (2, 13), (2, 14), (2, 15), (2,
    16), (2, 17), (2, 18), (2, 19), (2, 20), (3, 4), (3, 20), (4, 5), (4, 16), (4,
    17), (4, 18), (4, 19), (4, 20), (4, 21), (5, 6), (5, 21), (6, 7), (6, 21), (7,
    8), (7, 21), (8, 9), (8, 21), (9, 10), (9, 21), (10, 11), (10, 21), (11, 12),
    (11, 21), (12, 13), (12, 21), (13, 14), (13, 21), (14, 15), (14, 21), (15, 16),
    (15, 21), (16, 17), (16, 21), (17, 18), (18, 19), (19, 20)])
t4c21_31minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (2,
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    16), (4, 17), (4, 18), (4, 19), (4, 20), (4, 21), (5, 6), (5, 21), (6, 7), (6,
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    (11, 12), (11, 21), (12, 13), (12, 20), (12, 21), (13, 14), (13, 20), (14, 15),
    (14, 20), (15, 16), (16, 17), (17, 18), (18, 19), (20, 21)])
t4c21_32minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (2,
    7), (2, 8), (2, 9), (2, 10), (2, 11), (2, 12), (2, 13), (2, 14), (2, 15), (2,
    16), (2, 17), (2, 18), (2, 19), (3, 4), (3, 19), (4, 5), (4, 14), (4, 15), (4,
    16), (4, 17), (4, 18), (4, 19), (4, 20), (4, 21), (5, 6), (5, 21), (6, 7), (6,
    21), (7, 8), (7, 21), (8, 9), (8, 21), (9, 10), (9, 21), (10, 11), (10, 21),
    (11, 12), (11, 21), (12, 13), (12, 21), (13, 14), (13, 20), (13, 21), (14, 15),
    (14, 20), (15, 16), (16, 17), (17, 18), (18, 19), (20, 21)])
t4c21_33minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (2,
    7), (2, 8), (2, 9), (2, 10), (2, 11), (2, 12), (2, 13), (2, 14), (2, 15), (2,
    16), (2, 17), (2, 18), (2, 19), (2, 20), (3, 4), (3, 20), (4, 5), (4, 13), (4,
    14), (4, 15), (4, 16), (4, 17), (4, 18), (4, 19), (4, 20), (4, 21), (5, 6), (5,
    21), (6, 7), (6, 21), (7, 8), (7, 21), (8, 9), (8, 21), (9, 10), (9, 21), (10,
    11), (10, 21), (11, 12), (11, 21), (12, 13), (12, 21), (13, 14), (13, 21), (14,
    15), (15, 16), (16, 17), (17, 18), (18, 19), (19, 20)])
t4c21_34minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (2,
    7), (2, 8), (2, 9), (2, 10), (2, 11), (2, 12), (2, 13), (3, 4), (3, 13), (3, 14)
    , (4, 5), (4, 6), (4, 7), (4, 8), (4, 9), (4, 10), (4, 11), (4, 12), (4, 14),
    (4, 15), (4, 16), (4, 17), (4, 18), (4, 19), (4, 20), (4, 21), (5, 6), (6, 7),
    (7, 8), (8, 9), (9, 10), (10, 11), (11, 12), (12, 13), (12, 21), (13, 14), (13,
    15), (13, 16), (13, 17), (13, 18), (13, 19), (13, 20), (13, 21), (14, 15), (15,
    16), (16, 17), (17, 18), (18, 19), (19, 20), (20, 21)])
t4c21_35minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (2,
    7), (2, 8), (2, 9), (2, 10), (2, 11), (2, 12), (2, 13), (2, 14), (2, 15), (2,
    16), (2, 17), (2, 18), (3, 4), (3, 18), (4, 5), (4, 6), (4, 7), (4, 8), (4, 9),
    (4, 10), (4, 11), (4, 12), (4, 13), (4, 14), (4, 18), (4, 19), (4, 20), (4, 21),
     (5, 6), (6, 7), (7, 8), (8, 9), (9, 10), (10, 11), (11, 12), (12, 13), (13, 14)
    , (14, 15), (14, 21), (15, 16), (15, 20), (15, 21), (16, 17), (16, 20), (17, 18)
    , (17, 19), (17, 20), (18, 19), (19, 20), (20, 21)])
t4c21_36minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (3,
    4), (3, 6), (3, 7), (3, 8), (3, 9), (3, 10), (3, 11), (3, 12), (3, 13), (3, 14),
     (3, 15), (3, 16), (3, 17), (3, 18), (3, 19), (4, 5), (4, 19), (5, 6), (5, 9),
    (5, 10), (5, 11), (5, 12), (5, 13), (5, 14), (5, 15), (5, 16), (5, 17), (5, 18),
     (5, 19), (5, 20), (5, 21), (6, 7), (6, 21), (7, 8), (7, 20), (7, 21), (8, 9),
    (8, 20), (9, 10), (9, 20), (10, 11), (11, 12), (12, 13), (13, 14), (14, 15),
    (15, 16), (16, 17), (17, 18), (18, 19), (20, 21)])
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t4c21_37minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (3,
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     (3, 15), (3, 16), (3, 17), (3, 18), (3, 19), (4, 5), (4, 19), (5, 6), (5, 8),
    (5, 10), (5, 11), (5, 12), (5, 13), (5, 14), (5, 15), (5, 16), (5, 17), (5, 18),
     (5, 19), (5, 20), (5, 21), (6, 7), (6, 21), (7, 8), (7, 21), (8, 9), (8, 20),
    (8, 21), (9, 10), (9, 20), (10, 11), (10, 20), (11, 12), (12, 13), (13, 14),
    (14, 15), (15, 16), (16, 17), (17, 18), (18, 19)])
t4c21_38minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (3,
    4), (3, 6), (3, 7), (3, 8), (3, 9), (3, 10), (3, 11), (3, 12), (3, 13), (3, 14),
     (3, 15), (3, 16), (3, 17), (3, 18), (3, 19), (3, 20), (4, 5), (4, 20), (5, 6),
    (5, 8), (5, 9), (5, 10), (5, 11), (5, 12), (5, 13), (5, 14), (5, 15), (5, 16),
    (5, 17), (5, 18), (5, 19), (5, 20), (5, 21), (6, 7), (6, 21), (7, 8), (7, 21),
    (8, 9), (8, 21), (9, 10), (10, 11), (11, 12), (12, 13), (13, 14), (14, 15), (15,
     16), (16, 17), (17, 18), (18, 19), (19, 20)])
t4c21_39minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (3,
    4), (3, 6), (3, 7), (3, 8), (3, 9), (3, 10), (3, 11), (3, 12), (3, 13), (3, 14),
     (3, 15), (3, 16), (3, 17), (3, 18), (3, 19), (3, 20), (3, 21), (4, 5), (4, 21),
     (5, 6), (5, 7), (5, 8), (5, 9), (5, 10), (5, 11), (5, 12), (5, 13), (5, 14),
    (5, 15), (5, 16), (5, 17), (5, 18), (5, 19), (5, 20), (5, 21), (6, 7), (7, 8),
    (8, 9), (9, 10), (10, 11), (11, 12), (12, 13), (13, 14), (14, 15), (15, 16),
    (16, 17), (17, 18), (18, 19), (19, 20), (20, 21)])
t4c22_1minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (2, 7)
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     (2, 17), (2, 18), (2, 19), (3, 4), (3, 19), (4, 5), (4, 19), (4, 20), (4, 21),
    (4, 22), (5, 6), (5, 22), (6, 7), (6, 21), (6, 22), (7, 8), (7, 21), (8, 9), (8,
     21), (9, 10), (9, 21), (10, 11), (10, 21), (11, 12), (11, 21), (12, 13), (12,
    21), (13, 14), (13, 21), (14, 15), (14, 21), (15, 16), (15, 21), (16, 17), (16,
    21), (17, 18), (17, 21), (18, 19), (18, 20), (18, 21), (19, 20), (20, 21), (21,
    22)])
t4c22_2minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 3), (2, 6), (2, 7)
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     (2, 17), (2, 18), (2, 19), (2, 20), (2, 21), (3, 4), (3, 21), (4, 5), (4, 21),
    (4, 22), (5, 6), (5, 7), (5, 22), (6, 7), (7, 8), (7, 22), (8, 9), (8, 22), (9,
    10), (9, 22), (10, 11), (10, 22), (11, 12), (11, 22), (12, 13), (12, 22), (13,
    14), (13, 22), (14, 15), (14, 22), (15, 16), (15, 22), (16, 17), (16, 22), (17,
    18), (17, 22), (18, 19), (18, 22), (19, 20), (19, 22), (20, 21), (20, 22), (21,
    22)])
t4c22_3minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (2, 7)
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     (2, 17), (2, 18), (2, 19), (2, 20), (3, 4), (3, 20), (4, 5), (4, 20), (4, 21),
    (4, 22), (5, 6), (5, 22), (6, 7), (6, 21), (6, 22), (7, 8), (7, 21), (8, 9), (8,
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21), (17, 18), (17, 21), (18, 19), (18, 21), (19, 20), (19, 21), (20, 21), (21,
22)])
t4c22_4minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (2, 7)
, (2, 8), (2, 9), (2, 10), (2, 11), (2, 12), (2, 13), (2, 14), (2, 15), (2, 16),
 (2, 17), (2, 18), (2, 19), (2, 20), (2, 21), (3, 4), (3, 21), (4, 5), (4, 21),
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18), (17, 22), (18, 19), (18, 22), (19, 20), (19, 22), (20, 21), (20, 22), (21,
22)])
t4c22_5minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (2, 7)
, (2, 8), (2, 9), (2, 10), (2, 11), (2, 12), (2, 13), (2, 14), (2, 15), (2, 16),
 (2, 17), (2, 18), (2, 19), (2, 20), (3, 4), (3, 20), (4, 5), (4, 20), (4, 21),
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21), (17, 18), (17, 21), (18, 19), (18, 21), (19, 20), (19, 21), (20, 21), (21,
22)])
t4c22_6minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (2, 7)
, (2, 8), (2, 9), (2, 10), (2, 11), (2, 12), (2, 13), (2, 14), (2, 15), (2, 16),
 (2, 17), (2, 18), (2, 19), (2, 20), (3, 4), (3, 20), (4, 5), (4, 20), (4, 21),
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22), (17, 18), (17, 21), (18, 19), (18, 21), (19, 20), (19, 21), (20, 21), (21,
22)])
t4c22_7minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (2, 7)
, (2, 8), (2, 9), (2, 10), (2, 11), (2, 12), (2, 13), (2, 14), (2, 15), (2, 16),
 (2, 17), (2, 18), (2, 19), (2, 20), (3, 4), (3, 20), (4, 5), (4, 20), (4, 21),
(4, 22), (5, 6), (5, 22), (6, 7), (6, 22), (7, 8), (7, 22), (8, 9), (8, 22), (9,
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21), (13, 14), (13, 21), (14, 15), (14, 21), (15, 16), (15, 21), (16, 17), (16,
21), (17, 18), (17, 21), (18, 19), (18, 21), (19, 20), (19, 21), (20, 21), (21,
22)])
t4c22_8minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (2, 7)
, (2, 8), (2, 9), (2, 10), (2, 11), (2, 12), (2, 13), (2, 14), (2, 15), (2, 16),
 (2, 17), (2, 18), (2, 19), (2, 20), (3, 4), (3, 20), (4, 5), (4, 20), (4, 21),
(4, 22), (5, 6), (5, 22), (6, 7), (6, 22), (7, 8), (7, 22), (8, 9), (8, 22), (9,
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    21), (17, 18), (17, 21), (18, 19), (18, 21), (19, 20), (19, 21), (20, 21), (21,
    22)])
t4c22_9minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (2, 7)
    , (2, 8), (3, 4), (3, 8), (3, 9), (4, 5), (4, 6), (4, 9), (4, 10), (4, 11), (4,
    12), (4, 13), (4, 14), (4, 15), (4, 16), (4, 17), (4, 18), (4, 19), (4, 20), (4,
     21), (4, 22), (5, 6), (6, 7), (6, 22), (7, 8), (7, 15), (7, 16), (7, 17), (7,
    18), (7, 19), (7, 20), (7, 21), (7, 22), (8, 9), (8, 10), (8, 11), (8, 12), (8,
    13), (8, 14), (8, 15), (9, 10), (10, 11), (11, 12), (12, 13), (13, 14), (14, 15)
    , (15, 16), (16, 17), (17, 18), (18, 19), (19, 20), (20, 21), (21, 22)])
t4c22_10minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (2,
    7), (2, 8), (2, 9), (2, 10), (2, 11), (2, 12), (2, 13), (2, 14), (2, 15), (2,
    16), (2, 17), (2, 18), (2, 19), (2, 20), (3, 4), (3, 20), (4, 5), (4, 6), (4, 7)
    , (4, 8), (4, 9), (4, 10), (4, 11), (4, 12), (4, 20), (4, 21), (4, 22), (5, 6),
    (6, 7), (7, 8), (8, 9), (9, 10), (10, 11), (11, 12), (12, 13), (12, 22), (13,
    14), (13, 22), (14, 15), (14, 22), (15, 16), (15, 22), (16, 17), (16, 21), (16,
    22), (17, 18), (17, 21), (18, 19), (18, 21), (19, 20), (19, 21), (20, 21), (21,
    22)])
t4c22_11minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (2,
    7), (2, 8), (2, 9), (2, 10), (2, 11), (2, 12), (2, 13), (2, 14), (2, 15), (2,
    16), (2, 17), (2, 18), (2, 19), (2, 20), (3, 4), (3, 20), (4, 5), (4, 6), (4, 7)
    , (4, 20), (4, 21), (4, 22), (5, 6), (6, 7), (7, 8), (7, 22), (8, 9), (8, 22),
    (9, 10), (9, 22), (10, 11), (10, 22), (11, 12), (11, 22), (12, 13), (12, 22),
    (13, 14), (13, 22), (14, 15), (14, 22), (15, 16), (15, 22), (16, 17), (16, 21),
    (16, 22), (17, 18), (17, 21), (18, 19), (18, 21), (19, 20), (19, 21), (20, 21),
    (21, 22)])
t4c22_12minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (2,
    7), (2, 8), (2, 9), (2, 10), (2, 11), (2, 12), (2, 13), (2, 14), (2, 15), (2,
    16), (2, 17), (2, 18), (2, 19), (2, 20), (3, 4), (3, 20), (4, 5), (4, 6), (4, 7)
    , (4, 8), (4, 9), (4, 10), (4, 20), (4, 21), (4, 22), (5, 6), (6, 7), (7, 8),
    (8, 9), (9, 10), (10, 11), (10, 22), (11, 12), (11, 22), (12, 13), (12, 22),
    (13, 14), (13, 22), (14, 15), (14, 22), (15, 16), (15, 22), (16, 17), (16, 21),
    (16, 22), (17, 18), (17, 21), (18, 19), (18, 21), (19, 20), (19, 21), (20, 21),
    (21, 22)])
t4c22_13minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (2,
    7), (2, 8), (2, 9), (2, 10), (2, 11), (2, 12), (3, 4), (3, 12), (4, 5), (4, 6),
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    (8, 17), (8, 18), (8, 19), (9, 10), (9, 13), (10, 11), (11, 12), (13, 14), (14,
    15), (15, 16), (16, 17), (17, 18), (18, 19), (19, 20), (20, 21), (21, 22)])
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t4c22_14minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (3,
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    (5, 10), (5, 11), (5, 12), (5, 13), (5, 14), (5, 15), (5, 16), (5, 17), (5, 18),
     (5, 19), (5, 20), (5, 21), (5, 22), (6, 7), (6, 22), (7, 8), (7, 15), (7, 16),
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    15), (15, 16), (16, 17), (17, 18), (18, 19), (19, 20), (20, 21), (21, 22)])
t4c22_15minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (2,
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    (4, 11), (4, 12), (4, 13), (4, 14), (4, 15), (4, 16), (4, 17), (4, 18), (4, 19),
     (4, 20), (4, 21), (4, 22), (5, 6), (6, 7), (6, 22), (7, 8), (7, 17), (7, 18),
    (7, 19), (7, 20), (7, 21), (7, 22), (8, 9), (8, 11), (8, 12), (8, 13), (8, 14),
    (8, 15), (8, 16), (8, 17), (9, 10), (9, 11), (11, 12), (12, 13), (13, 14), (14,
    15), (15, 16), (16, 17), (17, 18), (18, 19), (19, 20), (20, 21), (21, 22)])
t4c22_16minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (3,
    4), (3, 6), (3, 7), (3, 8), (3, 9), (3, 10), (3, 11), (3, 12), (3, 13), (3, 14),
     (3, 15), (3, 16), (3, 17), (3, 18), (3, 19), (3, 20), (4, 5), (4, 20), (5, 6),
    (5, 7), (5, 8), (5, 20), (5, 21), (5, 22), (6, 7), (7, 8), (8, 9), (8, 22), (9,
    10), (9, 22), (10, 11), (10, 22), (11, 12), (11, 22), (12, 13), (12, 22), (13,
    14), (13, 22), (14, 15), (14, 21), (14, 22), (15, 16), (15, 21), (16, 17), (16,
    21), (17, 18), (17, 21), (18, 19), (18, 21), (19, 20), (19, 21), (20, 21), (21,
    22)])
t4c22_17minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 3), (2, 6), (2,
    7), (2, 8), (2, 9), (2, 10), (2, 11), (2, 12), (2, 13), (2, 14), (2, 15), (2,
    16), (2, 17), (2, 18), (2, 19), (2, 20), (3, 4), (3, 20), (4, 5), (4, 11), (4,
    12), (4, 13), (4, 14), (4, 15), (4, 16), (4, 17), (4, 18), (4, 19), (4, 20), (4,
     21), (4, 22), (5, 6), (5, 22), (6, 7), (6, 22), (7, 8), (7, 22), (8, 9), (8,
    22), (9, 10), (9, 22), (10, 11), (10, 21), (10, 22), (11, 12), (11, 21), (12,
    13), (13, 14), (14, 15), (15, 16), (16, 17), (17, 18), (18, 19), (19, 20), (21,
    22)])
t4c22_18minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (2,
    7), (3, 4), (3, 7), (3, 8), (3, 9), (3, 10), (3, 11), (3, 12), (3, 13), (3, 14),
     (3, 15), (3, 16), (3, 17), (3, 18), (3, 19), (3, 20), (3, 21), (4, 5), (4, 21),
     (5, 6), (5, 11), (5, 12), (5, 13), (5, 14), (5, 15), (5, 16), (5, 17), (5, 18),
     (5, 19), (5, 20), (5, 21), (5, 22), (6, 7), (6, 8), (6, 9), (6, 22), (7, 8),
    (8, 9), (9, 10), (9, 22), (10, 11), (10, 22), (11, 12), (11, 22), (12, 13), (13,
     14), (14, 15), (15, 16), (16, 17), (17, 18), (18, 19), (19, 20), (20, 21)])
t4c22_19minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (2,
    7), (3, 4), (3, 7), (3, 8), (3, 9), (3, 10), (3, 11), (3, 12), (3, 13), (3, 14),
     (3, 15), (3, 16), (3, 17), (3, 18), (3, 19), (3, 20), (3, 21), (4, 5), (4, 21),
     (5, 6), (5, 11), (5, 12), (5, 13), (5, 14), (5, 15), (5, 16), (5, 17), (5, 18),
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     (5, 19), (5, 20), (5, 21), (5, 22), (6, 7), (6, 8), (6, 9), (6, 10), (6, 22),
    (7, 8), (8, 9), (9, 10), (10, 11), (10, 22), (11, 12), (11, 22), (12, 13), (13,
    14), (14, 15), (15, 16), (16, 17), (17, 18), (18, 19), (19, 20), (20, 21)])
t4c22_20minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (2,
    7), (3, 4), (3, 7), (3, 8), (3, 9), (3, 10), (3, 11), (3, 12), (3, 13), (3, 14),
     (3, 15), (3, 16), (3, 17), (3, 18), (3, 19), (3, 20), (3, 21), (3, 22), (4, 5),
     (4, 22), (5, 6), (5, 10), (5, 11), (5, 12), (5, 13), (5, 14), (5, 15), (5, 16),
     (5, 17), (5, 18), (5, 19), (5, 20), (5, 21), (5, 22), (6, 7), (6, 8), (6, 9),
    (6, 10), (7, 8), (8, 9), (9, 10), (10, 11), (11, 12), (12, 13), (13, 14), (14,
    15), (15, 16), (16, 17), (17, 18), (18, 19), (19, 20), (20, 21), (21, 22)])
t4c22_21minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (2,
    7), (2, 8), (2, 9), (2, 10), (2, 11), (2, 12), (2, 13), (2, 14), (2, 15), (2,
    16), (2, 17), (2, 18), (2, 19), (3, 4), (3, 19), (4, 5), (4, 17), (4, 18), (4,
    19), (4, 20), (4, 21), (4, 22), (5, 6), (5, 22), (6, 7), (6, 21), (6, 22), (7,
    8), (7, 21), (8, 9), (8, 21), (9, 10), (9, 21), (10, 11), (10, 21), (11, 12),
    (11, 21), (12, 13), (12, 21), (13, 14), (13, 21), (14, 15), (14, 21), (15, 16),
    (15, 21), (16, 17), (16, 20), (16, 21), (17, 18), (17, 20), (18, 19), (20, 21),
    (21, 22)])
t4c22_22minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (2,
    7), (2, 8), (2, 9), (2, 10), (2, 11), (2, 12), (2, 13), (2, 14), (2, 15), (2,
    16), (2, 17), (2, 18), (2, 19), (2, 20), (3, 4), (3, 20), (4, 5), (4, 18), (4,
    19), (4, 20), (4, 21), (4, 22), (5, 6), (5, 22), (6, 7), (6, 22), (7, 8), (7,
    22), (8, 9), (8, 22), (9, 10), (9, 22), (10, 11), (10, 22), (11, 12), (11, 22),
    (12, 13), (12, 22), (13, 14), (13, 22), (14, 15), (14, 22), (15, 16), (15, 22),
    (16, 17), (16, 22), (17, 18), (17, 21), (17, 22), (18, 19), (18, 21), (19, 20),
    (21, 22)])
t4c22_23minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (2,
    7), (2, 8), (2, 9), (2, 10), (2, 11), (2, 12), (2, 13), (2, 14), (2, 15), (2,
    16), (2, 17), (2, 18), (2, 19), (2, 20), (3, 4), (3, 20), (4, 5), (4, 19), (4,
    20), (4, 21), (4, 22), (5, 6), (5, 22), (6, 7), (6, 22), (7, 8), (7, 22), (8, 9)
    , (8, 22), (9, 10), (9, 22), (10, 11), (10, 22), (11, 12), (11, 22), (12, 13),
    (12, 22), (13, 14), (13, 22), (14, 15), (14, 22), (15, 16), (15, 22), (16, 17),
    (16, 22), (17, 18), (17, 21), (17, 22), (18, 19), (18, 21), (19, 20), (19, 21),
    (21, 22)])
t4c22_24minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (2,
    7), (2, 8), (2, 9), (2, 10), (2, 11), (2, 12), (2, 13), (2, 14), (2, 15), (2,
    16), (2, 17), (2, 18), (2, 19), (2, 20), (2, 21), (3, 4), (3, 21), (4, 5), (4,
    19), (4, 20), (4, 21), (4, 22), (5, 6), (5, 22), (6, 7), (6, 22), (7, 8), (7,
    22), (8, 9), (8, 22), (9, 10), (9, 22), (10, 11), (10, 22), (11, 12), (11, 22),
    (12, 13), (12, 22), (13, 14), (13, 22), (14, 15), (14, 22), (15, 16), (15, 22),
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    (16, 17), (16, 22), (17, 18), (17, 22), (18, 19), (18, 22), (19, 20), (19, 22),
    (20, 21)])
t4c22_25minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 3), (2, 6), (2,
    7), (2, 8), (2, 9), (2, 10), (2, 11), (2, 12), (2, 13), (2, 14), (2, 15), (2,
    16), (2, 17), (2, 18), (2, 19), (2, 20), (3, 4), (3, 20), (4, 5), (4, 13), (4,
    14), (4, 15), (4, 16), (4, 17), (4, 18), (4, 19), (4, 20), (4, 21), (4, 22), (5,
     6), (5, 22), (6, 7), (6, 22), (7, 8), (7, 22), (8, 9), (8, 22), (9, 10), (9,
    22), (10, 11), (10, 22), (11, 12), (11, 22), (12, 13), (12, 21), (12, 22), (13,
    14), (13, 21), (14, 15), (15, 16), (16, 17), (17, 18), (18, 19), (19, 20), (21,
    22)])
t4c22_26minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (2,
    7), (3, 4), (3, 7), (3, 8), (3, 9), (3, 10), (3, 11), (3, 12), (3, 13), (3, 14),
     (3, 15), (3, 16), (3, 17), (3, 18), (3, 19), (3, 20), (3, 21), (4, 5), (4, 21),
     (5, 6), (5, 13), (5, 14), (5, 15), (5, 16), (5, 17), (5, 18), (5, 19), (5, 20),
     (5, 21), (5, 22), (6, 7), (6, 8), (6, 9), (6, 10), (6, 11), (6, 22), (7, 8),
    (8, 9), (9, 10), (10, 11), (11, 12), (11, 22), (12, 13), (12, 22), (13, 14),
    (13, 22), (14, 15), (15, 16), (16, 17), (17, 18), (18, 19), (19, 20), (20, 21)])
t4c22_27minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (2,
    7), (3, 4), (3, 7), (3, 8), (3, 9), (3, 10), (3, 11), (3, 12), (3, 13), (3, 14),
     (3, 15), (3, 16), (3, 17), (3, 18), (3, 19), (3, 20), (3, 21), (4, 5), (4, 21),
     (5, 6), (5, 13), (5, 14), (5, 15), (5, 16), (5, 17), (5, 18), (5, 19), (5, 20),
     (5, 21), (5, 22), (6, 7), (6, 8), (6, 9), (6, 10), (6, 11), (6, 12), (6, 22),
    (7, 8), (8, 9), (9, 10), (10, 11), (11, 12), (12, 13), (12, 22), (13, 14), (13,
    22), (14, 15), (15, 16), (16, 17), (17, 18), (18, 19), (19, 20), (20, 21)])
t4c22_28minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (2,
    7), (3, 4), (3, 7), (3, 8), (3, 9), (3, 10), (3, 11), (3, 12), (3, 13), (3, 14),
     (3, 15), (3, 16), (3, 17), (3, 18), (3, 19), (3, 20), (3, 21), (3, 22), (4, 5),
     (4, 22), (5, 6), (5, 12), (5, 13), (5, 14), (5, 15), (5, 16), (5, 17), (5, 18),
     (5, 19), (5, 20), (5, 21), (5, 22), (6, 7), (6, 8), (6, 9), (6, 10), (6, 11),
    (6, 12), (7, 8), (8, 9), (9, 10), (10, 11), (11, 12), (12, 13), (13, 14), (14,
    15), (15, 16), (16, 17), (17, 18), (18, 19), (19, 20), (20, 21), (21, 22)])
t4c22_29minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (2,
    7), (2, 8), (2, 9), (2, 10), (2, 11), (2, 12), (2, 13), (2, 14), (2, 15), (2,
    16), (2, 17), (2, 18), (2, 19), (3, 4), (3, 19), (4, 5), (4, 15), (4, 16), (4,
    17), (4, 18), (4, 19), (4, 20), (4, 21), (4, 22), (5, 6), (5, 22), (6, 7), (6,
    21), (6, 22), (7, 8), (7, 21), (8, 9), (8, 21), (9, 10), (9, 21), (10, 11), (10,
     21), (11, 12), (11, 21), (12, 13), (12, 21), (13, 14), (13, 21), (14, 15), (14,
     20), (14, 21), (15, 16), (15, 20), (16, 17), (17, 18), (18, 19), (20, 21), (21,
     22)])
t4c22_30minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (2,
    7), (2, 8), (2, 9), (2, 10), (2, 11), (2, 12), (2, 13), (2, 14), (2, 15), (2,
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    17), (4, 18), (4, 19), (4, 20), (4, 21), (4, 22), (5, 6), (5, 22), (6, 7), (6,
    22), (7, 8), (7, 22), (8, 9), (8, 22), (9, 10), (9, 22), (10, 11), (10, 22),
    (11, 12), (11, 22), (12, 13), (12, 22), (13, 14), (13, 22), (14, 15), (14, 22),
    (15, 16), (15, 21), (15, 22), (16, 17), (16, 21), (17, 18), (18, 19), (19, 20),
    (21, 22)])
t4c22_31minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (2,
    7), (2, 8), (2, 9), (2, 10), (2, 11), (2, 12), (2, 13), (2, 14), (2, 15), (2,
    16), (2, 17), (2, 18), (2, 19), (2, 20), (3, 4), (3, 20), (4, 5), (4, 17), (4,
    18), (4, 19), (4, 20), (4, 21), (4, 22), (5, 6), (5, 22), (6, 7), (6, 22), (7,
    8), (7, 22), (8, 9), (8, 22), (9, 10), (9, 22), (10, 11), (10, 22), (11, 12),
    (11, 22), (12, 13), (12, 22), (13, 14), (13, 22), (14, 15), (14, 22), (15, 16),
    (15, 21), (15, 22), (16, 17), (16, 21), (17, 18), (17, 21), (18, 19), (19, 20),
    (21, 22)])
t4c22_32minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (2,
    7), (2, 8), (2, 9), (2, 10), (2, 11), (2, 12), (2, 13), (2, 14), (2, 15), (2,
    16), (2, 17), (2, 18), (2, 19), (2, 20), (2, 21), (3, 4), (3, 21), (4, 5), (4,
    17), (4, 18), (4, 19), (4, 20), (4, 21), (4, 22), (5, 6), (5, 22), (6, 7), (6,
    22), (7, 8), (7, 22), (8, 9), (8, 22), (9, 10), (9, 22), (10, 11), (10, 22),
    (11, 12), (11, 22), (12, 13), (12, 22), (13, 14), (13, 22), (14, 15), (14, 22),
    (15, 16), (15, 22), (16, 17), (16, 22), (17, 18), (17, 22), (18, 19), (19, 20),
    (20, 21)])
t4c22_33minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (2,
    7), (3, 4), (3, 7), (3, 8), (3, 9), (3, 10), (3, 11), (3, 12), (3, 13), (3, 14),
     (3, 15), (3, 16), (3, 17), (3, 18), (3, 19), (3, 20), (3, 21), (4, 5), (4, 21),
     (5, 6), (5, 15), (5, 16), (5, 17), (5, 18), (5, 19), (5, 20), (5, 21), (5, 22),
     (6, 7), (6, 8), (6, 9), (6, 10), (6, 11), (6, 12), (6, 13), (6, 22), (7, 8),
    (8, 9), (9, 10), (10, 11), (11, 12), (12, 13), (13, 14), (13, 22), (14, 15),
    (14, 22), (15, 16), (15, 22), (16, 17), (17, 18), (18, 19), (19, 20), (20, 21)])
t4c22_34minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (2,
    7), (3, 4), (3, 7), (3, 8), (3, 9), (3, 10), (3, 11), (3, 12), (3, 13), (3, 14),
     (3, 15), (3, 16), (3, 17), (3, 18), (3, 19), (3, 20), (3, 21), (4, 5), (4, 21),
     (5, 6), (5, 15), (5, 16), (5, 17), (5, 18), (5, 19), (5, 20), (5, 21), (5, 22),
     (6, 7), (6, 8), (6, 9), (6, 10), (6, 11), (6, 12), (6, 13), (6, 14), (6, 22),
    (7, 8), (8, 9), (9, 10), (10, 11), (11, 12), (12, 13), (13, 14), (14, 15), (14,
    22), (15, 16), (15, 22), (16, 17), (17, 18), (18, 19), (19, 20), (20, 21)])
t4c22_35minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (2,
    7), (3, 4), (3, 7), (3, 8), (3, 9), (3, 10), (3, 11), (3, 12), (3, 13), (3, 14),
     (3, 15), (3, 16), (3, 17), (3, 18), (3, 19), (3, 20), (3, 21), (3, 22), (4, 5),
     (4, 22), (5, 6), (5, 14), (5, 15), (5, 16), (5, 17), (5, 18), (5, 19), (5, 20),
     (5, 21), (5, 22), (6, 7), (6, 8), (6, 9), (6, 10), (6, 11), (6, 12), (6, 13),
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    (6, 14), (7, 8), (8, 9), (9, 10), (10, 11), (11, 12), (12, 13), (13, 14), (14,
    15), (15, 16), (16, 17), (17, 18), (18, 19), (19, 20), (20, 21), (21, 22)])
t4c22_36minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (2,
    7), (2, 8), (2, 9), (2, 10), (2, 11), (2, 12), (2, 13), (3, 4), (3, 13), (3, 14)
    , (4, 5), (4, 6), (4, 7), (4, 8), (4, 9), (4, 10), (4, 11), (4, 14), (4, 15),
    (4, 16), (4, 17), (4, 18), (4, 19), (4, 20), (4, 21), (4, 22), (5, 6), (6, 7),
    (7, 8), (8, 9), (9, 10), (10, 11), (11, 12), (11, 22), (12, 13), (12, 21), (12,
    22), (13, 14), (13, 15), (13, 16), (13, 17), (13, 18), (13, 19), (13, 20), (13,
    21), (14, 15), (15, 16), (16, 17), (17, 18), (18, 19), (19, 20), (20, 21), (21,
    22)])
t4c22_37minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (2,
    7), (2, 8), (2, 9), (2, 10), (2, 11), (2, 12), (2, 13), (3, 4), (3, 13), (3, 14)
    , (4, 5), (4, 6), (4, 7), (4, 8), (4, 9), (4, 10), (4, 11), (4, 12), (4, 14),
    (4, 15), (4, 16), (4, 17), (4, 18), (4, 19), (4, 20), (4, 21), (4, 22), (5, 6),
    (6, 7), (7, 8), (8, 9), (9, 10), (10, 11), (11, 12), (12, 13), (12, 22), (13,
    14), (13, 15), (13, 16), (13, 17), (13, 18), (13, 19), (13, 20), (13, 21), (13,
    22), (14, 15), (15, 16), (16, 17), (17, 18), (18, 19), (19, 20), (20, 21), (21,
    22)])
t4c22_38minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (2,
    7), (2, 8), (2, 9), (2, 10), (2, 11), (2, 12), (2, 13), (2, 14), (2, 15), (2,
    16), (2, 17), (2, 18), (2, 19), (3, 4), (3, 19), (4, 5), (4, 6), (4, 7), (4, 8),
     (4, 9), (4, 10), (4, 11), (4, 12), (4, 13), (4, 14), (4, 15), (4, 19), (4, 20),
     (4, 21), (4, 22), (5, 6), (6, 7), (7, 8), (8, 9), (9, 10), (10, 11), (11, 12),
    (12, 13), (13, 14), (14, 15), (15, 16), (15, 22), (16, 17), (16, 21), (16, 22),
    (17, 18), (17, 21), (18, 19), (18, 20), (18, 21), (19, 20), (20, 21), (21, 22)])
t4c22_39minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (2,
    7), (2, 8), (2, 9), (2, 10), (2, 11), (2, 12), (2, 13), (2, 14), (2, 15), (2,
    16), (2, 17), (2, 18), (2, 19), (2, 20), (3, 4), (3, 20), (4, 5), (4, 6), (4, 7)
    , (4, 8), (4, 9), (4, 10), (4, 11), (4, 12), (4, 13), (4, 14), (4, 15), (4, 16),
     (4, 17), (4, 20), (4, 21), (4, 22), (5, 6), (6, 7), (7, 8), (8, 9), (9, 10),
    (10, 11), (11, 12), (12, 13), (13, 14), (14, 15), (15, 16), (16, 17), (17, 18),
    (17, 22), (18, 19), (18, 21), (18, 22), (19, 20), (19, 21), (20, 21), (21, 22)])
t4c22_40minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (2,
    7), (2, 8), (2, 9), (2, 10), (2, 11), (2, 12), (2, 13), (2, 14), (2, 15), (2,
    16), (2, 17), (2, 18), (2, 19), (2, 20), (3, 4), (3, 20), (4, 5), (4, 6), (4, 7)
    , (4, 8), (4, 9), (4, 10), (4, 11), (4, 12), (4, 13), (4, 14), (4, 15), (4, 16),
     (4, 18), (4, 20), (4, 21), (4, 22), (5, 6), (6, 7), (7, 8), (8, 9), (9, 10),
    (10, 11), (11, 12), (12, 13), (13, 14), (14, 15), (15, 16), (16, 17), (16, 22),
    (17, 18), (17, 22), (18, 19), (18, 21), (18, 22), (19, 20), (19, 21), (20, 21)])
t4c22_41minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (2,
    7), (2, 8), (2, 9), (2, 10), (2, 11), (2, 12), (2, 13), (2, 14), (2, 15), (2,
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    16), (2, 17), (2, 18), (2, 19), (2, 20), (2, 21), (3, 4), (3, 21), (4, 5), (4,
    6), (4, 7), (4, 8), (4, 9), (4, 10), (4, 11), (4, 12), (4, 13), (4, 14), (4, 15)
    , (4, 16), (4, 17), (4, 18), (4, 19), (4, 21), (4, 22), (5, 6), (6, 7), (7, 8),
    (8, 9), (9, 10), (10, 11), (11, 12), (12, 13), (13, 14), (14, 15), (15, 16),
    (16, 17), (17, 18), (18, 19), (19, 20), (19, 22), (20, 21), (20, 22), (21, 22)])
t4c22_42minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (2,
    7), (2, 8), (2, 9), (2, 10), (2, 11), (2, 12), (2, 13), (2, 14), (2, 15), (2,
    16), (2, 17), (2, 18), (2, 19), (2, 20), (2, 21), (2, 22), (3, 4), (3, 22), (4,
    5), (4, 6), (4, 7), (4, 8), (4, 9), (4, 10), (4, 11), (4, 12), (4, 13), (4, 14),
     (4, 15), (4, 16), (4, 17), (4, 18), (4, 19), (4, 20), (4, 21), (4, 22), (5, 6),
     (6, 7), (7, 8), (8, 9), (9, 10), (10, 11), (11, 12), (12, 13), (13, 14), (14,
    15), (15, 16), (16, 17), (17, 18), (18, 19), (19, 20), (20, 21), (21, 22)])

#5-connected triangulations which maximize the WI
t5c12maxwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 3), (2, 6), (2, 7),
    (2, 8), (3, 4), (3, 8), (3, 9), (4, 5), (4, 9), (4, 10), (5, 6), (5, 10), (5,
    11), (6, 7), (6, 11), (7, 8), (7, 11), (7, 12), (8, 9), (8, 12), (9, 10), (9,
    12), (10, 11), (10, 12), (11, 12)])
t5c14maxwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 3), (2, 6), (2, 7),
    (2, 8), (3, 4), (3, 8), (3, 9), (4, 5), (4, 9), (4, 10), (5, 6), (5, 10), (5,
    11), (6, 7), (6, 11), (6, 12), (7, 8), (7, 12), (7, 13), (8, 9), (8, 13), (9,
    10), (9, 13), (9, 14), (10, 11), (10, 14), (11, 12), (11, 14), (12, 13), (12,
    14), (13, 14)])
t5c15maxwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (1, 7), (2, 3), (2, 7),
    (2, 8), (2, 9), (3, 4), (3, 9), (3, 10), (4, 5), (4, 10), (4, 11), (5, 6), (5,
    11), (5, 12), (6, 7), (6, 12), (6, 13), (7, 8), (7, 13), (8, 9), (8, 13), (8,
    14), (9, 10), (9, 14), (10, 11), (10, 14), (10, 15), (11, 12), (11, 15), (12,
    13), (12, 15), (13, 14), (13, 15), (14, 15)])
t5c16_1maxwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 3), (2, 6), (2, 7)
    , (2, 8), (3, 4), (3, 8), (3, 9), (3, 10), (4, 5), (4, 10), (4, 11), (5, 6), (5,
     11), (5, 12), (6, 7), (6, 12), (6, 13), (7, 8), (7, 13), (7, 14), (8, 9), (8,
    14), (9, 10), (9, 14), (9, 15), (10, 11), (10, 15), (11, 12), (11, 15), (11, 16)
    , (12, 13), (12, 16), (13, 14), (13, 16), (14, 15), (14, 16), (15, 16)])
t5c16_2maxwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 3), (2, 6), (2, 7)
    , (2, 8), (3, 4), (3, 8), (3, 9), (3, 10), (4, 5), (4, 10), (4, 11), (5, 6), (5,
     11), (5, 12), (6, 7), (6, 12), (6, 13), (7, 8), (7, 13), (7, 14), (8, 9), (8,
    14), (9, 10), (9, 14), (9, 15), (10, 11), (10, 15), (10, 16), (11, 12), (11, 16)
    , (12, 13), (12, 16), (13, 14), (13, 15), (13, 16), (14, 15), (15, 16)])
t5c17maxwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 3), (2, 6), (2, 7),
    (2, 8), (2, 9), (3, 4), (3, 9), (3, 10), (4, 5), (4, 10), (4, 11), (5, 6), (5,
    11), (5, 12), (6, 7), (6, 12), (6, 13), (7, 8), (7, 13), (7, 14), (8, 9), (8,
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    14), (8, 15), (9, 10), (9, 15), (9, 16), (10, 11), (10, 16), (11, 12), (11, 16),
     (12, 13), (12, 16), (12, 17), (13, 14), (13, 17), (14, 15), (14, 17), (15, 16),
     (15, 17), (16, 17)])
t5c18maxwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (1, 7), (2, 3), (2, 7),
    (2, 8), (2, 9), (2, 10), (3, 4), (3, 10), (3, 11), (4, 5), (4, 11), (4, 12), (4,
     13), (5, 6), (5, 13), (5, 14), (6, 7), (6, 14), (6, 15), (7, 8), (7, 15), (8,
    9), (8, 15), (8, 16), (8, 17), (9, 10), (9, 17), (9, 18), (10, 11), (10, 18),
    (11, 12), (11, 18), (12, 13), (12, 17), (12, 18), (13, 14), (13, 16), (13, 17),
    (14, 15), (14, 16), (15, 16), (16, 17), (17, 18)])
t5c19maxwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 3), (2, 6), (2, 7),
    (2, 8), (2, 9), (3, 4), (3, 9), (3, 10), (4, 5), (4, 10), (4, 11), (4, 12), (5,
    6), (5, 12), (5, 13), (5, 14), (6, 7), (6, 14), (7, 8), (7, 14), (7, 15), (8, 9)
    , (8, 15), (8, 16), (9, 10), (9, 16), (10, 11), (10, 16), (10, 17), (11, 12),
    (11, 17), (11, 18), (12, 13), (12, 18), (13, 14), (13, 18), (13, 19), (14, 15),
    (14, 19), (15, 16), (15, 17), (15, 19), (16, 17), (17, 18), (17, 19), (18, 19)])
t5c20maxwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 3), (2, 6), (2, 7),
    (2, 8), (2, 9), (3, 4), (3, 9), (3, 10), (3, 11), (4, 5), (4, 11), (4, 12), (5,
    6), (5, 12), (5, 13), (5, 14), (6, 7), (6, 14), (7, 8), (7, 14), (7, 15), (7,
    16), (8, 9), (8, 16), (8, 17), (9, 10), (9, 17), (10, 11), (10, 17), (10, 18),
    (11, 12), (11, 18), (11, 19), (12, 13), (12, 19), (13, 14), (13, 15), (13, 19),
    (14, 15), (15, 16), (15, 19), (15, 20), (16, 17), (16, 20), (17, 18), (17, 20),
    (18, 19), (18, 20), (19, 20)])
t5c21maxwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (1, 7), (2, 3), (2, 7),
    (2, 8), (2, 9), (2, 10), (3, 4), (3, 10), (3, 11), (4, 5), (4, 11), (4, 12), (5,
     6), (5, 12), (5, 13), (6, 7), (6, 13), (6, 14), (6, 15), (7, 8), (7, 15), (7,
    16), (8, 9), (8, 16), (8, 17), (9, 10), (9, 17), (9, 18), (9, 19), (10, 11),
    (10, 19), (10, 20), (11, 12), (11, 20), (12, 13), (12, 20), (13, 14), (13, 19),
    (13, 20), (14, 15), (14, 18), (14, 19), (14, 21), (15, 16), (15, 21), (16, 17),
    (16, 21), (17, 18), (17, 21), (18, 19), (18, 21), (19, 20)])
t5c22_1maxwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 3), (2, 6), (2, 7)
    , (2, 8), (2, 9), (3, 4), (3, 9), (3, 10), (4, 5), (4, 10), (4, 11), (5, 6), (5,
     11), (5, 12), (6, 7), (6, 12), (7, 8), (7, 12), (7, 13), (7, 14), (7, 15), (8,
    9), (8, 15), (8, 16), (8, 17), (9, 10), (9, 17), (9, 18), (9, 19), (10, 11),
    (10, 19), (11, 12), (11, 19), (12, 13), (12, 19), (13, 14), (13, 18), (13, 19),
    (13, 20), (14, 15), (14, 20), (14, 21), (15, 16), (15, 21), (16, 17), (16, 21),
    (16, 22), (17, 18), (17, 22), (18, 19), (18, 20), (18, 22), (20, 21), (20, 22),
    (21, 22)])
t5c22_2maxwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 3), (2, 6), (2, 7)
    , (2, 8), (2, 9), (3, 4), (3, 9), (3, 10), (4, 5), (4, 10), (4, 11), (5, 6), (5,
     11), (5, 12), (6, 7), (6, 12), (6, 13), (7, 8), (7, 13), (7, 14), (7, 15), (8,
    9), (8, 15), (8, 16), (8, 17), (9, 10), (9, 17), (9, 18), (10, 11), (10, 18),
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    (11, 12), (11, 18), (12, 13), (12, 18), (12, 19), (13, 14), (13, 19), (13, 20),
    (14, 15), (14, 20), (14, 21), (15, 16), (15, 21), (16, 17), (16, 21), (16, 22),
    (17, 18), (17, 19), (17, 22), (18, 19), (19, 20), (19, 22), (20, 21), (20, 22),
    (21, 22)])
t5c23maxwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 3), (2, 6), (2, 7),
    (2, 8), (2, 9), (3, 4), (3, 9), (3, 10), (4, 5), (4, 10), (4, 11), (5, 6), (5,
    11), (5, 12), (6, 7), (6, 12), (6, 13), (6, 14), (6, 15), (7, 8), (7, 15), (7,
    16), (7, 17), (8, 9), (8, 17), (8, 18), (8, 19), (9, 10), (9, 19), (9, 20), (10,
     11), (10, 20), (11, 12), (11, 20), (12, 13), (12, 19), (12, 20), (13, 14), (13,
     18), (13, 19), (13, 21), (14, 15), (14, 21), (14, 22), (15, 16), (15, 22), (16,
     17), (16, 22), (16, 23), (17, 18), (17, 23), (18, 19), (18, 21), (18, 23), (19,
     20), (21, 22), (21, 23), (22, 23)])
t5c24maxwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 3), (2, 6), (2, 7),
    (2, 8), (2, 9), (2, 10), (3, 4), (3, 10), (3, 11), (4, 5), (4, 11), (4, 12), (5,
     6), (5, 12), (5, 13), (6, 7), (6, 13), (6, 14), (7, 8), (7, 14), (7, 15), (7,
    16), (8, 9), (8, 16), (8, 17), (8, 18), (9, 10), (9, 18), (9, 19), (9, 20), (10,
     11), (10, 20), (10, 21), (11, 12), (11, 21), (12, 13), (12, 21), (13, 14), (13,
     20), (13, 21), (14, 15), (14, 19), (14, 20), (14, 22), (15, 16), (15, 22), (15,
     23), (16, 17), (16, 23), (17, 18), (17, 23), (17, 24), (18, 19), (18, 24), (19,
     20), (19, 22), (19, 24), (20, 21), (22, 23), (22, 24), (23, 24)])
t5c25maxwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 3), (2, 6), (2, 7),
    (2, 8), (2, 9), (2, 10), (3, 4), (3, 10), (3, 11), (4, 5), (4, 11), (4, 12), (5,
     6), (5, 12), (5, 13), (6, 7), (6, 13), (6, 14), (7, 8), (7, 14), (7, 15), (7,
    16), (7, 17), (8, 9), (8, 17), (8, 18), (8, 19), (9, 10), (9, 19), (9, 20), (9,
    21), (10, 11), (10, 21), (10, 22), (11, 12), (11, 22), (12, 13), (12, 22), (13,
    14), (13, 22), (14, 15), (14, 21), (14, 22), (15, 16), (15, 20), (15, 21), (15,
    23), (16, 17), (16, 23), (16, 24), (17, 18), (17, 24), (18, 19), (18, 24), (18,
    25), (19, 20), (19, 25), (20, 21), (20, 23), (20, 25), (21, 22), (23, 24), (23,
    25), (24, 25)])
t5c26maxwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 3), (2, 6), (2, 7),
    (2, 8), (2, 9), (3, 4), (3, 9), (3, 10), (4, 5), (4, 10), (4, 11), (5, 6), (5,
    11), (5, 12), (6, 7), (6, 12), (6, 13), (6, 14), (7, 8), (7, 14), (7, 15), (7,
    16), (8, 9), (8, 16), (8, 17), (8, 18), (9, 10), (9, 18), (10, 11), (10, 18),
    (11, 12), (11, 18), (12, 13), (12, 17), (12, 18), (13, 14), (13, 17), (13, 19),
    (13, 20), (13, 21), (14, 15), (14, 21), (14, 22), (15, 16), (15, 22), (15, 23),
    (15, 24), (16, 17), (16, 19), (16, 24), (17, 18), (17, 19), (19, 20), (19, 24),
    (19, 25), (20, 21), (20, 25), (20, 26), (21, 22), (21, 26), (22, 23), (22, 26),
    (23, 24), (23, 25), (23, 26), (24, 25), (25, 26)])
t5c27maxwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 3), (2, 6), (2, 7),
    (2, 8), (2, 9), (2, 10), (3, 4), (3, 10), (3, 11), (4, 5), (4, 11), (4, 12), (5,
     6), (5, 12), (5, 13), (6, 7), (6, 13), (6, 14), (7, 8), (7, 14), (7, 15), (7,
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    16), (8, 9), (8, 16), (8, 17), (8, 18), (9, 10), (9, 18), (9, 19), (9, 20), (10,
     11), (10, 20), (10, 21), (11, 12), (11, 21), (12, 13), (12, 21), (13, 14), (13,
     20), (13, 21), (14, 15), (14, 19), (14, 20), (15, 16), (15, 19), (15, 22), (15,
     23), (16, 17), (16, 23), (16, 24), (17, 18), (17, 24), (17, 25), (18, 19), (18,
     25), (18, 26), (19, 20), (19, 22), (19, 26), (20, 21), (22, 23), (22, 26), (22,
     27), (23, 24), (23, 27), (24, 25), (24, 27), (25, 26), (25, 27), (26, 27)])
t5c28maxwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 3), (2, 6), (2, 7),
    (2, 8), (3, 4), (3, 8), (3, 9), (3, 10), (4, 5), (4, 10), (4, 11), (5, 6), (5,
    11), (5, 12), (6, 7), (6, 12), (7, 8), (7, 12), (7, 13), (8, 9), (8, 13), (8,
    14), (8, 15), (9, 10), (9, 15), (9, 16), (9, 17), (10, 11), (10, 17), (10, 18),
    (11, 12), (11, 13), (11, 18), (12, 13), (13, 14), (13, 18), (14, 15), (14, 18),
    (14, 19), (14, 20), (15, 16), (15, 20), (15, 21), (15, 22), (16, 17), (16, 22),
    (16, 23), (16, 24), (17, 18), (17, 19), (17, 24), (18, 19), (19, 20), (19, 24),
    (19, 25), (20, 21), (20, 25), (20, 26), (21, 22), (21, 26), (21, 27), (22, 23),
    (22, 27), (23, 24), (23, 27), (23, 28), (24, 25), (24, 28), (25, 26), (25, 28),
    (26, 27), (26, 28), (27, 28)])
t5c29maxwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 3), (2, 6), (2, 7),
    (2, 8), (2, 9), (2, 10), (3, 4), (3, 10), (3, 11), (4, 5), (4, 11), (4, 12), (5,
     6), (5, 12), (5, 13), (6, 7), (6, 13), (6, 14), (7, 8), (7, 14), (7, 15), (7,
    16), (8, 9), (8, 16), (8, 17), (8, 18), (9, 10), (9, 18), (9, 19), (9, 20), (10,
     11), (10, 20), (10, 21), (11, 12), (11, 21), (12, 13), (12, 21), (13, 14), (13,
     20), (13, 21), (14, 15), (14, 19), (14, 20), (15, 16), (15, 19), (15, 22), (15,
     23), (15, 24), (16, 17), (16, 24), (16, 25), (17, 18), (17, 25), (17, 26), (17,
     27), (18, 19), (18, 22), (18, 27), (19, 20), (19, 22), (20, 21), (22, 23), (22,
     27), (22, 28), (23, 24), (23, 28), (23, 29), (24, 25), (24, 29), (25, 26), (25,
     29), (26, 27), (26, 28), (26, 29), (27, 28), (28, 29)])
t5c30maxwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 3), (2, 6), (2, 7),
    (2, 8), (2, 9), (2, 10), (3, 4), (3, 10), (3, 11), (4, 5), (4, 11), (4, 12), (5,
     6), (5, 12), (5, 13), (6, 7), (6, 13), (6, 14), (7, 8), (7, 14), (7, 15), (7,
    16), (8, 9), (8, 16), (8, 17), (8, 18), (9, 10), (9, 18), (9, 19), (9, 20), (10,
     11), (10, 20), (10, 21), (11, 12), (11, 21), (12, 13), (12, 21), (13, 14), (13,
     20), (13, 21), (14, 15), (14, 19), (14, 20), (15, 16), (15, 19), (15, 22), (15,
     23), (16, 17), (16, 23), (16, 24), (17, 18), (17, 24), (17, 25), (17, 26), (18,
     19), (18, 22), (18, 26), (18, 27), (19, 20), (19, 22), (20, 21), (22, 23), (22,
     27), (22, 28), (23, 24), (23, 28), (23, 29), (24, 25), (24, 29), (25, 26), (25,
     29), (25, 30), (26, 27), (26, 30), (27, 28), (27, 30), (28, 29), (28, 30), (29,
     30)])
t5c31maxwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 3), (2, 6), (2, 7),
    (2, 8), (2, 9), (3, 4), (3, 9), (3, 10), (4, 5), (4, 10), (4, 11), (5, 6), (5,
    11), (5, 12), (6, 7), (6, 12), (6, 13), (6, 14), (7, 8), (7, 14), (7, 15), (7,
    16), (8, 9), (8, 16), (8, 17), (8, 18), (9, 10), (9, 18), (10, 11), (10, 18),
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    (11, 12), (11, 18), (12, 13), (12, 17), (12, 18), (13, 14), (13, 17), (13, 19),
    (13, 20), (14, 15), (14, 20), (14, 21), (15, 16), (15, 21), (15, 22), (15, 23),
    (16, 17), (16, 19), (16, 23), (17, 18), (17, 19), (19, 20), (19, 23), (19, 24),
    (20, 21), (20, 24), (20, 25), (20, 26), (21, 22), (21, 26), (21, 27), (22, 23),
    (22, 27), (22, 28), (22, 29), (23, 24), (23, 29), (24, 25), (24, 29), (24, 30),
    (25, 26), (25, 30), (25, 31), (26, 27), (26, 31), (27, 28), (27, 31), (28, 29),
    (28, 30), (28, 31), (29, 30), (30, 31)])
t5c32maxwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 3), (2, 6), (2, 7),
    (2, 8), (2, 9), (2, 10), (3, 4), (3, 10), (3, 11), (4, 5), (4, 11), (4, 12), (5,
     6), (5, 12), (5, 13), (6, 7), (6, 13), (6, 14), (7, 8), (7, 14), (7, 15), (7,
    16), (8, 9), (8, 16), (8, 17), (8, 18), (9, 10), (9, 18), (9, 19), (9, 20), (10,
     11), (10, 20), (10, 21), (11, 12), (11, 21), (12, 13), (12, 21), (13, 14), (13,
     20), (13, 21), (14, 15), (14, 19), (14, 20), (15, 16), (15, 19), (15, 22), (15,
     23), (16, 17), (16, 23), (16, 24), (17, 18), (17, 24), (17, 25), (17, 26), (18,
     19), (18, 22), (18, 26), (19, 20), (19, 22), (20, 21), (22, 23), (22, 26), (22,
     27), (22, 28), (23, 24), (23, 28), (23, 29), (24, 25), (24, 29), (24, 30), (25,
     26), (25, 30), (25, 31), (26, 27), (26, 31), (27, 28), (27, 31), (27, 32), (28,
     29), (28, 32), (29, 30), (29, 32), (30, 31), (30, 32), (31, 32)])

#5-connected triangulations which minimize the WI
t5c12minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 3), (2, 6), (2, 7),
    (2, 8), (3, 4), (3, 8), (3, 9), (4, 5), (4, 9), (4, 10), (5, 6), (5, 10), (5,
    11), (6, 7), (6, 11), (7, 8), (7, 11), (7, 12), (8, 9), (8, 12), (9, 10), (9,
    12), (10, 11), (10, 12), (11, 12)])
t5c14minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 3), (2, 6), (2, 7),
    (2, 8), (3, 4), (3, 8), (3, 9), (4, 5), (4, 9), (4, 10), (5, 6), (5, 10), (5,
    11), (6, 7), (6, 11), (6, 12), (7, 8), (7, 12), (7, 13), (8, 9), (8, 13), (9,
    10), (9, 13), (9, 14), (10, 11), (10, 14), (11, 12), (11, 14), (12, 13), (12,
    14), (13, 14)])
t5c15minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (1, 7), (2, 3), (2, 7),
    (2, 8), (2, 9), (3, 4), (3, 9), (3, 10), (4, 5), (4, 10), (4, 11), (5, 6), (5,
    11), (5, 12), (6, 7), (6, 12), (6, 13), (7, 8), (7, 13), (8, 9), (8, 13), (8,
    14), (9, 10), (9, 14), (10, 11), (10, 14), (10, 15), (11, 12), (11, 15), (12,
    13), (12, 15), (13, 14), (13, 15), (14, 15)])
t5c16minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 3), (2, 6), (2, 7),
    (2, 8), (2, 9), (2, 10), (3, 4), (3, 10), (3, 11), (4, 5), (4, 11), (4, 12), (5,
     6), (5, 12), (5, 13), (6, 7), (6, 13), (7, 8), (7, 13), (7, 14), (8, 9), (8,
    14), (8, 15), (9, 10), (9, 15), (9, 16), (10, 11), (10, 16), (11, 12), (11, 16),
     (12, 13), (12, 14), (12, 15), (12, 16), (13, 14), (14, 15), (15, 16)])
t5c17minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (1, 7), (2, 3), (2, 7),
    (2, 8), (2, 9), (3, 4), (3, 9), (3, 10), (4, 5), (4, 10), (4, 11), (5, 6), (5,
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    11), (5, 12), (6, 7), (6, 12), (6, 13), (7, 8), (7, 13), (7, 14), (8, 9), (8,
    14), (8, 15), (8, 16), (9, 10), (9, 16), (10, 11), (10, 16), (11, 12), (11, 15),
     (11, 16), (11, 17), (12, 13), (12, 17), (13, 14), (13, 17), (14, 15), (14, 17),
     (15, 16), (15, 17)])
t5c18minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 3), (2, 6), (2, 7),
    (2, 8), (3, 4), (3, 8), (3, 9), (4, 5), (4, 9), (4, 10), (4, 11), (4, 12), (4,
    13), (5, 6), (5, 13), (5, 14), (6, 7), (6, 14), (7, 8), (7, 14), (7, 15), (7,
    16), (7, 17), (7, 18), (8, 9), (8, 18), (9, 10), (9, 18), (10, 11), (10, 17),
    (10, 18), (11, 12), (11, 16), (11, 17), (12, 13), (12, 15), (12, 16), (13, 14),
    (13, 15), (14, 15), (15, 16), (16, 17), (17, 18)])
t5c19_1minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (1, 7), (2, 3), (2, 7)
    , (2, 8), (2, 9), (3, 4), (3, 9), (3, 10), (4, 5), (4, 10), (4, 11), (5, 6), (5,
     11), (5, 12), (6, 7), (6, 12), (6, 13), (7, 8), (7, 13), (7, 14), (7, 15), (8,
    9), (8, 15), (8, 16), (8, 17), (9, 10), (9, 17), (10, 11), (10, 17), (11, 12),
    (11, 16), (11, 17), (11, 18), (11, 19), (12, 13), (12, 19), (13, 14), (13, 19),
    (14, 15), (14, 18), (14, 19), (15, 16), (15, 18), (16, 17), (16, 18), (18, 19)])
t5c19_2minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 3), (2, 6), (2, 7)
    , (2, 8), (2, 9), (2, 10), (3, 4), (3, 10), (3, 11), (4, 5), (4, 11), (4, 12),
    (5, 6), (5, 12), (5, 13), (6, 7), (6, 13), (6, 14), (6, 15), (7, 8), (7, 15),
    (7, 16), (8, 9), (8, 16), (8, 17), (9, 10), (9, 17), (9, 18), (10, 11), (10, 18)
    , (11, 12), (11, 18), (12, 13), (12, 18), (13, 14), (13, 18), (14, 15), (14, 18)
    , (14, 19), (15, 16), (15, 19), (16, 17), (16, 19), (17, 18), (17, 19), (18, 19)
    ])
t5c20minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 3), (2, 6), (2, 7),
    (2, 8), (3, 4), (3, 8), (3, 9), (4, 5), (4, 9), (4, 10), (4, 11), (4, 12), (4,
    13), (4, 14), (5, 6), (5, 14), (5, 15), (6, 7), (6, 15), (7, 8), (7, 15), (7,
    16), (7, 17), (7, 18), (7, 19), (7, 20), (8, 9), (8, 20), (9, 10), (9, 20), (10,
     11), (10, 19), (10, 20), (11, 12), (11, 18), (11, 19), (12, 13), (12, 17), (12,
     18), (13, 14), (13, 16), (13, 17), (14, 15), (14, 16), (15, 16), (16, 17), (17,
     18), (18, 19), (19, 20)])
t5c21minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (1, 7), (2, 3), (2, 7),
    (2, 8), (2, 9), (3, 4), (3, 9), (3, 10), (4, 5), (4, 10), (4, 11), (5, 6), (5,
    11), (5, 12), (6, 7), (6, 12), (6, 13), (7, 8), (7, 13), (7, 14), (7, 15), (7,
    16), (8, 9), (8, 16), (8, 17), (8, 18), (9, 10), (9, 18), (10, 11), (10, 18),
    (11, 12), (11, 17), (11, 18), (11, 19), (11, 20), (11, 21), (12, 13), (12, 21),
    (13, 14), (13, 21), (14, 15), (14, 20), (14, 21), (15, 16), (15, 19), (15, 20),
    (16, 17), (16, 19), (17, 18), (17, 19), (19, 20), (20, 21)])
t5c22minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 3), (2, 6), (2, 7),
    (2, 8), (3, 4), (3, 8), (3, 9), (4, 5), (4, 9), (4, 10), (4, 11), (4, 12), (4,
    13), (4, 14), (4, 15), (5, 6), (5, 15), (5, 16), (6, 7), (6, 16), (7, 8), (7,
    16), (7, 17), (7, 18), (7, 19), (7, 20), (7, 21), (7, 22), (8, 9), (8, 22), (9,
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    10), (9, 22), (10, 11), (10, 21), (10, 22), (11, 12), (11, 20), (11, 21), (12,
    13), (12, 19), (12, 20), (13, 14), (13, 18), (13, 19), (14, 15), (14, 17), (14,
    18), (15, 16), (15, 17), (16, 17), (17, 18), (18, 19), (19, 20), (20, 21), (21,
    22)])
t5c23minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (1, 7), (2, 3), (2, 7),
    (2, 8), (2, 9), (3, 4), (3, 9), (3, 10), (4, 5), (4, 10), (4, 11), (5, 6), (5,
    11), (5, 12), (5, 13), (5, 14), (5, 15), (5, 16), (6, 7), (6, 16), (6, 17), (7,
    8), (7, 17), (8, 9), (8, 17), (8, 18), (8, 19), (8, 20), (8, 21), (8, 22), (8,
    23), (9, 10), (9, 23), (10, 11), (10, 23), (11, 12), (11, 22), (11, 23), (12,
    13), (12, 21), (12, 22), (13, 14), (13, 20), (13, 21), (14, 15), (14, 19), (14,
    20), (15, 16), (15, 18), (15, 19), (16, 17), (16, 18), (17, 18), (18, 19), (19,
    20), (20, 21), (21, 22), (22, 23)])
t5c24minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 3), (2, 6), (2, 7),
    (2, 8), (3, 4), (3, 8), (3, 9), (4, 5), (4, 9), (4, 10), (4, 11), (4, 12), (4,
    13), (4, 14), (4, 15), (4, 16), (5, 6), (5, 16), (5, 17), (6, 7), (6, 17), (7,
    8), (7, 17), (7, 18), (7, 19), (7, 20), (7, 21), (7, 22), (7, 23), (7, 24), (8,
    9), (8, 24), (9, 10), (9, 24), (10, 11), (10, 23), (10, 24), (11, 12), (11, 22),
     (11, 23), (12, 13), (12, 21), (12, 22), (13, 14), (13, 20), (13, 21), (14, 15),
     (14, 19), (14, 20), (15, 16), (15, 18), (15, 19), (16, 17), (16, 18), (17, 18),
     (18, 19), (19, 20), (20, 21), (21, 22), (22, 23), (23, 24)])
t5c25minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (1, 7), (2, 3), (2, 7),
    (2, 8), (2, 9), (3, 4), (3, 9), (3, 10), (4, 5), (4, 10), (4, 11), (5, 6), (5,
    11), (5, 12), (5, 13), (5, 14), (5, 15), (5, 16), (5, 17), (6, 7), (6, 17), (6,
    18), (7, 8), (7, 18), (8, 9), (8, 18), (8, 19), (8, 20), (8, 21), (8, 22), (8,
    23), (8, 24), (8, 25), (9, 10), (9, 25), (10, 11), (10, 25), (11, 12), (11, 24),
     (11, 25), (12, 13), (12, 23), (12, 24), (13, 14), (13, 22), (13, 23), (14, 15),
     (14, 21), (14, 22), (15, 16), (15, 20), (15, 21), (16, 17), (16, 19), (16, 20),
     (17, 18), (17, 19), (18, 19), (19, 20), (20, 21), (21, 22), (22, 23), (23, 24),
     (24, 25)])
t5c26minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 3), (2, 6), (2, 7),
    (2, 8), (3, 4), (3, 8), (3, 9), (4, 5), (4, 9), (4, 10), (4, 11), (4, 12), (4,
    13), (4, 14), (4, 15), (4, 16), (4, 17), (5, 6), (5, 17), (5, 18), (6, 7), (6,
    18), (7, 8), (7, 18), (7, 19), (7, 20), (7, 21), (7, 22), (7, 23), (7, 24), (7,
    25), (7, 26), (8, 9), (8, 26), (9, 10), (9, 26), (10, 11), (10, 25), (10, 26),
    (11, 12), (11, 24), (11, 25), (12, 13), (12, 23), (12, 24), (13, 14), (13, 22),
    (13, 23), (14, 15), (14, 21), (14, 22), (15, 16), (15, 20), (15, 21), (16, 17),
    (16, 19), (16, 20), (17, 18), (17, 19), (18, 19), (19, 20), (20, 21), (21, 22),
    (22, 23), (23, 24), (24, 25), (25, 26)])
t5c27minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (1, 7), (2, 3), (2, 7),
    (2, 8), (2, 9), (3, 4), (3, 9), (3, 10), (4, 5), (4, 10), (4, 11), (5, 6), (5,
    11), (5, 12), (5, 13), (5, 14), (5, 15), (5, 16), (5, 17), (5, 18), (6, 7), (6,
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    18), (6, 19), (7, 8), (7, 19), (8, 9), (8, 19), (8, 20), (8, 21), (8, 22), (8,
    23), (8, 24), (8, 25), (8, 26), (8, 27), (9, 10), (9, 27), (10, 11), (10, 27),
    (11, 12), (11, 26), (11, 27), (12, 13), (12, 25), (12, 26), (13, 14), (13, 24),
    (13, 25), (14, 15), (14, 23), (14, 24), (15, 16), (15, 22), (15, 23), (16, 17),
    (16, 21), (16, 22), (17, 18), (17, 20), (17, 21), (18, 19), (18, 20), (19, 20),
    (20, 21), (21, 22), (22, 23), (23, 24), (24, 25), (25, 26), (26, 27)])
t5c28minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 3), (2, 6), (2, 7),
    (2, 8), (3, 4), (3, 8), (3, 9), (4, 5), (4, 9), (4, 10), (4, 11), (4, 12), (4,
    13), (4, 14), (4, 15), (4, 16), (4, 17), (4, 18), (5, 6), (5, 18), (5, 19), (6,
    7), (6, 19), (7, 8), (7, 19), (7, 20), (7, 21), (7, 22), (7, 23), (7, 24), (7,
    25), (7, 26), (7, 27), (7, 28), (8, 9), (8, 28), (9, 10), (9, 28), (10, 11),
    (10, 27), (10, 28), (11, 12), (11, 26), (11, 27), (12, 13), (12, 25), (12, 26),
    (13, 14), (13, 24), (13, 25), (14, 15), (14, 23), (14, 24), (15, 16), (15, 22),
    (15, 23), (16, 17), (16, 21), (16, 22), (17, 18), (17, 20), (17, 21), (18, 19),
    (18, 20), (19, 20), (20, 21), (21, 22), (22, 23), (23, 24), (24, 25), (25, 26),
    (26, 27), (27, 28)])
t5c29minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (1, 7), (2, 3), (2, 7),
    (2, 8), (2, 9), (3, 4), (3, 9), (3, 10), (4, 5), (4, 10), (4, 11), (5, 6), (5,
    11), (5, 12), (5, 13), (5, 14), (5, 15), (5, 16), (5, 17), (5, 18), (5, 19), (6,
     7), (6, 19), (6, 20), (7, 8), (7, 20), (8, 9), (8, 20), (8, 21), (8, 22), (8,
    23), (8, 24), (8, 25), (8, 26), (8, 27), (8, 28), (8, 29), (9, 10), (9, 29),
    (10, 11), (10, 29), (11, 12), (11, 28), (11, 29), (12, 13), (12, 27), (12, 28),
    (13, 14), (13, 26), (13, 27), (14, 15), (14, 25), (14, 26), (15, 16), (15, 24),
    (15, 25), (16, 17), (16, 23), (16, 24), (17, 18), (17, 22), (17, 23), (18, 19),
    (18, 21), (18, 22), (19, 20), (19, 21), (20, 21), (21, 22), (22, 23), (23, 24),
    (24, 25), (25, 26), (26, 27), (27, 28), (28, 29)])
t5c30minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 3), (2, 6), (2, 7),
    (2, 8), (3, 4), (3, 8), (3, 9), (4, 5), (4, 9), (4, 10), (4, 11), (4, 12), (4,
    13), (4, 14), (4, 15), (4, 16), (4, 17), (4, 18), (4, 19), (5, 6), (5, 19), (5,
    20), (6, 7), (6, 20), (7, 8), (7, 20), (7, 21), (7, 22), (7, 23), (7, 24), (7,
    25), (7, 26), (7, 27), (7, 28), (7, 29), (7, 30), (8, 9), (8, 30), (9, 10), (9,
    30), (10, 11), (10, 29), (10, 30), (11, 12), (11, 28), (11, 29), (12, 13), (12,
    27), (12, 28), (13, 14), (13, 26), (13, 27), (14, 15), (14, 25), (14, 26), (15,
    16), (15, 24), (15, 25), (16, 17), (16, 23), (16, 24), (17, 18), (17, 22), (17,
    23), (18, 19), (18, 21), (18, 22), (19, 20), (19, 21), (20, 21), (21, 22), (22,
    23), (23, 24), (24, 25), (25, 26), (26, 27), (27, 28), (28, 29), (29, 30)])
t5c31minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (1, 7), (2, 3), (2, 7),
    (2, 8), (2, 9), (3, 4), (3, 9), (3, 10), (4, 5), (4, 10), (4, 11), (5, 6), (5,
    11), (5, 12), (5, 13), (5, 14), (5, 15), (5, 16), (5, 17), (5, 18), (5, 19), (5,
     20), (6, 7), (6, 20), (6, 21), (7, 8), (7, 21), (8, 9), (8, 21), (8, 22), (8,
    23), (8, 24), (8, 25), (8, 26), (8, 27), (8, 28), (8, 29), (8, 30), (8, 31), (9,
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     10), (9, 31), (10, 11), (10, 31), (11, 12), (11, 30), (11, 31), (12, 13), (12,
    29), (12, 30), (13, 14), (13, 28), (13, 29), (14, 15), (14, 27), (14, 28), (15,
    16), (15, 26), (15, 27), (16, 17), (16, 25), (16, 26), (17, 18), (17, 24), (17,
    25), (18, 19), (18, 23), (18, 24), (19, 20), (19, 22), (19, 23), (20, 21), (20,
    22), (21, 22), (22, 23), (23, 24), (24, 25), (25, 26), (26, 27), (27, 28), (28,
    29), (29, 30), (30, 31)])
t5c32minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 3), (2, 6), (2, 7),
    (2, 8), (3, 4), (3, 8), (3, 9), (4, 5), (4, 9), (4, 10), (4, 11), (4, 12), (4,
    13), (4, 14), (4, 15), (4, 16), (4, 17), (4, 18), (4, 19), (4, 20), (5, 6), (5,
    20), (5, 21), (6, 7), (6, 21), (7, 8), (7, 21), (7, 22), (7, 23), (7, 24), (7,
    25), (7, 26), (7, 27), (7, 28), (7, 29), (7, 30), (7, 31), (7, 32), (8, 9), (8,
    32), (9, 10), (9, 32), (10, 11), (10, 31), (10, 32), (11, 12), (11, 30), (11,
    31), (12, 13), (12, 29), (12, 30), (13, 14), (13, 28), (13, 29), (14, 15), (14,
    27), (14, 28), (15, 16), (15, 26), (15, 27), (16, 17), (16, 25), (16, 26), (17,
    18), (17, 24), (17, 25), (18, 19), (18, 23), (18, 24), (19, 20), (19, 22), (19,
    23), (20, 21), (20, 22), (21, 22), (22, 23), (23, 24), (24, 25), (25, 26), (26,
    27), (27, 28), (28, 29), (29, 30), (30, 31), (31, 32)])

#general triangulations which maximize the prox
t4p = Graph([(1, 2), (1, 3), (1, 4), (2, 3), (2, 4), (3, 4)])
t5p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 4), (2, 5), (3, 4), (4, 5)
    ])
t6p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (3, 4), (3, 6),
     (4, 5), (4, 6), (5, 6)])
t7_1p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 3), (2, 6), (2, 7), (3,
    4), (3, 7), (4, 5), (4, 7), (5, 6), (5, 7), (6, 7)])
t7_2p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (2, 7), (3,
    4), (3, 7), (4, 5), (4, 7), (5, 6), (5, 7), (6, 7)])
t8_1p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 3), (2, 6), (2, 7), (2,
    8), (3, 4), (3, 5), (3, 8), (4, 5), (5, 6), (5, 8), (6, 7), (6, 8), (7, 8)])
t8_2p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (2, 7), (3,
    4), (3, 7), (3, 8), (4, 5), (4, 6), (4, 8), (5, 6), (6, 7), (6, 8), (7, 8)])
t9p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (2, 7), (3, 4),
     (3, 7), (3, 8), (4, 5), (4, 8), (4, 9), (5, 6), (5, 9), (6, 7), (6, 9), (7, 8),
     (7, 9), (8, 9)])
t10p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (2, 7), (3, 4)
    , (3, 7), (3, 8), (4, 5), (4, 8), (4, 9), (5, 6), (5, 9), (6, 7), (6, 9), (6,
    10), (7, 8), (7, 10), (8, 9), (8, 10), (9, 10)])
t11_1p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (1, 7), (2, 3), (2, 6), (2,
    7), (2, 8), (2, 9), (3, 4), (3, 5), (3, 6), (4, 5), (5, 6), (6, 7), (6, 9), (7,
    8), (7, 9), (7, 10), (8, 9), (8, 10), (8, 11), (9, 10), (9, 11), (10, 11)])
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t11_2p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (1, 7), (2, 3), (2, 7), (2,
    8), (2, 9), (2, 10), (3, 4), (3, 8), (3, 9), (3, 10), (4, 5), (4, 8), (5, 6),
    (5, 7), (5, 8), (5, 11), (6, 7), (6, 11), (7, 8), (7, 11), (8, 9), (9, 10)])
t11_3p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (1, 7), (2, 3), (2, 7), (2,
    8), (2, 9), (2, 10), (3, 4), (3, 10), (3, 11), (4, 5), (4, 11), (5, 6), (5, 11),
     (6, 7), (6, 10), (6, 11), (7, 8), (7, 9), (7, 10), (8, 9), (9, 10), (10, 11)])
t11_4p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (1, 7), (2, 3), (2, 7), (2,
    8), (2, 9), (2, 10), (3, 4), (3, 10), (3, 11), (4, 5), (4, 10), (4, 11), (5, 6),
     (5, 9), (5, 10), (6, 7), (6, 9), (7, 8), (7, 9), (8, 9), (9, 10), (10, 11)])
t11_5p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (1, 7), (2, 3), (2, 7), (2,
    8), (2, 9), (3, 4), (3, 9), (3, 10), (4, 5), (4, 9), (4, 10), (5, 6), (5, 8),
    (5, 9), (6, 7), (6, 8), (6, 11), (7, 8), (7, 11), (8, 9), (8, 11), (9, 10)])
t11_6p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (1, 7), (2, 3), (2, 7), (2,
    8), (2, 9), (2, 10), (3, 4), (3, 10), (4, 5), (4, 10), (4, 11), (5, 6), (5, 9),
    (5, 10), (5, 11), (6, 7), (6, 9), (7, 8), (7, 9), (8, 9), (9, 10), (10, 11)])
t11_7p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (1, 7), (2, 3), (2, 7), (2,
    8), (2, 9), (2, 10), (3, 4), (3, 10), (4, 5), (4, 10), (5, 6), (5, 9), (5, 10),
    (6, 7), (6, 9), (6, 11), (7, 8), (7, 11), (8, 9), (8, 11), (9, 10), (9, 11)])
t11_8p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (1, 7), (2, 3), (2, 7), (2,
    8), (3, 4), (3, 8), (3, 9), (4, 5), (4, 9), (4, 10), (4, 11), (5, 6), (5, 11),
    (6, 7), (6, 11), (7, 8), (7, 11), (8, 9), (8, 10), (8, 11), (9, 10), (10, 11)])
t11_9p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (1, 7), (2, 3), (2, 7), (2,
    8), (3, 4), (3, 8), (4, 5), (4, 8), (4, 9), (4, 10), (5, 6), (5, 10), (6, 7),
    (6, 10), (7, 8), (7, 10), (7, 11), (8, 9), (8, 11), (9, 10), (9, 11), (10, 11)])
t11_10p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (1, 7), (2, 3), (2, 7), (2,
     8), (2, 9), (3, 4), (3, 9), (3, 10), (4, 5), (4, 10), (5, 6), (5, 10), (5, 11),
     (6, 7), (6, 11), (7, 8), (7, 9), (7, 11), (8, 9), (9, 10), (9, 11), (10, 11)])
t11_11p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (1, 7), (2, 3), (2, 7), (2,
     8), (3, 4), (3, 8), (3, 9), (4, 5), (4, 9), (5, 6), (5, 9), (5, 10), (6, 7),
    (6, 10), (7, 8), (7, 10), (7, 11), (8, 9), (8, 10), (8, 11), (9, 10), (10, 11)])
t11_12p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (1, 7), (2, 3), (2, 7), (2,
     8), (3, 4), (3, 8), (3, 9), (4, 5), (4, 9), (5, 6), (5, 9), (5, 10), (6, 7),
    (6, 10), (7, 8), (7, 10), (8, 9), (8, 10), (8, 11), (9, 10), (9, 11), (10, 11)])
t11_13p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (1, 7), (2, 3), (2, 7), (2,
     8), (2, 9), (3, 4), (3, 9), (4, 5), (4, 9), (4, 10), (5, 6), (5, 10), (5, 11),
    (6, 7), (6, 11), (7, 8), (7, 11), (8, 9), (8, 10), (8, 11), (9, 10), (10, 11)])
t11_14p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (3, 4), (3,
     6), (4, 5), (4, 6), (4, 7), (4, 8), (5, 6), (5, 8), (5, 9), (6, 7), (6, 9), (7,
     8), (7, 9), (7, 10), (7, 11), (8, 9), (8, 10), (8, 11), (9, 10), (10, 11)])
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t11_15p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 3), (2, 4), (2, 6), (2,
     7), (3, 4), (4, 5), (4, 7), (5, 6), (5, 7), (5, 8), (5, 9), (6, 7), (6, 9), (6,
     10), (7, 8), (7, 10), (8, 9), (8, 10), (8, 11), (9, 10), (9, 11), (10, 11)])
t11_16p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (1, 7), (2, 3), (2, 5), (2,
     7), (2, 8), (2, 9), (3, 4), (3, 5), (4, 5), (5, 6), (5, 9), (6, 7), (6, 9), (6,
     10), (7, 8), (7, 10), (7, 11), (8, 9), (8, 11), (9, 10), (9, 11), (10, 11)])
t11_17p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (1, 7), (2, 3), (2, 4), (2,
     7), (2, 8), (2, 9), (3, 4), (4, 5), (4, 6), (4, 9), (5, 6), (6, 7), (6, 9), (6,
     10), (7, 8), (7, 10), (7, 11), (8, 9), (8, 11), (9, 10), (9, 11), (10, 11)])
t11_18p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (3, 4), (3,
     6), (3, 7), (4, 5), (4, 7), (4, 8), (4, 9), (5, 6), (5, 9), (6, 7), (6, 9), (6,
     10), (7, 8), (7, 10), (8, 9), (8, 10), (8, 11), (9, 10), (9, 11), (10, 11)])
t11_19p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 3), (2, 6), (2, 7), (2,
     8), (3, 4), (3, 5), (3, 8), (3, 9), (4, 5), (5, 6), (5, 7), (5, 9), (6, 7), (7,
     8), (7, 9), (7, 10), (8, 9), (8, 10), (8, 11), (9, 10), (9, 11), (10, 11)])
t11_20p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (2, 7), (3,
     4), (3, 7), (4, 5), (4, 7), (4, 8), (5, 6), (5, 8), (5, 9), (6, 7), (6, 9), (6,
     10), (6, 11), (7, 8), (7, 11), (8, 9), (8, 10), (8, 11), (9, 10), (10, 11)])
t11_21p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (1, 7), (2, 3), (2, 5), (2,
     7), (2, 8), (2, 9), (3, 4), (3, 9), (3, 10), (4, 5), (4, 8), (4, 10), (5, 6),
    (5, 7), (5, 8), (6, 7), (8, 9), (8, 10), (8, 11), (9, 10), (9, 11), (10, 11)])
t11_22p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (1, 7), (2, 3), (2, 7), (2,
     8), (2, 9), (3, 4), (3, 5), (3, 9), (3, 10), (4, 5), (5, 6), (5, 7), (5, 10),
    (6, 7), (7, 8), (7, 10), (8, 9), (8, 10), (8, 11), (9, 10), (9, 11), (10, 11)])
t11_23p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (1, 7), (2, 3), (2, 4), (2,
     7), (2, 8), (3, 4), (4, 5), (4, 8), (4, 9), (5, 6), (5, 7), (5, 9), (5, 10),
    (6, 7), (7, 8), (7, 10), (8, 9), (8, 10), (8, 11), (9, 10), (9, 11), (10, 11)])
t11_24p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (1, 7), (2, 3), (2, 7), (2,
     8), (2, 9), (2, 10), (3, 4), (3, 10), (4, 5), (4, 9), (4, 10), (4, 11), (5, 6),
     (5, 7), (5, 8), (5, 11), (6, 7), (7, 8), (8, 9), (8, 11), (9, 10), (9, 11)])
t11_25p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 3), (2, 6), (2, 7), (2,
     8), (2, 9), (3, 4), (3, 8), (3, 9), (4, 5), (4, 8), (4, 10), (4, 11), (5, 6),
    (5, 11), (6, 7), (6, 10), (6, 11), (7, 8), (7, 10), (8, 9), (8, 10), (10, 11)])
t11_26p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 3), (2, 6), (2, 7), (2,
     8), (3, 4), (3, 8), (4, 5), (4, 8), (4, 9), (5, 6), (5, 9), (5, 10), (5, 11),
    (6, 7), (6, 11), (7, 8), (7, 10), (7, 11), (8, 9), (8, 10), (9, 10), (10, 11)])
t11_27p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 3), (2, 6), (2, 7), (2,
     8), (2, 9), (3, 4), (3, 9), (4, 5), (4, 9), (4, 10), (4, 11), (5, 6), (5, 7),
    (5, 11), (6, 7), (7, 8), (7, 10), (7, 11), (8, 9), (8, 10), (9, 10), (10, 11)])
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t11_28p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 3), (2, 6), (2, 7), (2,
     8), (2, 9), (3, 4), (3, 9), (3, 10), (3, 11), (4, 5), (4, 6), (4, 7), (4, 11),
    (5, 6), (6, 7), (7, 8), (7, 10), (7, 11), (8, 9), (8, 10), (9, 10), (10, 11)])
t11_29p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (2, 7), (3,
     4), (3, 7), (4, 5), (4, 7), (4, 8), (4, 9), (5, 6), (5, 9), (5, 10), (6, 7),
    (6, 8), (6, 10), (7, 8), (8, 9), (8, 10), (8, 11), (9, 10), (9, 11), (10, 11)])
t11_30p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (2, 7), (3,
     4), (3, 7), (4, 5), (4, 7), (4, 8), (4, 9), (5, 6), (5, 9), (5, 10), (6, 7),
    (6, 8), (6, 10), (6, 11), (7, 8), (8, 9), (8, 10), (8, 11), (9, 10), (10, 11)])
t11_31p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (2, 7), (2,
     8), (3, 4), (3, 8), (3, 9), (4, 5), (4, 9), (5, 6), (5, 9), (5, 10), (6, 7),
    (6, 10), (7, 8), (7, 10), (8, 9), (8, 10), (8, 11), (9, 10), (9, 11), (10, 11)])
t11_32p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (2, 7), (2,
     8), (3, 4), (3, 8), (3, 9), (4, 5), (4, 9), (4, 10), (5, 6), (5, 10), (5, 11),
    (6, 7), (6, 8), (6, 10), (6, 11), (7, 8), (8, 9), (8, 10), (9, 10), (10, 11)])
t11_33p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (2, 7), (2,
     8), (3, 4), (3, 8), (3, 9), (3, 10), (4, 5), (4, 9), (4, 10), (4, 11), (5, 6),
    (5, 11), (6, 7), (6, 8), (6, 11), (7, 8), (8, 9), (8, 11), (9, 10), (9, 11)])
t11_34p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 3), (2, 6), (2, 7), (2,
     8), (2, 9), (3, 4), (3, 9), (3, 10), (4, 5), (4, 6), (4, 10), (4, 11), (5, 6),
    (6, 7), (6, 11), (7, 8), (7, 9), (7, 11), (8, 9), (9, 10), (9, 11), (10, 11)])
t11_35p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (2, 7), (2,
     8), (3, 4), (3, 7), (3, 8), (3, 9), (4, 5), (4, 9), (4, 10), (4, 11), (5, 6),
    (5, 11), (6, 7), (6, 11), (7, 8), (7, 9), (7, 11), (9, 10), (9, 11), (10, 11)])
t11_36p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (2, 7), (3,
     4), (3, 7), (3, 8), (3, 9), (4, 5), (4, 9), (4, 10), (4, 11), (5, 6), (5, 11),
    (6, 7), (6, 11), (7, 8), (7, 11), (8, 9), (8, 10), (8, 11), (9, 10), (10, 11)])
t11_37p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (2, 7), (2,
     8), (3, 4), (3, 7), (3, 8), (3, 9), (4, 5), (4, 9), (4, 10), (5, 6), (5, 10),
    (6, 7), (6, 9), (6, 10), (6, 11), (7, 8), (7, 9), (7, 11), (9, 10), (9, 11)])
t11_38p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 3), (2, 6), (2, 7), (2,
     8), (2, 9), (3, 4), (3, 8), (3, 9), (3, 10), (4, 5), (4, 6), (4, 10), (4, 11),
    (5, 6), (6, 7), (6, 11), (7, 8), (7, 10), (7, 11), (8, 9), (8, 10), (10, 11)])
t11_39p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (2, 7), (2,
     8), (3, 4), (3, 7), (3, 8), (3, 9), (4, 5), (4, 9), (4, 10), (4, 11), (5, 6),
    (5, 10), (5, 11), (6, 7), (6, 9), (6, 10), (7, 8), (7, 9), (9, 10), (10, 11)])
t11_40p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 3), (2, 6), (2, 7), (2,
     8), (3, 4), (3, 8), (3, 9), (4, 5), (4, 9), (4, 10), (4, 11), (5, 6), (5, 11),
    (6, 7), (6, 10), (6, 11), (7, 8), (7, 9), (7, 10), (8, 9), (9, 10), (10, 11)])
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t11_41p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (2, 7), (2,
     8), (3, 4), (3, 7), (3, 8), (3, 9), (4, 5), (4, 9), (4, 10), (4, 11), (5, 6),
    (5, 10), (5, 11), (6, 7), (6, 10), (7, 8), (7, 9), (7, 10), (9, 10), (10, 11)])
t11_42p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (2, 7), (2,
     8), (3, 4), (3, 8), (3, 9), (3, 10), (4, 5), (4, 10), (4, 11), (5, 6), (5, 11),
     (6, 7), (6, 9), (6, 11), (7, 8), (7, 9), (8, 9), (9, 10), (9, 11), (10, 11)])
t11_43p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (2, 7), (3,
     4), (3, 7), (3, 8), (4, 5), (4, 8), (4, 9), (5, 6), (5, 9), (6, 7), (6, 9), (6,
     10), (6, 11), (7, 8), (7, 10), (7, 11), (8, 9), (8, 10), (9, 10), (10, 11)])
t11_44p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (2, 7), (3,
     4), (3, 7), (3, 8), (3, 9), (4, 5), (4, 9), (4, 10), (5, 6), (5, 10), (6, 7),
    (6, 10), (6, 11), (7, 8), (7, 9), (7, 11), (8, 9), (9, 10), (9, 11), (10, 11)])
t12p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 3), (2, 6), (2, 7), (2, 8)
    , (3, 4), (3, 8), (3, 9), (4, 5), (4, 9), (4, 10), (5, 6), (5, 10), (5, 11), (6,
     7), (6, 11), (7, 8), (7, 11), (7, 12), (8, 9), (8, 12), (9, 10), (9, 12), (10,
    11), (10, 12), (11, 12)])
t13_1p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (1, 7), (2, 3), (2, 7), (2,
    8), (2, 9), (3, 4), (3, 9), (4, 5), (4, 9), (4, 10), (5, 6), (5, 10), (5, 11),
    (6, 7), (6, 11), (6, 12), (7, 8), (7, 12), (8, 9), (8, 12), (8, 13), (9, 10),
    (9, 13), (10, 11), (10, 13), (11, 12), (11, 13), (12, 13)])
t13_2p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (1, 7), (2, 3), (2, 4), (2,
    7), (2, 8), (2, 9), (3, 4), (4, 5), (4, 9), (4, 10), (5, 6), (5, 10), (5, 11),
    (6, 7), (6, 11), (6, 12), (7, 8), (7, 12), (8, 9), (8, 12), (8, 13), (9, 10),
    (9, 13), (10, 11), (10, 13), (11, 12), (11, 13), (12, 13)])
t14p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (1, 7), (2, 3), (2, 6), (2, 7)
    , (2, 8), (2, 9), (3, 4), (3, 9), (3, 10), (4, 5), (4, 10), (4, 11), (5, 6), (5,
     11), (5, 12), (6, 7), (6, 8), (6, 12), (8, 9), (8, 12), (8, 13), (9, 10), (9,
    13), (10, 11), (10, 13), (10, 14), (11, 12), (11, 13), (11, 14), (12, 13), (13,
    14)])
t15_1p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (1, 7), (2, 3), (2, 6), (2,
    7), (2, 8), (2, 9), (3, 4), (3, 5), (3, 6), (4, 5), (5, 6), (6, 7), (6, 9), (7,
    8), (7, 9), (7, 10), (8, 9), (8, 10), (8, 11), (8, 12), (9, 10), (9, 12), (10,
    11), (10, 12), (10, 13), (11, 12), (11, 13), (11, 14), (11, 15), (12, 13), (12,
    15), (13, 14), (13, 15), (14, 15)])
t15_2p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (1, 7), (2, 3), (2, 7), (2,
    8), (2, 9), (3, 4), (3, 9), (3, 10), (4, 5), (4, 10), (4, 11), (5, 6), (5, 11),
    (5, 12), (6, 7), (6, 12), (6, 13), (7, 8), (7, 13), (8, 9), (8, 13), (8, 14),
    (8, 15), (9, 10), (9, 15), (10, 11), (10, 15), (11, 12), (11, 14), (11, 15),
    (12, 13), (12, 14), (13, 14), (14, 15)])
t16_1p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (1, 7), (2, 3), (2, 6), (2,
    7), (2, 8), (2, 9), (3, 4), (3, 5), (3, 6), (4, 5), (5, 6), (6, 7), (6, 9), (7,
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    8), (7, 9), (7, 10), (8, 9), (8, 10), (8, 11), (8, 12), (9, 10), (9, 12), (10,
    11), (10, 12), (10, 13), (11, 12), (11, 13), (11, 14), (11, 15), (12, 13), (12,
    15), (13, 14), (13, 15), (13, 16), (14, 15), (14, 16), (15, 16)])
t16_2p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (1, 7), (2, 3), (2, 7), (2,
    8), (2, 9), (3, 4), (3, 9), (3, 10), (4, 5), (4, 10), (4, 11), (5, 6), (5, 11),
    (5, 12), (6, 7), (6, 12), (6, 13), (7, 8), (7, 13), (8, 9), (8, 13), (8, 14),
    (9, 10), (9, 14), (9, 15), (10, 11), (10, 15), (11, 12), (11, 15), (11, 16),
    (12, 13), (12, 16), (13, 14), (13, 16), (14, 15), (14, 16), (15, 16)])
t17p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (1, 7), (2, 3), (2, 6), (2, 7)
    , (2, 8), (2, 9), (3, 4), (3, 5), (3, 6), (4, 5), (5, 6), (6, 7), (6, 9), (7, 8)
    , (7, 9), (7, 10), (8, 9), (8, 10), (8, 11), (8, 12), (9, 10), (9, 12), (10, 11)
    , (10, 12), (10, 13), (11, 12), (11, 13), (11, 14), (11, 15), (12, 13), (12, 15)
    , (13, 14), (13, 15), (13, 16), (14, 15), (14, 16), (14, 17), (15, 16), (15, 17)
    , (16, 17)])
t18_1p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (1, 7), (2, 3), (2, 6), (2,
    7), (2, 8), (2, 9), (3, 4), (3, 5), (3, 6), (4, 5), (5, 6), (6, 7), (6, 9), (7,
    8), (7, 9), (7, 10), (8, 9), (8, 10), (8, 11), (8, 12), (9, 10), (9, 12), (10,
    11), (10, 12), (10, 13), (11, 12), (11, 13), (11, 14), (11, 15), (12, 13), (12,
    15), (13, 14), (13, 15), (13, 16), (14, 15), (14, 16), (14, 17), (14, 18), (15,
    16), (15, 17), (15, 18), (16, 17), (17, 18)])
t18_2p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (1, 7), (2, 3), (2, 6), (2,
    7), (2, 8), (2, 9), (3, 4), (3, 5), (3, 6), (4, 5), (5, 6), (6, 7), (6, 9), (7,
    8), (7, 9), (7, 10), (8, 9), (8, 10), (8, 11), (8, 12), (9, 10), (9, 12), (10,
    11), (10, 12), (10, 13), (11, 12), (11, 13), (11, 14), (11, 15), (12, 13), (12,
    15), (13, 14), (13, 15), (13, 16), (14, 15), (14, 16), (14, 17), (15, 16), (15,
    17), (15, 18), (16, 17), (16, 18), (17,
18)])
t18_3p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (1, 7), (2, 3), (2, 6), (2,
    7), (2, 8), (2, 9), (3, 4), (3, 5), (3, 6), (4, 5), (5, 6), (6, 7), (6, 9), (7,
    8), (7, 9), (7, 10), (8, 9), (8, 10), (8, 11), (8, 12), (9, 10), (9, 12), (10,
    11), (10, 12), (10, 13), (11, 12), (11, 13), (11, 14), (11, 15), (12, 13), (12,
    15), (13, 14), (13, 15), (13, 16), (14, 15), (14, 16), (14, 17), (14, 18), (15,
    16), (15, 18), (16, 17), (16, 18), (17, 18)])

#4-connected triangulations which maximize the prox
t4c6p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (3, 4), (3,
    6), (4, 5), (4, 6), (5, 6)])
t4c7p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (3, 4), (3,
    6), (3, 7), (4, 5), (4, 7), (5, 6), (5, 7), (6, 7)])
t4c8p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (2, 7), (3,
    4), (3, 7), (3, 8), (4, 5), (4, 8), (5, 6), (5, 8), (6, 7), (6, 8), (7, 8)])
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t4c9p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 3), (2, 6), (2, 7), (2,
    8), (3, 4), (3, 8), (3, 9), (4, 5), (4, 9), (5, 6), (5, 7), (5, 9), (6, 7), (7,
    8), (7, 9), (8, 9)])
t4c10p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 3), (2, 6), (2, 7), (2,
    8), (3, 4), (3, 8), (3, 9), (4, 5), (4, 9), (5, 6), (5, 9), (5, 10), (6, 7), (6,
     10), (7, 8), (7, 10), (8, 9), (8, 10), (9, 10)])
t411_1p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 3), (2, 6), (2, 7), (2,
     8), (3, 4), (3, 8), (3, 9), (4, 5), (4, 9), (4, 10), (4, 11), (5, 6), (5, 11),
    (6, 7), (6, 11), (7, 8), (7, 10), (7, 11), (8, 9), (8, 10), (9, 10), (10, 11)])
t4c11_2p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (2, 7),
    (2, 8), (3, 4), (3, 8), (3, 9), (3, 10), (4, 5), (4, 10), (5, 6), (5, 10), (6,
    7), (6, 10), (6, 11), (7, 8), (7, 11), (8, 9), (8, 11), (9, 10), (9, 11), (10,
    11)])
t4c11_3p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (2, 7),
    (2, 8), (3, 4), (3, 8), (3, 9), (3, 10), (4, 5), (4, 10), (5, 6), (5, 10), (6,
    7), (6, 9), (6, 10), (6, 11), (7, 8), (7, 11), (8, 9), (8, 11), (9, 10), (9, 11)
    ])
t4c11_4p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 3), (2, 6), (2, 7),
    (2, 8), (2, 9), (3, 4), (3, 9), (4, 5), (4, 9), (4, 10), (4, 11), (5, 6), (5,
    11), (6, 7), (6, 11), (7, 8), (7, 10), (7, 11), (8, 9), (8, 10), (9, 10), (10,
    11)])
t4c11_5p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (2, 7),
    (3, 4), (3, 7), (3, 8), (3, 9), (4, 5), (4, 9), (5, 6), (5, 9), (5, 10), (6, 7),
     (6, 10), (6, 11), (7, 8), (7, 11), (8, 9), (8, 10), (8, 11), (9, 10), (10, 11)
    ])
t4c11_6p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (2, 7),
    (3, 4), (3, 7), (3, 8), (3, 9), (4, 5), (4, 9), (5, 6), (5, 9), (5, 10), (6, 7),
     (6, 10), (6, 11), (7, 8), (7, 11), (8, 9), (8, 11), (9, 10), (9, 11), (10, 11)
    ])
t4c11_7p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (2, 7),
    (2, 8), (3, 4), (3, 8), (4, 5), (4, 8), (4, 9), (4, 10), (5, 6), (5, 10), (6, 7)
    , (6, 9), (6, 10), (6, 11), (7, 8), (7, 11), (8, 9), (8, 11), (9, 10), (9, 11)])
t4c11_8p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (2, 7),
    (2, 8), (3, 4), (3, 8), (3, 9), (3, 10), (4, 5), (4, 6), (4, 7), (4, 10), (5, 6)
    , (6, 7), (7, 8), (7, 10), (7, 11), (8, 9), (8, 11), (9, 10), (9, 11), (10, 11)
    ])
t4c11_9p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (1, 7), (2, 3), (2, 7),
    (2, 8), (3, 4), (3, 8), (3, 9), (3, 10), (4, 5), (4, 10), (5, 6), (5, 10), (6,
    7), (6, 8), (6, 10), (6, 11), (7, 8), (8, 9), (8, 11), (9, 10), (9, 11), (10,
    11)])
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t4c11_10p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 3), (2, 6), (2, 7),
    (3, 4), (3, 7), (3, 8), (3, 9), (4, 5), (4, 9), (4, 10), (5, 6), (5, 10), (6, 7)
    , (6, 10), (6, 11), (7, 8), (7, 11), (8, 9), (8, 10), (8, 11), (9, 10), (10, 11)
    ])
t4c12p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 3), (2, 6), (2, 7), (2,
    8), (3, 4), (3, 8), (3, 9), (4, 5), (4, 9), (4, 10), (5, 6), (5, 10), (5, 11),
    (6, 7), (6, 11), (7, 8), (7, 11), (7, 12), (8, 9), (8, 12), (9, 10), (9, 12),
    (10, 11), (10, 12), (11, 12)])
t4c13p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 3), (2, 6), (2, 7), (2,
    8), (3, 4), (3, 8), (3, 9), (4, 5), (4, 9), (4, 10), (4, 11), (5, 6), (5, 11),
    (5, 12), (6, 7), (6, 12), (7, 8), (7, 12), (7, 13), (8, 9), (8, 13), (9, 10),
    (9, 13), (10, 11), (10, 12), (10, 13), (11, 12), (12, 13)])
t4c14_1p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 3), (2, 6), (2, 7),
    (2, 8), (3, 4), (3, 8), (3, 9), (4, 5), (4, 9), (4, 10), (4, 11), (5, 6), (5,
    11), (5, 12), (6, 7), (6, 12), (7, 8), (7, 12), (7, 13), (8, 9), (8, 13), (9,
    10), (9, 13), (10, 11), (10, 13), (10, 14), (11, 12), (11, 14), (12, 13), (12,
    14), (13, 14)])
t4c14_2p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 3), (2, 6), (2, 7),
    (2, 8), (3, 4), (3, 8), (3, 9), (4, 5), (4, 9), (4, 10), (4, 11), (5, 6), (5,
    11), (5, 12), (5, 13), (6, 7), (6, 13), (7, 8), (7, 13), (7, 14), (8, 9), (8,
    14), (9, 10), (9, 14), (10, 11), (10, 12), (10, 13), (10, 14), (11, 12), (12,
    13), (13, 14)])
t4c14_3p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (1, 7), (2, 3), (2, 7),
    (2, 8), (2, 9), (3, 4), (3, 9), (3, 10), (3, 11), (4, 5), (4, 11), (5, 6), (5,
    11), (5, 12), (5, 13), (6, 7), (6, 13), (7, 8), (7, 13), (8, 9), (8, 13), (8,
    14), (9, 10), (9, 14), (10, 11), (10, 12), (10, 14), (11, 12), (12, 13), (12,
    14), (13, 14)])
t4c14_4p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (1, 7), (2, 3), (2, 7),
    (2, 8), (3, 4), (3, 8), (3, 9), (4, 5), (4, 9), (4, 10), (5, 6), (5, 10), (5,
    11), (5, 12), (6, 7), (6, 12), (7, 8), (7, 12), (7, 13), (8, 9), (8, 13), (9,
    10), (9, 13), (9, 14), (10, 11), (10, 14), (11, 12), (11, 14), (12, 13), (12,
    14), (13, 14)])
t4c14_5p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 3), (2, 6), (2, 7),
    (2, 8), (3, 4), (3, 8), (3, 9), (4, 5), (4, 9), (4, 10), (4, 11), (5, 6), (5,
    11), (5, 12), (6, 7), (6, 12), (6, 13), (7, 8), (7, 13), (8, 9), (8, 13), (8,
    14), (9, 10), (9, 14), (10, 11), (10, 12), (10, 14), (11, 12), (12, 13), (12,
    14), (13, 14)])
t4c14_6p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 3), (2, 6), (2, 7),
    (2, 8), (2, 9), (3, 4), (3, 9), (3, 10), (4, 5), (4, 10), (4, 11), (5, 6), (5,
    11), (5, 12), (6, 7), (6, 12), (6, 13), (7, 8), (7, 13), (7, 14), (8, 9), (8,
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    10), (8, 14), (9, 10), (10, 11), (10, 14), (11, 12), (11, 13), (11, 14), (12,
    13), (13, 14)])
t4c14_7p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (1, 7), (2, 3), (2, 7),
    (2, 8), (3, 4), (3, 8), (3, 9), (3, 10), (4, 5), (4, 10), (5, 6), (5, 10), (5,
    11), (5, 12), (6, 7), (6, 12), (7, 8), (7, 12), (7, 13), (8, 9), (8, 13), (9,
    10), (9, 13), (9, 14), (10, 11), (10, 14), (11, 12), (11, 13), (11, 14), (12,
    13), (13, 14)])
t4c14_8p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 3), (2, 6), (2, 7),
    (2, 8), (3, 4), (3, 8), (3, 9), (3, 10), (4, 5), (4, 10), (5, 6), (5, 10), (5,
    11), (5, 12), (6, 7), (6, 12), (6, 13), (7, 8), (7, 13), (7, 14), (8, 9), (8,
    14), (9, 10), (9, 11), (9, 14), (10, 11), (11, 12), (11, 13), (11, 14), (12, 13)
    , (13, 14)])
t4c14_9p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 3), (2, 6), (2, 7),
    (2, 8), (3, 4), (3, 8), (3, 9), (4, 5), (4, 9), (4, 10), (4, 11), (5, 6), (5,
    11), (5, 12), (6, 7), (6, 12), (7, 8), (7, 12), (7, 13), (7, 14), (8, 9), (8,
    14), (9, 10), (9, 14), (10, 11), (10, 13), (10, 14), (11, 12), (11, 13), (12,
    13), (13, 14)])
t4c14_10p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 3), (2, 6), (2, 7),
    (3, 4), (3, 7), (3, 8), (3, 9), (4, 5), (4, 9), (4, 10), (4, 11), (5, 6), (5,
    11), (5, 12), (6, 7), (6, 12), (6, 13), (7, 8), (7, 13), (7, 14), (8, 9), (8,
    10), (8, 14), (9, 10), (10, 11), (10, 12), (10, 14), (11, 12), (12, 13), (12,
    14), (13, 14)])
t4c14_11p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (2, 7),
    (2, 8), (3, 4), (3, 8), (3, 9), (4, 5), (4, 9), (4, 10), (4, 11), (5, 6), (5,
    11), (6, 7), (6, 11), (6, 12), (7, 8), (7, 12), (7, 13), (7, 14), (8, 9), (8,
    14), (9, 10), (9, 13), (9, 14), (10, 11), (10, 12), (10, 13), (11, 12), (12, 13)
    , (13, 14)])
t4c14_12p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (2, 7),
    (2, 8), (3, 4), (3, 8), (3, 9), (3, 10), (4, 5), (4, 10), (4, 11), (5, 6), (5,
    11), (5, 12), (6, 7), (6, 12), (7, 8), (7, 9), (7, 12), (7, 13), (8, 9), (9, 10)
    , (9, 13), (9, 14), (10, 11), (10, 14), (11, 12), (11, 14), (12, 13), (12, 14),
    (13, 14)])
t4c14_13p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (2, 7),
    (2, 8), (3, 4), (3, 8), (3, 9), (3, 10), (4, 5), (4, 10), (4, 11), (4, 12), (5,
    6), (5, 12), (5, 13), (6, 7), (6, 13), (7, 8), (7, 9), (7, 13), (7, 14), (8, 9),
     (9, 10), (9, 11), (9, 14), (10, 11), (11, 12), (11, 13), (11, 14), (12, 13),
    (13, 14)])
t4c15p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 3), (2, 6), (2, 7), (2,
    8), (3, 4), (3, 8), (3, 9), (4, 5), (4, 9), (4, 10), (4, 11), (5, 6), (5, 11),
    (5, 12), (6, 7), (6, 12), (7, 8), (7, 12), (7, 13), (8, 9), (8, 13), (8, 14),
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    (9, 10), (9, 14), (10, 11), (10, 14), (10, 15), (11, 12), (11, 15), (12, 13),
    (12, 15), (13, 14), (13, 15), (14, 15)])
t4c16p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 3), (2, 6), (2, 7), (2,
    8), (3, 4), (3, 8), (3, 9), (4, 5), (4, 9), (4, 10), (4, 11), (5, 6), (5, 11),
    (5, 12), (6, 7), (6, 12), (6, 13), (7, 8), (7, 13), (7, 14), (8, 9), (8, 14),
    (8, 15), (9, 10), (9, 15), (10, 11), (10, 15), (10, 16), (11, 12), (11, 16),
    (12, 13), (12, 16), (13, 14), (13, 16), (14, 15), (14, 16), (15, 16)])
t4c17_1p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 3), (2, 6), (2, 7),
    (2, 8), (2, 9), (3, 4), (3, 9), (3, 10), (4, 5), (4, 10), (4, 11), (4, 12), (5,
    6), (5, 12), (5, 13), (5, 14), (6, 7), (6, 14), (7, 8), (7, 14), (7, 15), (8, 9)
    , (8, 15), (8, 16), (9, 10), (9, 16), (10, 11), (10, 16), (10, 17), (11, 12),
    (11, 13), (11, 17), (12, 13), (13, 14), (13, 15), (13, 17), (14, 15), (15, 16),
    (15, 17), (16, 17)])
t4c17_2p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 3), (2, 6), (2, 7),
    (2, 8), (2, 9), (3, 4), (3, 9), (3, 10), (4, 5), (4, 10), (4, 11), (4, 12), (5,
    6), (5, 12), (5, 13), (6, 7), (6, 13), (7, 8), (7, 13), (7, 14), (8, 9), (8, 14)
    , (8, 15), (9, 10), (9, 15), (10, 11), (10, 15), (10, 16), (11, 12), (11, 16),
    (12, 13), (12, 16), (12, 17), (13, 14), (13, 17), (14, 15), (14, 16), (14, 17),
    (15, 16), (16, 17)])
t4c17_3p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (1, 7), (2, 3), (2, 7),
    (2, 8), (3, 4), (3, 8), (3, 9), (3, 10), (4, 5), (4, 10), (5, 6), (5, 10), (5,
    11), (5, 12), (6, 7), (6, 12), (6, 13), (7, 8), (7, 13), (7, 14), (8, 9), (8,
    14), (8, 15), (9, 10), (9, 15), (9, 16), (10, 11), (10, 16), (11, 12), (11, 16),
     (12, 13), (12, 16), (12, 17), (13, 14), (13, 17), (14, 15), (14, 17), (15, 16),
     (15, 17), (16, 17)])
t4c17_4p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 3), (2, 6), (2, 7),
    (2, 8), (2, 9), (3, 4), (3, 9), (3, 10), (4, 5), (4, 10), (4, 11), (5, 6), (5,
    11), (5, 12), (6, 7), (6, 12), (6, 13), (7, 8), (7, 13), (8, 9), (8, 13), (8,
    14), (8, 15), (9, 10), (9, 15), (10, 11), (10, 15), (10, 16), (11, 12), (11, 16)
    , (11, 17), (12, 13), (12, 17), (13, 14), (13, 17), (14, 15), (14, 16), (14, 17)
    , (15, 16), (16, 17)])
t4c17_5p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 3), (2, 6), (2, 7),
    (2, 8), (3, 4), (3, 8), (3, 9), (4, 5), (4, 9), (4, 10), (4, 11), (5, 6), (5,
    11), (5, 12), (6, 7), (6, 12), (6, 13), (7, 8), (7, 13), (7, 14), (8, 9), (8,
    14), (8, 15), (9, 10), (9, 15), (10, 11), (10, 15), (10, 16), (11, 12), (11, 16)
    , (12, 13), (12, 16), (12, 17), (13, 14), (13, 17), (14, 15), (14, 17), (15, 16)
    , (15, 17), (16, 17)])
t4c18_1p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 3), (2, 6), (2, 7),
    (2, 8), (2, 9), (3, 4), (3, 9), (3, 10), (4, 5), (4, 10), (4, 11), (4, 12), (5,
    6), (5, 12), (5, 13), (6, 7), (6, 13), (6, 14), (7, 8), (7, 14), (8, 9), (8, 14)
    , (8, 15), (8, 16), (9, 10), (9, 16), (10, 11), (10, 16), (10, 17), (11, 12),
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    (11, 17), (12, 13), (12, 17), (12, 18), (13, 14), (13, 18), (14, 15), (14, 18),
    (15, 16), (15, 17), (15, 18), (16, 17), (17, 18)])
t4c18_2p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (1, 7), (2, 3), (2, 7),
    (2, 8), (2, 9), (2, 10), (3, 4), (3, 10), (3, 11), (4, 5), (4, 11), (4, 12), (5,
     6), (5, 12), (5, 13), (6, 7), (6, 13), (6, 14), (7, 8), (7, 14), (8, 9), (8,
    14), (8, 15), (9, 10), (9, 15), (9, 16), (10, 11), (10, 16), (11, 12), (11, 16),
     (11, 17), (12, 13), (12, 17), (12, 18), (13, 14), (13, 18), (14, 15), (14, 18),
     (15, 16), (15, 17), (15, 18), (16, 17), (17, 18)])
t4c19_1p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (1, 7), (2, 3), (2, 7),
    (2, 8), (2, 9), (2, 10), (3, 4), (3, 10), (4, 5), (4, 10), (4, 11), (4, 12), (5,
     6), (5, 12), (5, 13), (6, 7), (6, 13), (6, 14), (7, 8), (7, 14), (7, 15), (8,
    9), (8, 15), (8, 16), (9, 10), (9, 11), (9, 16), (9, 17), (10, 11), (11, 12),
    (11, 17), (12, 13), (12, 17), (12, 18), (13, 14), (13, 18), (13, 19), (14, 15),
    (14, 19), (15, 16), (15, 19), (16, 17), (16, 19), (17, 18), (17, 19), (18, 19)])
t4c19_2p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (1, 7), (2, 3), (2, 7),
    (2, 8), (3, 4), (3, 8), (3, 9), (3, 10), (4, 5), (4, 10), (4, 11), (4, 12), (5,
    6), (5, 12), (5, 13), (6, 7), (6, 13), (6, 14), (6, 15), (7, 8), (7, 15), (8, 9)
    , (8, 15), (8, 16), (9, 10), (9, 16), (9, 17), (10, 11), (10, 17), (11, 12),
    (11, 17), (11, 18), (12, 13), (12, 18), (13, 14), (13, 18), (13, 19), (14, 15),
    (14, 16), (14, 19), (15, 16), (16, 17), (16, 19), (17, 18), (17, 19), (18, 19)])
t4c19_3p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (1, 7), (2, 3), (2, 7),
    (2, 8), (2, 9), (3, 4), (3, 9), (3, 10), (3, 11), (4, 5), (4, 11), (5, 6), (5,
    11), (5, 12), (5, 13), (6, 7), (6, 13), (6, 14), (7, 8), (7, 14), (7, 15), (8,
    9), (8, 15), (8, 16), (9, 10), (9, 16), (9, 17), (10, 11), (10, 12), (10, 17),
    (11, 12), (12, 13), (12, 17), (12, 18), (13, 14), (13, 18), (13, 19), (14, 15),
    (14, 19), (15, 16), (15, 19), (16, 17), (16, 19), (17, 18), (17, 19), (18, 19)])
t4c19_4p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (1, 7), (2, 3), (2, 7),
    (2, 8), (2, 9), (3, 4), (3, 9), (3, 10), (3, 11), (4, 5), (4, 11), (5, 6), (5,
    11), (5, 12), (5, 13), (6, 7), (6, 13), (6, 14), (7, 8), (7, 14), (7, 15), (8,
    9), (8, 15), (8, 16), (9, 10), (9, 16), (9, 17), (10, 11), (10, 17), (10, 18),
    (11, 12), (11, 18), (12, 13), (12, 18), (13, 14), (13, 18), (13, 19), (14, 15),
    (14, 19), (15, 16), (15, 19), (16, 17), (16, 19), (17, 18), (17, 19), (18, 19)])
t4c19_5p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 3), (2, 6), (2, 7),
    (2, 8), (2, 9), (3, 4), (3, 9), (3, 10), (3, 11), (4, 5), (4, 11), (4, 12), (4,
    13), (5, 6), (5, 13), (5, 14), (6, 7), (6, 14), (6, 15), (7, 8), (7, 15), (7,
    16), (8, 9), (8, 10), (8, 16), (9, 10), (10, 11), (10, 16), (10, 17), (11, 12),
    (11, 17), (12, 13), (12, 17), (12, 18), (13, 14), (13, 18), (14, 15), (14, 18),
    (14, 19), (15, 16), (15, 19), (16, 17), (16, 19), (17, 18), (17, 19), (18, 19)])
t4c19_6p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 3), (2, 6), (2, 7),
    (2, 8), (2, 9), (3, 4), (3, 9), (3, 10), (4, 5), (4, 10), (4, 11), (4, 12), (5,
    6), (5, 12), (5, 13), (5, 14), (6, 7), (6, 14), (6, 15), (7, 8), (7, 15), (7,
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    16), (8, 9), (8, 16), (9, 10), (9, 16), (9, 17), (10, 11), (10, 17), (11, 12),
    (11, 17), (11, 18), (12, 13), (12, 18), (13, 14), (13, 15), (13, 18), (13, 19),
    (14, 15), (15, 16), (15, 19), (16, 17), (16, 19), (17, 18), (17, 19), (18, 19)])
t4c19_7p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 3), (2, 6), (2, 7),
    (2, 8), (2, 9), (3, 4), (3, 9), (3, 10), (4, 5), (4, 10), (4, 11), (4, 12), (5,
    6), (5, 12), (5, 13), (6, 7), (6, 13), (6, 14), (7, 8), (7, 14), (8, 9), (8, 14)
    , (8, 15), (8, 16), (9, 10), (9, 16), (9, 17), (10, 11), (10, 17), (11, 12),
    (11, 17), (11, 18), (12, 13), (12, 18), (13, 14), (13, 18), (13, 19), (14, 15),
    (14, 19), (15, 16), (15, 17), (15, 18), (15, 19), (16, 17), (17, 18), (18, 19)])
t4c19_8p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 3), (2, 6), (2, 7),
    (2, 8), (3, 4), (3, 8), (3, 9), (4, 5), (4, 9), (4, 10), (4, 11), (5, 6), (5,
    11), (5, 12), (5, 13), (6, 7), (6, 13), (6, 14), (7, 8), (7, 14), (7, 15), (7,
    16), (8, 9), (8, 16), (8, 17), (9, 10), (9, 17), (10, 11), (10, 17), (10, 18),
    (11, 12), (11, 18), (12, 13), (12, 14), (12, 18), (12, 19), (13, 14), (14, 15),
    (14, 19), (15, 16), (15, 19), (16, 17), (16, 19), (17, 18), (17, 19), (18, 19)])
t4c19_9p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 3), (2, 6), (2, 7),
    (2, 8), (2, 9), (3, 4), (3, 9), (3, 10), (3, 11), (4, 5), (4, 11), (4, 12), (4,
    13), (5, 6), (5, 13), (5, 14), (5, 15), (6, 7), (6, 15), (7, 8), (7, 15), (7,
    16), (7, 17), (8, 9), (8, 10), (8, 17), (9, 10), (10, 11), (10, 17), (10, 18),
    (11, 12), (11, 18), (12, 13), (12, 14), (12, 18), (12, 19), (13, 14), (14, 15),
    (14, 19), (15, 16), (15, 19), (16, 17), (16, 18), (16, 19), (17, 18), (18, 19)])
t4c19_10p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (1, 7), (2, 3), (2, 7),
    (2, 8), (2, 9), (3, 4), (3, 9), (3, 10), (4, 5), (4, 10), (4, 11), (4, 12), (5,
    6), (5, 12), (6, 7), (6, 12), (6, 13), (7, 8), (7, 13), (7, 14), (8, 9), (8, 14)
    , (8, 15), (8, 16), (9, 10), (9, 16), (9, 17), (10, 11), (10, 17), (10, 18),
    (11, 12), (11, 18), (11, 19), (12, 13), (12, 19), (13, 14), (13, 15), (13, 19),
    (14, 15), (15, 16), (15, 18), (15, 19), (16, 17), (16, 18), (17, 18), (18, 19)])
t4c19_11p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (1, 7), (2, 3), (2, 7),
    (2, 8), (2, 9), (3, 4), (3, 9), (3, 10), (3, 11), (4, 5), (4, 11), (5, 6), (5,
    11), (5, 12), (5, 13), (6, 7), (6, 13), (6, 14), (7, 8), (7, 14), (7, 15), (8,
    9), (8, 15), (8, 16), (9, 10), (9, 16), (9, 17), (10, 11), (10, 17), (10, 18),
    (11, 12), (11, 18), (12, 13), (12, 18), (13, 14), (13, 18), (13, 19), (14, 15),
    (14, 16), (14, 19), (15, 16), (16, 17), (16, 19), (17, 18), (17, 19), (18, 19)])
t4c19_12p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (1, 7), (2, 3), (2, 7),
    (2, 8), (2, 9), (3, 4), (3, 9), (3, 10), (4, 5), (4, 10), (4, 11), (4, 12), (5,
    6), (5, 12), (5, 13), (6, 7), (6, 13), (6, 14), (7, 8), (7, 14), (7, 15), (8, 9)
    , (8, 15), (8, 16), (9, 10), (9, 16), (9, 17), (10, 11), (10, 17), (10, 18),
    (11, 12), (11, 18), (12, 13), (12, 18), (13, 14), (13, 18), (13, 19), (14, 15),
    (14, 16), (14, 19), (15, 16), (16, 17), (16, 19), (17, 18), (17, 19), (18, 19)])
t4c19_13p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (1, 7), (2, 3), (2, 7),
    (2, 8), (2, 9), (2, 10), (3, 4), (3, 10), (3, 11), (4, 5), (4, 11), (4, 12), (4,
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    9), (8, 15), (9, 10), (9, 15), (9, 16), (10, 11), (10, 16), (10, 17), (11, 12),
    (11, 17), (12, 13), (12, 17), (12, 18), (13, 14), (13, 18), (13, 19), (14, 15),
    (14, 19), (15, 16), (15, 19), (16, 17), (16, 18), (16, 19), (17, 18), (18, 19)])
t4c19_14p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (1, 7), (2, 3), (2, 7),
    (2, 8), (2, 9), (2, 10), (3, 4), (3, 10), (4, 5), (4, 10), (4, 11), (5, 6), (5,
    11), (5, 12), (5, 13), (6, 7), (6, 13), (6, 14), (7, 8), (7, 14), (7, 15), (8,
    9), (8, 15), (9, 10), (9, 15), (9, 16), (9, 17), (10, 11), (10, 17), (11, 12),
    (11, 17), (11, 18), (12, 13), (12, 18), (13, 14), (13, 18), (13, 19), (14, 15),
    (14, 19), (15, 16), (15, 19), (16, 17), (16, 18), (16, 19), (17, 18), (18, 19)])
t4c19_15p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 3), (2, 6), (2, 7),
    (2, 8), (2, 9), (3, 4), (3, 9), (3, 10), (3, 11), (4, 5), (4, 11), (4, 12), (5,
    6), (5, 12), (5, 13), (6, 7), (6, 13), (7, 8), (7, 13), (7, 14), (7, 15), (8, 9)
    , (8, 15), (9, 10), (9, 15), (9, 16), (10, 11), (10, 16), (11, 12), (11, 16),
    (11, 17), (12, 13), (12, 17), (12, 18), (13, 14), (13, 18), (14, 15), (14, 18),
    (14, 19), (15, 16), (15, 19), (16, 17), (16, 19), (17, 18), (17, 19), (18, 19)])
t4c19_16p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 3), (2, 6), (2, 7),
    (2, 8), (2, 9), (3, 4), (3, 9), (3, 10), (4, 5), (4, 10), (4, 11), (5, 6), (5,
    11), (5, 12), (6, 7), (6, 12), (6, 13), (7, 8), (7, 13), (7, 14), (8, 9), (8,
    14), (8, 15), (8, 16), (9, 10), (9, 16), (10, 11), (10, 16), (10, 17), (11, 12),
     (11, 17), (11, 18), (12, 13), (12, 18), (13, 14), (13, 18), (13, 19), (14, 15),
     (14, 19), (15, 16), (15, 19), (16, 17), (16, 19), (17, 18), (17, 19), (18, 19)
    ])
t4c19_17p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (1, 7), (2, 3), (2, 7),
    (2, 8), (2, 9), (2, 10), (3, 4), (3, 10), (3, 11), (4, 5), (4, 11), (4, 12), (5,
     6), (5, 12), (5, 13), (6, 7), (6, 8), (6, 13), (6, 14), (7, 8), (8, 9), (8, 14)
    , (8, 15), (9, 10), (9, 15), (9, 16), (9, 17), (10, 11), (10, 17), (11, 12),
    (11, 17), (11, 18), (12, 13), (12, 18), (12, 19), (13, 14), (13, 19), (14, 15),
    (14, 16), (14, 19), (15, 16), (16, 17), (16, 18), (16, 19), (17, 18), (18, 19)])
t4c19_18p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 3), (2, 6), (2, 7),
    (2, 8), (2, 9), (3, 4), (3, 9), (3, 10), (4, 5), (4, 10), (4, 11), (4, 12), (5,
    6), (5, 12), (5, 13), (6, 7), (6, 13), (6, 14), (7, 8), (7, 14), (7, 15), (8, 9)
    , (8, 15), (8, 16), (8, 17), (9, 10), (9, 17), (10, 11), (10, 17), (10, 18),
    (11, 12), (11, 18), (12, 13), (12, 18), (12, 19), (13, 14), (13, 19), (14, 15),
    (14, 16), (14, 19), (15, 16), (16, 17), (16, 18), (16, 19), (17, 18), (18, 19)])
t4c19_19p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 3), (2, 6), (2, 7),
    (2, 8), (3, 4), (3, 8), (3, 9), (4, 5), (4, 9), (4, 10), (4, 11), (5, 6), (5,
    11), (5, 12), (6, 7), (6, 12), (6, 13), (7, 8), (7, 13), (7, 14), (7, 15), (8,
    9), (8, 15), (9, 10), (9, 15), (9, 16), (10, 11), (10, 16), (10, 17), (11, 12),
    (11, 17), (11, 18), (12, 13), (12, 18), (13, 14), (13, 18), (13, 19), (14, 15),
    (14, 19), (15, 16), (15, 19), (16, 17), (16, 19), (17, 18), (17, 19), (18, 19)])
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t4c19_20p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 3), (2, 6), (2, 7),
    (2, 8), (2, 9), (3, 4), (3, 9), (3, 10), (4, 5), (4, 10), (4, 11), (4, 12), (5,
    6), (5, 12), (5, 13), (6, 7), (6, 13), (6, 14), (7, 8), (7, 14), (8, 9), (8, 14)
    , (8, 15), (8, 16), (9, 10), (9, 16), (10, 11), (10, 16), (10, 17), (11, 12),
    (11, 17), (11, 18), (12, 13), (12, 18), (12, 19), (13, 14), (13, 19), (14, 15),
    (14, 19), (15, 16), (15, 17), (15, 18), (15, 19), (16, 17), (17, 18), (18, 19)])
t4c19_21p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (1, 7), (2, 3), (2, 7),
    (2, 8), (2, 9), (2, 10), (3, 4), (3, 10), (4, 5), (4, 10), (4, 11), (4, 12), (5,
     6), (5, 12), (5, 13), (5, 14), (6, 7), (6, 14), (7, 8), (7, 14), (7, 15), (8,
    9), (8, 15), (8, 16), (9, 10), (9, 11), (9, 16), (9, 17), (10, 11), (11, 12),
    (11, 17), (11, 18), (12, 13), (12, 18), (13, 14), (13, 15), (13, 18), (13, 19),
    (14, 15), (15, 16), (15, 19), (16, 17), (16, 18), (16, 19), (17, 18), (18, 19)])
t4c19_22p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (1, 7), (2, 3), (2, 7),
    (2, 8), (3, 4), (3, 8), (3, 9), (3, 10), (4, 5), (4, 10), (4, 11), (5, 6), (5,
    11), (5, 12), (6, 7), (6, 12), (6, 13), (7, 8), (7, 13), (7, 14), (8, 9), (8,
    14), (8, 15), (9, 10), (9, 15), (9, 16), (10, 11), (10, 16), (10, 17), (11, 12),
     (11, 17), (11, 18), (12, 13), (12, 18), (13, 14), (13, 18), (13, 19), (14, 15),
     (14, 19), (15, 16), (15, 19), (16, 17), (16, 19), (17, 18), (17, 19), (18, 19)
    ])
t4c19_23p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 3), (2, 6), (2, 7),
    (2, 8), (2, 9), (3, 4), (3, 9), (3, 10), (4, 5), (4, 10), (4, 11), (5, 6), (5,
    11), (5, 12), (5, 13), (6, 7), (6, 13), (6, 14), (7, 8), (7, 14), (7, 15), (8,
    9), (8, 15), (8, 16), (8, 17), (9, 10), (9, 17), (10, 11), (10, 17), (10, 18),
    (11, 12), (11, 18), (12, 13), (12, 18), (12, 19), (13, 14), (13, 15), (13, 19),
    (14, 15), (15, 16), (15, 19), (16, 17), (16, 18), (16, 19), (17, 18), (18, 19)])
t4c19_24p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 3), (2, 6), (2, 7),
    (2, 8), (2, 9), (3, 4), (3, 9), (3, 10), (4, 5), (4, 10), (4, 11), (5, 6), (5,
    11), (5, 12), (5, 13), (6, 7), (6, 13), (6, 14), (7, 8), (7, 14), (8, 9), (8,
    14), (8, 15), (8, 16), (9, 10), (9, 16), (10, 11), (10, 16), (10, 17), (11, 12),
     (11, 17), (11, 18), (12, 13), (12, 18), (12, 19), (13, 14), (13, 19), (14, 15),
     (14, 19), (15, 16), (15, 18), (15, 19), (16, 17), (16, 18), (17, 18), (18, 19)
    ])
t4c19_25p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 3), (2, 6), (2, 7),
    (2, 8), (3, 4), (3, 8), (3, 9), (4, 5), (4, 9), (4, 10), (4, 11), (5, 6), (5,
    11), (5, 12), (6, 7), (6, 12), (6, 13), (7, 8), (7, 13), (7, 14), (7, 15), (8,
    9), (8, 15), (9, 10), (9, 15), (9, 16), (10, 11), (10, 16), (10, 17), (11, 12),
    (11, 17), (11, 18), (12, 13), (12, 18), (13, 14), (13, 18), (13, 19), (14, 15),
    (14, 16), (14, 19), (15, 16), (16, 17), (16, 19), (17, 18), (17, 19), (18, 19)])
t4c19_26p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (1, 7), (2, 3), (2, 7),
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t4c19_27p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (1, 7), (2, 3), (2, 7),
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t4c19_28p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (1, 7), (2, 3), (2, 7),
    (2, 8), (2, 9), (2, 10), (3, 4), (3, 10), (3, 11), (4, 5), (4, 11), (4, 12), (5,
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    (11, 17), (12, 13), (12, 17), (12, 18), (13, 14), (13, 18), (13, 19), (14, 15),
    (14, 19), (15, 16), (15, 19), (16, 17), (16, 18), (16, 19), (17, 18), (18, 19)])
t4c20_1p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (1, 7), (2, 3), (2, 7),
    (2, 8), (2, 9), (3, 4), (3, 9), (3, 10), (3, 11), (4, 5), (4, 11), (5, 6), (5,
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    (14, 15), (14, 16), (14, 20), (15, 16), (16, 17), (16, 18), (16, 20), (17, 18),
    (18, 19), (18, 20), (19, 20)])
t4c20_2p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 3), (2, 6), (2, 7),
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     (11, 17), (11, 18), (12, 13), (12, 18), (13, 14), (13, 18), (13, 19), (14, 15),
     (14, 19), (14, 20), (15, 16), (15, 20), (16, 17), (16, 20), (17, 18), (17, 20),
     (18, 19), (18, 20), (19, 20)])
t4c20_3p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (1, 7), (2, 3), (2, 7),
    (2, 8), (2, 9), (2, 10), (3, 4), (3, 10), (3, 11), (4, 5), (4, 11), (4, 12), (4,
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     15), (14, 20), (15, 16), (15, 17), (15, 20), (16, 17), (17, 18), (17, 19), (17,
     20), (18, 19), (19, 20)])
t4c20_4p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 3), (2, 6), (2, 7),
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    (17, 18), (18, 19), (19, 20)])
t4c20_5p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (1, 7), (2, 3), (2, 7),
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    (11, 17), (11, 18), (12, 13), (12, 18), (12, 19), (13, 14), (13, 19), (14, 15),
    (14, 19), (14, 20), (15, 16), (15, 20), (16, 17), (16, 18), (16, 20), (17, 18),
    (18, 19), (18, 20), (19, 20)])
t4c20_6p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (1, 7), (2, 3), (2, 7),
    (2, 8), (2, 9), (3, 4), (3, 9), (3, 10), (4, 5), (4, 10), (4, 11), (4, 12), (5,
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    (18, 19), (18, 20), (19, 20)])
t4c20_7p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (1, 7), (2, 3), (2, 7),
    (2, 8), (2, 9), (2, 10), (3, 4), (3, 10), (3, 11), (4, 5), (4, 11), (4, 12), (5,
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    (14, 15), (14, 20), (15, 16), (15, 20), (16, 17), (16, 20), (17, 18), (17, 19),
    (17, 20), (18, 19), (19, 20)])
t4c20_8p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 3), (2, 6), (2, 7),
    (2, 8), (3, 4), (3, 8), (3, 9), (4, 5), (4, 9), (4, 10), (5, 6), (5, 10), (5,
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     (14, 19), (14, 20), (15, 16), (15, 20), (16, 17), (16, 20), (17, 18), (17, 19),
     (17, 20), (18, 19), (19, 20)])
t4c20_9p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 3), (2, 6), (2, 7),
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    (13, 20), (14, 15), (15, 16), (15, 20), (16, 17), (16, 18), (16, 20), (17, 18),
    (18, 19), (18, 20), (19, 20)])
t4c20_10p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (1, 7), (2, 3), (2, 7),
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    (18, 19), (18, 20), (19, 20)])
t4c20_11p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (1, 7), (2, 3), (2, 7),
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     (18, 19), (18, 20), (19, 20)])
t4c20_12p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (1, 7), (2, 3), (2, 7),
    (2, 8), (2, 9), (3, 4), (3, 9), (3, 10), (4, 5), (4, 10), (4, 11), (5, 6), (5,
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    (17, 19), (18, 19), (19, 20)])
t4c20_13p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (1, 7), (2, 3), (2, 7),
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    14), (8, 15), (9, 10), (9, 15), (9, 16), (10, 11), (10, 16), (11, 12), (11, 16),
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     (18, 19), (18, 20), (19, 20)])
t4c21_1p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 3), (2, 6), (2, 7),
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    (11, 17), (11, 18), (12, 13), (12, 18), (12, 19), (13, 14), (13, 19), (13, 20),
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    (17, 21), (18, 19), (18, 21), (19, 20), (19, 21), (20, 21)])
t4c21_2p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 3), (2, 6), (2, 7),
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t4c21_3p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (1, 7), (2, 3), (2, 7),
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    (17, 19), (17, 21), (18, 19), (19, 20), (19, 21), (20, 21)])
t4c21_4p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (1, 7), (2, 3), (2, 7),
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t4c21_5p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 3), (2, 6), (2, 7),
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    (17, 19), (17, 21), (18, 19), (19, 20), (19, 21), (20, 21)])
t4c21_6p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (1, 7), (2, 3), (2, 7),
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    (11, 12), (11, 18), (11, 19), (12, 13), (12, 19), (13, 14), (13, 19), (13, 20),
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    (17, 21), (18, 19), (18, 21), (19, 20), (19, 21), (20, 21)])
t4c22_1p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (1, 7), (2, 3), (2, 7),
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    (21, 22)])
t4c22_2p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (1, 7), (2, 3), (2, 7),
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(2, 9), (2, 10), (3, 4), (3, 10), (4, 5), (4, 10), (4, 11), (4, 12), (5, 6), (5, 12)
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    22), (18, 19), (18, 22), (19, 20), (19, 21), (19, 22), (20, 21), (21, 22)])
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t4c22_3p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (1, 7), (2, 3), (2, 7),
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t4c22_4p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (1, 7), (2, 3), (2, 7),
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    (21, 22)])
t4c22_5p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (1, 7), (2, 3), (2, 7),
    (2, 8), (2, 9), (2, 10), (3, 4), (3, 10), (3, 11), (4, 5), (4, 11), (4, 12), (5,
     6), (5, 12), (5, 13), (6, 7), (6, 13), (6, 14), (6, 15), (7, 8), (7, 15), (8,
    9), (8, 15), (8, 16), (8, 17), (9, 10), (9, 17), (9, 18), (10, 11), (10, 18),
    (11, 12), (11, 18), (11, 19), (12, 13), (12, 19), (12, 20), (13, 14), (13, 20),
    (14, 15), (14, 20), (14, 21), (15, 16), (15, 21), (16, 17), (16, 21), (16, 22),
    (17, 18), (17, 22), (18, 19), (18, 22), (19, 20), (19, 22), (20, 21), (20, 22),
    (21, 22)])

#5-connected triangulations which maximize the prox
t5c12p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 3), (2, 6), (2, 7), (2,
    8), (3, 4), (3, 8), (3, 9), (4, 5), (4, 9), (4, 10), (5, 6), (5, 10), (5, 11),
    (6, 7), (6, 11), (7, 8), (7, 11), (7, 12), (8, 9), (8, 12), (9, 10), (9, 12),
    (10, 11), (10, 12), (11, 12)])
t5c14p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 3), (2, 6), (2, 7), (2,
    8), (3, 4), (3, 8), (3, 9), (4, 5), (4, 9), (4, 10), (5, 6), (5, 10), (5, 11),
    (6, 7), (6, 11), (6, 12), (7, 8), (7, 12), (7, 13), (8, 9), (8, 13), (9, 10),
    (9, 13), (9, 14), (10, 11), (10, 14), (11, 12), (11, 14), (12, 13), (12, 14),
    (13, 14)])
t5c15p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (1, 7), (2, 3), (2, 7), (2,
    8), (2, 9), (3, 4), (3, 9), (3, 10), (4, 5), (4, 10), (4, 11), (5, 6), (5, 11),
    (5, 12), (6, 7), (6, 12), (6, 13), (7, 8), (7, 13), (8, 9), (8, 13), (8, 14),
    (9, 10), (9, 14), (10, 11), (10, 14), (10, 15), (11, 12), (11, 15), (12, 13),
    (12, 15), (13, 14), (13, 15), (14, 15)])
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t5c16p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 3), (2, 6), (2, 7), (2,
    8), (3, 4), (3, 8), (3, 9), (3, 10), (4, 5), (4, 10), (4, 11), (5, 6), (5, 11),
    (5, 12), (6, 7), (6, 12), (6, 13), (7, 8), (7, 13), (7, 14), (8, 9), (8, 14),
    (9, 10), (9, 14), (9, 15), (10, 11), (10, 15), (11, 12), (11, 15), (11, 16),
    (12, 13), (12, 16), (13, 14), (13, 16), (14, 15), (14, 16), (15, 16)])
t5c17p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 3), (2, 6), (2, 7), (2,
    8), (3, 4), (3, 8), (3, 9), (4, 5), (4, 9), (4, 10), (4, 11), (5, 6), (5, 11),
    (5, 12), (6, 7), (6, 12), (6, 13), (7, 8), (7, 13), (7, 14), (8, 9), (8, 14),
    (8, 15), (9, 10), (9, 15), (10, 11), (10, 15), (10, 16), (11, 12), (11, 16),
    (11, 17), (12, 13), (12, 17), (13, 14), (13, 17), (14, 15), (14, 16), (14, 17),
    (15, 16), (16, 17)])
t5c18p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 3), (2, 6), (2, 7), (2,
    8), (2, 9), (3, 4), (3, 9), (3, 10), (3, 11), (4, 5), (4, 11), (4, 12), (5, 6),
    (5, 12), (5, 13), (5, 14), (6, 7), (6, 14), (7, 8), (7, 14), (7, 15), (8, 9),
    (8, 15), (8, 16), (9, 10), (9, 16), (10, 11), (10, 16), (10, 17), (11, 12), (11,
     17), (12, 13), (12, 17), (13, 14), (13, 17), (13, 18), (14, 15), (14, 18), (15,
     16), (15, 18), (16, 17), (16, 18), (17, 18)])
t5c19_1p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 3), (2, 6), (2, 7),
    (2, 8), (3, 4), (3, 8), (3, 9), (4, 5), (4, 9), (4, 10), (4, 11), (5, 6), (5,
    11), (5, 12), (6, 7), (6, 12), (6, 13), (7, 8), (7, 13), (7, 14), (8, 9), (8,
    14), (8, 15), (9, 10), (9, 15), (10, 11), (10, 15), (10, 16), (11, 12), (11, 16)
    , (11, 17), (12, 13), (12, 17), (12, 18), (13, 14), (13, 18), (14, 15), (14, 18)
    , (14, 19), (15, 16), (15, 19), (16, 17), (16, 19), (17, 18), (17, 19), (18, 19)
    ])
t5c19_2p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 3), (2, 6), (2, 7),
    (2, 8), (2, 9), (3, 4), (3, 9), (3, 10), (4, 5), (4, 10), (4, 11), (4, 12), (5,
    6), (5, 12), (5, 13), (5, 14), (6, 7), (6, 14), (7, 8), (7, 14), (7, 15), (8, 9)
    , (8, 15), (8, 16), (9, 10), (9, 16), (9, 17), (10, 11), (10, 17), (11, 12),
    (11, 17), (11, 18), (12, 13), (12, 18), (13, 14), (13, 18), (13, 19), (14, 15),
    (14, 19), (15, 16), (15, 19), (16, 17), (16, 19), (17, 18), (17, 19), (18, 19)])
t5c19_3p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 3), (2, 6), (2, 7),
    (2, 8), (2, 9), (3, 4), (3, 9), (3, 10), (4, 5), (4, 10), (4, 11), (4, 12), (5,
    6), (5, 12), (5, 13), (5, 14), (6, 7), (6, 14), (7, 8), (7, 14), (7, 15), (8, 9)
    , (8, 15), (8, 16), (9, 10), (9, 16), (9, 17), (10, 11), (10, 17), (11, 12),
    (11, 17), (11, 18), (12, 13), (12, 18), (13, 14), (13, 18), (13, 19), (14, 15),
    (14, 19), (15, 16), (15, 19), (16, 17), (16, 18), (16, 19), (17, 18), (18, 19)])
t5c19_4p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 3), (2, 6), (2, 7),
    (2, 8), (2, 9), (3, 4), (3, 9), (3, 10), (4, 5), (4, 10), (4, 11), (4, 12), (5,
    6), (5, 12), (5, 13), (5, 14), (6, 7), (6, 14), (7, 8), (7, 14), (7, 15), (8, 9)
    , (8, 15), (8, 16), (9, 10), (9, 16), (10, 11), (10, 16), (10, 17), (11, 12),
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    (11, 17), (11, 18), (12, 13), (12, 18), (13, 14), (13, 18), (13, 19), (14, 15),
    (14, 19), (15, 16), (15, 17), (15, 19), (16, 17), (17, 18), (17, 19), (18, 19)])
t5c20_1p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 3), (2, 6), (2, 7),
    (2, 8), (2, 9), (3, 4), (3, 9), (3, 10), (3, 11), (4, 5), (4, 11), (4, 12), (5,
    6), (5, 12), (5, 13), (5, 14), (6, 7), (6, 14), (7, 8), (7, 14), (7, 15), (7,
    16), (8, 9), (8, 16), (8, 17), (9, 10), (9, 17), (10, 11), (10, 17), (10, 18),
    (11, 12), (11, 18), (11, 19), (12, 13), (12, 19), (13, 14), (13, 15), (13, 19),
    (14, 15), (15, 16), (15, 19), (15, 20), (16, 17), (16, 20), (17, 18), (17, 20),
    (18, 19), (18, 20), (19, 20)])
t5c20_1p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 3), (2, 6), (2, 7),
    (2, 8), (2, 9), (3, 4), (3, 9), (3, 10), (3, 11), (4, 5), (4, 11), (4, 12), (5,
    6), (5, 12), (5, 13), (5, 14), (6, 7), (6, 14), (7, 8), (7, 14), (7, 15), (7,
    16), (8, 9), (8, 16), (8, 17), (9, 10), (9, 17), (10, 11), (10, 17), (10, 18),
    (11, 12), (11, 18), (11, 19), (12, 13), (12, 19), (13, 14), (13, 15), (13, 19),
    (13, 20), (14, 15), (15, 16), (15, 20), (16, 17), (16, 20), (17, 18), (17, 20),
    (18, 19), (18, 20), (19, 20)])
t5c20_1p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (1, 7), (2, 3), (2, 7),
    (2, 8), (2, 9), (3, 4), (3, 9), (3, 10), (3, 11), (4, 5), (4, 11), (4, 12), (5,
    6), (5, 12), (5, 13), (6, 7), (6, 13), (6, 14), (6, 15), (7, 8), (7, 15), (8, 9)
    , (8, 15), (8, 16), (8, 17), (9, 10), (9, 17), (10, 11), (10, 17), (10, 18),
    (11, 12), (11, 18), (11, 19), (12, 13), (12, 19), (13, 14), (13, 19), (14, 15),
    (14, 16), (14, 19), (14, 20), (15, 16), (16, 17), (16, 20), (17, 18), (17, 20),
    (18, 19), (18, 20), (19, 20)])
t5c20_1p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (1, 7), (2, 3), (2, 7),
    (2, 8), (2, 9), (3, 4), (3, 9), (3, 10), (4, 5), (4, 10), (4, 11), (4, 12), (5,
    6), (5, 12), (5, 13), (6, 7), (6, 13), (6, 14), (6, 15), (7, 8), (7, 15), (8, 9)
    , (8, 15), (8, 16), (8, 17), (9, 10), (9, 17), (10, 11), (10, 17), (10, 18),
    (11, 12), (11, 18), (11, 19), (12, 13), (12, 19), (13, 14), (13, 19), (14, 15),
    (14, 16), (14, 19), (14, 20), (15, 16), (16, 17), (16, 20), (17, 18), (17, 20),
    (18, 19), (18, 20), (19, 20)])
t5c20_1p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (1, 7), (2, 3), (2, 7),
    (2, 8), (2, 9), (2, 10), (3, 4), (3, 10), (3, 11), (4, 5), (4, 11), (4, 12), (4,
     13), (5, 6), (5, 13), (5, 14), (6, 7), (6, 14), (6, 15), (7, 8), (7, 15), (8,
    9), (8, 15), (8, 16), (9, 10), (9, 16), (9, 17), (10, 11), (10, 17), (10, 18),
    (11, 12), (11, 18), (12, 13), (12, 18), (12, 19), (13, 14), (13, 19), (13, 20),
    (14, 15), (14, 16), (14, 20), (15, 16), (16, 17), (16, 20), (17, 18), (17, 19),
    (17, 20), (18, 19), (19, 20)])
t5c20_1p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 3), (2, 6), (2, 7),
    (2, 8), (2, 9), (3, 4), (3, 9), (3, 10), (4, 5), (4, 10), (4, 11), (4, 12), (5,
    6), (5, 12), (5, 13), (6, 7), (6, 13), (6, 14), (7, 8), (7, 14), (7, 15), (8, 9)
    , (8, 15), (8, 16), (8, 17), (9, 10), (9, 17), (10, 11), (10, 17), (10, 18),
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    (11, 12), (11, 18), (11, 19), (12, 13), (12, 19), (13, 14), (13, 19), (14, 15),
    (14, 19), (14, 20), (15, 16), (15, 20), (16, 17), (16, 18), (16, 20), (17, 18),
    (18, 19), (18, 20), (19, 20)])
t5c21_1p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (1, 7), (2, 3), (2, 7),
    (2, 8), (2, 9), (3, 4), (3, 9), (3, 10), (4, 5), (4, 10), (4, 11), (4, 12), (5,
    6), (5, 12), (5, 13), (6, 7), (6, 13), (6, 14), (7, 8), (7, 14), (7, 15), (8, 9)
    , (8, 15), (8, 16), (8, 17), (9, 10), (9, 17), (10, 11), (10, 17), (10, 18),
    (11, 12), (11, 18), (11, 19), (12, 13), (12, 19), (13, 14), (13, 19), (13, 20),
    (14, 15), (14, 20), (15, 16), (15, 20), (16, 17), (16, 20), (16, 21), (17, 18),
    (17, 21), (18, 19), (18, 21), (19, 20), (19, 21), (20, 21)])
t5c21_2p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (1, 7), (2, 3), (2, 7),
    (2, 8), (2, 9), (3, 4), (3, 9), (3, 10), (3, 11), (4, 5), (4, 11), (4, 12), (5,
    6), (5, 12), (5, 13), (6, 7), (6, 13), (6, 14), (6, 15), (7, 8), (7, 15), (8, 9)
    , (8, 15), (8, 16), (8, 17), (8, 18), (9, 10), (9, 18), (10, 11), (10, 18), (10,
     19), (11, 12), (11, 19), (11, 20), (12, 13), (12, 20), (13, 14), (13, 20), (13,
     21), (14, 15), (14, 16), (14, 21), (15, 16), (16, 17), (16, 21), (17, 18), (17,
     19), (17, 21), (18, 19), (19, 20), (19, 21), (20, 21)])
t5c21_3p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 3), (2, 6), (2, 7),
    (2, 8), (2, 9), (3, 4), (3, 9), (3, 10), (4, 5), (4, 10), (4, 11), (4, 12), (5,
    6), (5, 12), (5, 13), (6, 7), (6, 13), (6, 14), (7, 8), (7, 14), (7, 15), (8, 9)
    , (8, 15), (8, 16), (8, 17), (9, 10), (9, 17), (10, 11), (10, 17), (10, 18),
    (11, 12), (11, 18), (11, 19), (12, 13), (12, 19), (12, 20), (13, 14), (13, 20),
    (14, 15), (14, 20), (14, 21), (15, 16), (15, 21), (16, 17), (16, 18), (16, 21),
    (17, 18), (18, 19), (18, 21), (19, 20), (19, 21), (20, 21)])
t5c22_1p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 3), (2, 6), (2, 7),
    (2, 8), (2, 9), (3, 4), (3, 9), (3, 10), (3, 11), (4, 5), (4, 11), (4, 12), (5,
    6), (5, 12), (5, 13), (5, 14), (6, 7), (6, 14), (7, 8), (7, 14), (7, 15), (8, 9)
    , (8, 15), (8, 16), (8, 17), (9, 10), (9, 17), (10, 11), (10, 17), (10, 18),
    (10, 19), (11, 12), (11, 19), (12, 13), (12, 19), (12, 20), (13, 14), (13, 20),
    (13, 21), (14, 15), (14, 21), (15, 16), (15, 21), (16, 17), (16, 18), (16, 21),
    (16, 22), (17, 18), (18, 19), (18, 22), (19, 20), (19, 22), (20, 21), (20, 22),
    (21, 22)])
t5c22_2p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 3), (2, 6), (2, 7),
    (2, 8), (2, 9), (3, 4), (3, 9), (3, 10), (4, 5), (4, 10), (4, 11), (4, 12), (5,
    6), (5, 12), (5, 13), (5, 14), (6, 7), (6, 14), (7, 8), (7, 14), (7, 15), (7,
    16), (8, 9), (8, 16), (8, 17), (9, 10), (9, 17), (9, 18), (10, 11), (10, 18),
    (11, 12), (11, 18), (11, 19), (11, 20), (12, 13), (12, 20), (13, 14), (13, 15),
    (13, 20), (13, 21), (14, 15), (15, 16), (15, 21), (16, 17), (16, 21), (16, 22),
    (17, 18), (17, 19), (17, 22), (18, 19), (19, 20), (19, 22), (20, 21), (20, 22),
    (21, 22)])
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t5c23_1p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (1, 7), (2, 3), (2, 7),
    (2, 8), (2, 9), (3, 4), (3, 9), (3, 10), (4, 5), (4, 10), (4, 11), (4, 12), (5,
    6), (5, 12), (5, 13), (6, 7), (6, 13), (6, 14), (7, 8), (7, 14), (7, 15), (8, 9)
    , (8, 15), (8, 16), (9, 10), (9, 16), (9, 17), (9, 18), (10, 11), (10, 18), (11,
     12), (11, 18), (11, 19), (11, 20), (12, 13), (12, 20), (13, 14), (13, 20), (13,
     21), (13, 22), (14, 15), (14, 22), (15, 16), (15, 22), (15, 23), (16, 17), (16,
     23), (17, 18), (17, 19), (17, 23), (18, 19), (19, 20), (19, 21), (19, 23), (20,
     21), (21, 22), (21, 23), (22, 23)])
t5c23_2p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 3), (2, 6), (2, 7),
    (2, 8), (2, 9), (3, 4), (3, 9), (3, 10), (3, 11), (4, 5), (4, 11), (4, 12), (5,
    6), (5, 12), (5, 13), (5, 14), (6, 7), (6, 14), (7, 8), (7, 14), (7, 15), (7,
    16), (8, 9), (8, 16), (8, 17), (9, 10), (9, 17), (10, 11), (10, 17), (10, 18),
    (10, 19), (11, 12), (11, 19), (12, 13), (12, 19), (12, 20), (13, 14), (13, 20),
    (13, 21), (14, 15), (14, 21), (15, 16), (15, 21), (15, 22), (16, 17), (16, 22),
    (17, 18), (17, 22), (18, 19), (18, 22), (18, 23), (19, 20), (19, 23), (20, 21),
    (20, 23), (21, 22), (21, 23), (22, 23)])
t5c23_3p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (1, 7), (2, 3), (2, 7),
    (2, 8), (2, 9), (2, 10), (3, 4), (3, 10), (3, 11), (3, 12), (4, 5), (4, 12), (4,
     13), (4, 14), (5, 6), (5, 14), (5, 15), (6, 7), (6, 15), (6, 16), (7, 8), (7,
    16), (8, 9), (8, 16), (8, 17), (9, 10), (9, 17), (9, 18), (10, 11), (10, 18),
    (11, 12), (11, 18), (11, 19), (12, 13), (12, 19), (13, 14), (13, 19), (13, 20),
    (14, 15), (14, 20), (14, 21), (15, 16), (15, 21), (15, 22), (16, 17), (16, 22),
    (17, 18), (17, 22), (17, 23), (18, 19), (18, 23), (19, 20), (19, 23), (20, 21),
    (20, 23), (21, 22), (21, 23), (22, 23)])
t5c23_4p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (1, 7), (2, 3), (2, 7),
    (2, 8), (2, 9), (3, 4), (3, 9), (3, 10), (3, 11), (4, 5), (4, 11), (4, 12), (5,
    6), (5, 12), (5, 13), (5, 14), (6, 7), (6, 14), (6, 15), (7, 8), (7, 15), (8, 9)
    , (8, 15), (8, 16), (8, 17), (9, 10), (9, 17), (9, 18), (10, 11), (10, 18), (10,
     19), (11, 12), (11, 19), (11, 20), (12, 13), (12, 20), (13, 14), (13, 20), (13,
     21), (13, 22), (14, 15), (14, 22), (15, 16), (15, 22), (16, 17), (16, 21), (16,
     22), (16, 23), (17, 18), (17, 23), (18, 19), (18, 23), (19, 20), (19, 23), (20,
     21), (20, 23), (21, 22), (21, 23)])
t5c23_5p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (1, 7), (2, 3), (2, 7),
    (2, 8), (2, 9), (2, 10), (3, 4), (3, 10), (3, 11), (4, 5), (4, 11), (4, 12), (4,
     13), (5, 6), (5, 13), (5, 14), (6, 7), (6, 14), (6, 15), (7, 8), (7, 15), (8,
    9), (8, 15), (8, 16), (9, 10), (9, 16), (9, 17), (9, 18), (10, 11), (10, 18),
    (11, 12), (11, 18), (11, 19), (12, 13), (12, 19), (12, 20), (12, 21), (13, 14),
    (13, 21), (14, 15), (14, 21), (14, 22), (15, 16), (15, 22), (16, 17), (16, 22),
    (16, 23), (17, 18), (17, 19), (17, 20), (17, 23), (18, 19), (19, 20), (20, 21),
    (20, 23), (21, 22), (21, 23), (22, 23)])
```

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t5c24_1p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (1, 7), (2, 3), (2, 7),
    (2, 8), (2, 9), (3, 4), (3, 9), (3, 10), (3, 11), (4, 5), (4, 11), (4, 12), (4,
    13), (5, 6), (5, 13), (5, 14), (6, 7), (6, 14), (6, 15), (7, 8), (7, 15), (7,
    16), (8, 9), (8, 16), (8, 17), (9, 10), (9, 17), (9, 18), (10, 11), (10, 18),
    (10, 19), (11, 12), (11, 19), (12, 13), (12, 19), (12, 20), (12, 21), (13, 14),
    (13, 21), (14, 15), (14, 21), (14, 22), (15, 16), (15, 22), (15, 23), (16, 17),
    (16, 23), (17, 18), (17, 23), (17, 24), (18, 19), (18, 20), (18, 24), (19, 20),
    (20, 21), (20, 24), (21, 22), (21, 24), (22, 23), (22, 24), (23, 24)])
t5c24_2p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 3), (2, 6), (2, 7),
    (2, 8), (2, 9), (3, 4), (3, 9), (3, 10), (3, 11), (4, 5), (4, 11), (4, 12), (4,
    13), (5, 6), (5, 13), (5, 14), (6, 7), (6, 14), (6, 15), (7, 8), (7, 15), (7,
    16), (8, 9), (8, 16), (8, 17), (8, 18), (9, 10), (9, 18), (10, 11), (10, 18),
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    (16, 23), (17, 18), (17, 19), (17, 23), (17, 24), (18, 19), (19, 20), (19, 21),
    (19, 24), (20, 21), (21, 22), (21, 24), (22, 23), (22, 24), (23, 24)])
t5c24_3p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (1, 7), (2, 3), (2, 7),
    (2, 8), (2, 9), (3, 4), (3, 9), (3, 10), (4, 5), (4, 10), (4, 11), (4, 12), (5,
    6), (5, 12), (5, 13), (5, 14), (6, 7), (6, 14), (6, 15), (7, 8), (7, 15), (8, 9)
    , (8, 15), (8, 16), (8, 17), (9, 10), (9, 17), (9, 18), (10, 11), (10, 18), (10,
     19), (11, 12), (11, 19), (11, 20), (12, 13), (12, 20), (13, 14), (13, 20), (13,
     21), (13, 22), (14, 15), (14, 22), (15, 16), (15, 22), (16, 17), (16, 22), (16,
     23), (17, 18), (17, 23), (17, 24), (18, 19), (18, 24), (19, 20), (19, 24), (20,
     21), (20, 24), (21, 22), (21, 23), (21, 24), (22, 23), (23, 24)])
t5c24_4p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (1, 7), (1, 8), (2, 3),
    (2, 8), (2, 9), (2, 10), (3, 4), (3, 10), (3, 11), (4, 5), (4, 11), (4, 12), (4,
     13), (5, 6), (5, 13), (5, 14), (6, 7), (6, 14), (6, 15), (7, 8), (7, 15), (7,
    16), (8, 9), (8, 16), (9, 10), (9, 16), (9, 17), (9, 18), (10, 11), (10, 18),
    (10, 19), (11, 12), (11, 19), (12, 13), (12, 19), (12, 20), (13, 14), (13, 20),
    (13, 21), (14, 15), (14, 21), (15, 16), (15, 21), (15, 22), (16, 17), (16, 22),
    (17, 18), (17, 22), (17, 23), (18, 19), (18, 23), (19, 20), (19, 23), (19, 24),
    (20, 21), (20, 24), (21, 22), (21, 24), (22, 23), (22, 24), (23, 24)])
t5c24_5p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (1, 7), (1, 8), (2, 3),
    (2, 8), (2, 9), (2, 10), (3, 4), (3, 10), (3, 11), (4, 5), (4, 11), (4, 12), (4,
     13), (5, 6), (5, 13), (5, 14), (6, 7), (6, 14), (6, 15), (7, 8), (7, 15), (7,
    16), (8, 9), (8, 16), (9, 10), (9, 16), (9, 17), (9, 18), (10, 11), (10, 18),
    (10, 19), (11, 12), (11, 19), (12, 13), (12, 19), (12, 20), (13, 14), (13, 20),
    (13, 21), (14, 15), (14, 21), (14, 22), (15, 16), (15, 17), (15, 22), (16, 17),
    (17, 18), (17, 22), (17, 23), (18, 19), (18, 23), (19, 20), (19, 23), (19, 24),
    (20, 21), (20, 24), (21, 22), (21, 24), (22, 23), (22, 24), (23, 24)])
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t5c24_6p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 3), (2, 6), (2, 7),
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    16), (8, 9), (8, 16), (8, 17), (9, 10), (9, 17), (10, 11), (10, 17), (10, 18),
    (10, 19), (11, 12), (11, 19), (12, 13), (12, 19), (12, 20), (13, 14), (13, 20),
    (13, 21), (14, 15), (14, 21), (15, 16), (15, 21), (15, 22), (16, 17), (16, 22),
    (16, 23), (17, 18), (17, 23), (18, 19), (18, 20), (18, 23), (18, 24), (19, 20),
    (20, 21), (20, 24), (21, 22), (21, 24), (22, 23), (22, 24), (23, 24)])
t5c24_7p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 3), (2, 6), (2, 7),
    (2, 8), (2, 9), (3, 4), (3, 9), (3, 10), (3, 11), (4, 5), (4, 11), (4, 12), (5,
    6), (5, 12), (5, 13), (5, 14), (5, 15), (6, 7), (6, 15), (7, 8), (7, 15), (7,
    16), (7, 17), (8, 9), (8, 17), (8, 18), (8, 19), (9, 10), (9, 19), (10, 11),
    (10, 19), (10, 20), (10, 21), (11, 12), (11, 21), (12, 13), (12, 21), (13, 14),
    (13, 21), (13, 22), (14, 15), (14, 16), (14, 22), (14, 23), (15, 16), (16, 17),
    (16, 23), (17, 18), (17, 23), (17, 24), (18, 19), (18, 20), (18, 24), (19, 20),
    (20, 21), (20, 22), (20, 24), (21, 22), (22, 23), (22, 24), (23, 24)])
t5c24_8p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 3), (2, 6), (2, 7),
    (2, 8), (2, 9), (3, 4), (3, 9), (3, 10), (3, 11), (4, 5), (4, 11), (4, 12), (4,
    13), (5, 6), (5, 13), (5, 14), (5, 15), (6, 7), (6, 15), (7, 8), (7, 15), (7,
    16), (8, 9), (8, 16), (8, 17), (8, 18), (8, 19), (9, 10), (9, 19), (10, 11),
    (10, 19), (10, 20), (11, 12), (11, 20), (11, 21), (12, 13), (12, 21), (12, 22),
    (13, 14), (13, 22), (14, 15), (14, 22), (14, 23), (15, 16), (15, 23), (16, 17),
    (16, 23), (17, 18), (17, 23), (17, 24), (18, 19), (18, 20), (18, 24), (19, 20),
    (20, 21), (20, 24), (21, 22), (21, 24), (22, 23), (22, 24), (23, 24)])
t5c24_9p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (1, 7), (2, 3), (2, 7),
    (2, 8), (2, 9), (3, 4), (3, 9), (3, 10), (4, 5), (4, 10), (4, 11), (4, 12), (5,
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    16), (8, 9), (8, 16), (8, 17), (8, 18), (9, 10), (9, 18), (9, 19), (10, 11),
    (10, 19), (11, 12), (11, 19), (11, 20), (11, 21), (12, 13), (12, 21), (12, 22),
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    (16, 23), (17, 18), (17, 23), (17, 24), (18, 19), (18, 20), (18, 24), (19, 20),
    (20, 21), (20, 24), (21, 22), (21, 24), (22, 23), (22, 24), (23, 24)])
t5c24_10p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 3), (2, 6), (2, 7),
    (2, 8), (2, 9), (3, 4), (3, 9), (3, 10), (3, 11), (4, 5), (4, 11), (4, 12), (5,
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    , (8, 15), (8, 16), (8, 17), (9, 10), (9, 17), (10, 11), (10, 17), (10, 18),
    (11, 12), (11, 18), (11, 19), (12, 13), (12, 19), (12, 20), (13, 14), (13, 20),
    (14, 15), (14, 20), (14, 21), (15, 16), (15, 21), (15, 22), (16, 17), (16, 22),
    (16, 23), (17, 18), (17, 23), (18, 19), (18, 23), (18, 24), (19, 20), (19, 24),
    (20, 21), (20, 24), (21, 22), (21, 24), (22, 23), (22, 24), (23, 24)])
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t5c24_11p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 3), (2, 6), (2, 7),
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    6), (5, 12), (5, 13), (5, 14), (6, 7), (6, 14), (7, 8), (7, 14), (7, 15), (7,
    16), (8, 9), (8, 16), (8, 17), (9, 10), (9, 17), (9, 18), (10, 11), (10, 18),
    (10, 19), (11, 12), (11, 19), (11, 20), (12, 13), (12, 20), (13, 14), (13, 20),
    (13, 21), (14, 15), (14, 21), (15, 16), (15, 21), (15, 22), (16, 17), (16, 22),
    (16, 23), (17, 18), (17, 23), (18, 19), (18, 23), (19, 20), (19, 23), (19, 24),
    (20, 21), (20, 24), (21, 22), (21, 24), (22, 23), (22, 24), (23, 24)])
t5c24_12p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 3), (2, 6), (2, 7),
    (2, 8), (2, 9), (3, 4), (3, 9), (3, 10), (4, 5), (4, 10), (4, 11), (4, 12), (5,
    6), (5, 12), (5, 13), (5, 14), (6, 7), (6, 14), (7, 8), (7, 14), (7, 15), (7,
    16), (8, 9), (8, 16), (8, 17), (9, 10), (9, 17), (9, 18), (10, 11), (10, 18),
    (10, 19), (11, 12), (11, 19), (11, 20), (12, 13), (12, 20), (13, 14), (13, 20),
    (13, 21), (13, 22), (14, 15), (14, 22), (15, 16), (15, 22), (15, 23), (16, 17),
    (16, 23), (16, 24), (17, 18), (17, 24), (18, 19), (18, 24), (19, 20), (19, 21),
    (19, 24), (20, 21), (21, 22), (21, 23), (21, 24), (22, 23), (23, 24)])
t5c24_13p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 3), (2, 6), (2, 7),
    (2, 8), (3, 4), (3, 8), (3, 9), (3, 10), (4, 5), (4, 10), (4, 11), (4, 12), (5,
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    16), (8, 9), (8, 16), (8, 17), (9, 10), (9, 17), (9, 18), (10, 11), (10, 18),
    (10, 19), (11, 12), (11, 19), (11, 20), (12, 13), (12, 20), (13, 14), (13, 20),
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    (16, 23), (16, 24), (17, 18), (17, 24), (18, 19), (18, 24), (19, 20), (19, 21),
    (19, 24), (20, 21), (21, 22), (21, 23), (21, 24), (22, 23), (23, 24)])
t5c24_14p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (1, 7), (2, 3), (2, 7),
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    6), (5, 12), (5, 13), (5, 14), (6, 7), (6, 14), (6, 15), (6, 16), (7, 8), (7,
    16), (8, 9), (8, 16), (8, 17), (8, 18), (9, 10), (9, 18), (9, 19), (10, 11),
    (10, 19), (10, 20), (10, 21), (11, 12), (11, 21), (12, 13), (12, 21), (12, 22),
    (13, 14), (13, 22), (13, 23), (14, 15), (14, 23), (15, 16), (15, 17), (15, 23),
    (16, 17), (17, 18), (17, 23), (17, 24), (18, 19), (18, 24), (19, 20), (19, 24),
    (20, 21), (20, 22), (20, 24), (21, 22), (22, 23), (22, 24), (23, 24)])
t5c24_15p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (1, 7), (2, 3), (2, 7),
    (2, 8), (2, 9), (2, 10), (2, 11), (3, 4), (3, 11), (3, 12), (4, 5), (4, 12), (4,
     13), (4, 14), (5, 6), (5, 14), (5, 15), (6, 7), (6, 15), (6, 16), (7, 8), (7,
    16), (8, 9), (8, 16), (8, 17), (9, 10), (9, 17), (9, 18), (10, 11), (10, 18),
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    (16, 23), (17, 18), (17, 23), (17, 24), (18, 19), (18, 20), (18, 24), (19, 20),
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t5c24_16p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (1, 7), (2, 3), (2, 7),
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    6), (5, 12), (5, 13), (6, 7), (6, 13), (6, 14), (6, 15), (7, 8), (7, 15), (8, 9)
    , (8, 15), (8, 16), (8, 17), (9, 10), (9, 17), (9, 18), (10, 11), (10, 18), (10,
     19), (11, 12), (11, 19), (11, 20), (12, 13), (12, 20), (12, 21), (13, 14), (13,
     21), (13, 22), (14, 15), (14, 16), (14, 22), (15, 16), (16, 17), (16, 22), (16,
     23), (17, 18), (17, 23), (18, 19), (18, 23), (18, 24), (19, 20), (19, 24), (20,
     21), (20, 24), (21, 22), (21, 23), (21, 24), (22, 23), (23, 24)])
t5c24_17p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (1, 7), (2, 3), (2, 7),
    (2, 8), (2, 9), (3, 4), (3, 9), (3, 10), (3, 11), (4, 5), (4, 11), (4, 12), (5,
    6), (5, 12), (5, 13), (5, 14), (6, 7), (6, 14), (6, 15), (7, 8), (7, 15), (8, 9)
    , (8, 15), (8, 16), (8, 17), (9, 10), (9, 17), (9, 18), (10, 11), (10, 18), (10,
     19), (11, 12), (11, 19), (11, 20), (12, 13), (12, 20), (13, 14), (13, 20), (13,
     21), (13, 22), (14, 15), (14, 22), (15, 16), (15, 22), (16, 17), (16, 22), (16,
     23), (17, 18), (17, 23), (18, 19), (18, 23), (18, 24), (19, 20), (19, 24), (20,
     21), (20, 24), (21, 22), (21, 23), (21, 24), (22, 23), (23, 24)])
t5c24_18p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (1, 7), (2, 3), (2, 7),
    (2, 8), (2, 9), (2, 10), (3, 4), (3, 10), (3, 11), (4, 5), (4, 11), (4, 12), (5,
     6), (5, 12), (5, 13), (5, 14), (6, 7), (6, 14), (6, 15), (7, 8), (7, 15), (8,
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    (13, 21), (13, 22), (14, 15), (14, 22), (15, 16), (15, 22), (16, 17), (16, 22),
    (16, 23), (17, 18), (17, 23), (17, 24), (18, 19), (18, 24), (19, 20), (19, 24),
    (20, 21), (20, 24), (21, 22), (21, 23), (21, 24), (22, 23), (23, 24)])
t5c24_19p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (1, 7), (2, 3), (2, 7),
    (2, 8), (2, 9), (2, 10), (3, 4), (3, 10), (3, 11), (4, 5), (4, 11), (4, 12), (4,
     13), (5, 6), (5, 13), (5, 14), (6, 7), (6, 14), (6, 15), (7, 8), (7, 15), (8,
    9), (8, 15), (8, 16), (8, 17), (9, 10), (9, 17), (9, 18), (10, 11), (10, 18),
    (11, 12), (11, 18), (11, 19), (12, 13), (12, 19), (12, 20), (12, 21), (13, 14),
    (13, 21), (14, 15), (14, 21), (14, 22), (15, 16), (15, 22), (16, 17), (16, 22),
    (16, 23), (16, 24), (17, 18), (17, 24), (18, 19), (18, 24), (19, 20), (19, 24),
    (20, 21), (20, 23), (20, 24), (21, 22), (21, 23), (22, 23), (23, 24)])
t5c25_1p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (1, 7), (2, 3), (2, 7),
    (2, 8), (2, 9), (3, 4), (3, 9), (3, 10), (3, 11), (4, 5), (4, 11), (4, 12), (4,
    13), (5, 6), (5, 13), (5, 14), (6, 7), (6, 14), (6, 15), (7, 8), (7, 15), (7,
    16), (8, 9), (8, 16), (8, 17), (9, 10), (9, 17), (9, 18), (10, 11), (10, 18),
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    (16, 23), (17, 18), (17, 23), (17, 24), (18, 19), (18, 20), (18, 24), (19, 20),
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    (23, 25), (24, 25)])
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t5c25_2p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (1, 7), (2, 3), (2, 7),
    (2, 8), (2, 9), (3, 4), (3, 9), (3, 10), (3, 11), (4, 5), (4, 11), (4, 12), (4,
    13), (5, 6), (5, 13), (5, 14), (6, 7), (6, 14), (6, 15), (7, 8), (7, 15), (7,
    16), (8, 9), (8, 16), (8, 17), (8, 18), (9, 10), (9, 18), (10, 11), (10, 18),
    (10, 19), (10, 20), (11, 12), (11, 20), (12, 13), (12, 20), (12, 21), (12, 22),
    (13, 14), (13, 22), (14, 15), (14, 22), (14, 23), (15, 16), (15, 23), (15, 24),
    (16, 17), (16, 24), (17, 18), (17, 19), (17, 24), (17, 25), (18, 19), (19, 20),
    (19, 21), (19, 25), (20, 21), (21, 22), (21, 25), (22, 23), (22, 25), (23, 24),
    (23, 25), (24, 25)])
t5c25_3p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (1, 7), (2, 3), (2, 7),
    (2, 8), (2, 9), (3, 4), (3, 9), (3, 10), (3, 11), (4, 5), (4, 11), (4, 12), (4,
    13), (5, 6), (5, 13), (5, 14), (6, 7), (6, 14), (6, 15), (7, 8), (7, 15), (7,
    16), (8, 9), (8, 16), (8, 17), (8, 18), (9, 10), (9, 18), (10, 11), (10, 18),
    (10, 19), (11, 12), (11, 19), (12, 13), (12, 19), (12, 20), (12, 21), (13, 14),
    (13, 21), (14, 15), (14, 21), (14, 22), (15, 16), (15, 22), (15, 23), (16, 17),
    (16, 23), (17, 18), (17, 23), (17, 24), (18, 19), (18, 24), (19, 20), (19, 24),
    (20, 21), (20, 24), (20, 25), (21, 22), (21, 25), (22, 23), (22, 25), (23, 24),
    (23, 25), (24, 25)])
t5c25_4p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (1, 7), (2, 3), (2, 7),
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    16), (8, 9), (8, 16), (8, 17), (9, 10), (9, 17), (9, 18), (10, 11), (10, 18),
    (10, 19), (11, 12), (11, 19), (11, 20), (12, 13), (12, 20), (13, 14), (13, 20),
    (13, 21), (13, 22), (13, 23), (14, 15), (14, 23), (15, 16), (15, 23), (15, 24),
    (16, 17), (16, 24), (17, 18), (17, 24), (17, 25), (18, 19), (18, 25), (19, 20),
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    (23, 24), (24, 25)])
t5c25_5p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (1, 7), (2, 3), (2, 7),
    (2, 8), (2, 9), (2, 10), (3, 4), (3, 10), (3, 11), (4, 5), (4, 11), (4, 12), (4,
     13), (5, 6), (5, 13), (5, 14), (6, 7), (6, 14), (6, 15), (6, 16), (7, 8), (7,
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    (16, 17), (17, 18), (17, 23), (17, 24), (18, 19), (18, 24), (18, 25), (19, 20),
    (19, 25), (20, 21), (20, 25), (21, 22), (21, 24), (21, 25), (22, 23), (22, 24),
    (23, 24), (24, 25)])
t5c25_6p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 3), (2, 6), (2, 7),
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    (16, 23), (16, 24), (17, 18), (17, 24), (18, 19), (18, 24), (18, 25), (19, 20),
    (19, 25), (20, 21), (20, 25), (21, 22), (21, 25), (22, 23), (22, 24), (22, 25),
    (23, 24), (24, 25)])
t5c25_7p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 3), (2, 6), (2, 7),
    (2, 8), (3, 4), (3, 8), (3, 9), (4, 5), (4, 9), (4, 10), (4, 11), (5, 6), (5,
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    (16, 23), (16, 24), (17, 18), (17, 24), (18, 19), (18, 24), (18, 25), (19, 20),
    (19, 25), (20, 21), (20, 25), (21, 22), (21, 25), (22, 23), (22, 25), (23, 24),
    (23, 25), (24, 25)])
t5c25_8p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 3), (2, 6), (2, 7),
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    (16, 23), (16, 24), (17, 18), (17, 24), (18, 19), (18, 20), (18, 24), (18, 25),
    (19, 20), (20, 21), (20, 25), (21, 22), (21, 25), (22, 23), (22, 25), (23, 24),
    (23, 25), (24, 25)])
t5c25_9p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 3), (2, 6), (2, 7),
    (2, 8), (3, 4), (3, 8), (3, 9), (4, 5), (4, 9), (4, 10), (4, 11), (5, 6), (5,
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    (16, 23), (17, 18), (17, 23), (17, 24), (18, 19), (18, 24), (19, 20), (19, 24),
    (19, 25), (20, 21), (20, 25), (21, 22), (21, 25), (22, 23), (22, 25), (23, 24),
    (23, 25), (24, 25)])
t5c25_10p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 3), (2, 6), (2, 7),
    (2, 8), (2, 9), (3, 4), (3, 9), (3, 10), (4, 5), (4, 10), (4, 11), (4, 12), (5,
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    (13, 21), (14, 15), (14, 21), (15, 16), (15, 21), (15, 22), (15, 23), (16, 17),
    (16, 23), (17, 18), (17, 23), (17, 24), (18, 19), (18, 24), (18, 25), (19, 20),
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    (23, 24), (24, 25)])
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t5c25_11p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 3), (2, 6), (2, 7),
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    16), (8, 9), (8, 16), (8, 17), (9, 10), (9, 17), (9, 18), (10, 11), (10, 18),
    (10, 19), (11, 12), (11, 19), (11, 20), (12, 13), (12, 20), (13, 14), (13, 20),
    (13, 21), (14, 15), (14, 21), (15, 16), (15, 21), (15, 22), (16, 17), (16, 22),
    (16, 23), (17, 18), (17, 23), (18, 19), (18, 23), (18, 24), (19, 20), (19, 24),
    (19, 25), (20, 21), (20, 25), (21, 22), (21, 25), (22, 23), (22, 24), (22, 25),
    (23, 24), (24, 25)])
t5c25_12p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 3), (2, 6), (2, 7),
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    (16, 23), (16, 24), (17, 18), (17, 24), (18, 19), (18, 24), (18, 25), (19, 20),
    (19, 25), (20, 21), (20, 22), (20, 25), (21, 22), (22, 23), (22, 25), (23, 24),
    (23, 25), (24, 25)])
t5c25_13p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (1, 7), (2, 3), (2, 7),
    (2, 8), (2, 9), (3, 4), (3, 9), (3, 10), (3, 11), (4, 5), (4, 11), (4, 12), (5,
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    16), (8, 9), (8, 16), (8, 17), (8, 18), (9, 10), (9, 18), (9, 19), (10, 11),
    (10, 19), (10, 20), (10, 21), (11, 12), (11, 21), (12, 13), (12, 21), (12, 22),
    (13, 14), (13, 22), (13, 23), (14, 15), (14, 23), (15, 16), (15, 17), (15, 23),
    (15, 24), (16, 17), (17, 18), (17, 24), (18, 19), (18, 24), (18, 25), (19, 20),
    (19, 25), (20, 21), (20, 22), (20, 25), (21, 22), (22, 23), (22, 25), (23, 24),
    (23, 25), (24, 25)])
t5c25_14p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (1, 7), (2, 3), (2, 7),
    (2, 8), (2, 9), (2, 10), (3, 4), (3, 10), (3, 11), (4, 5), (4, 11), (4, 12), (5,
     6), (5, 12), (5, 13), (5, 14), (6, 7), (6, 14), (6, 15), (7, 8), (7, 15), (8,
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    (10, 19), (11, 12), (11, 19), (11, 20), (12, 13), (12, 20), (12, 21), (13, 14),
    (13, 21), (13, 22), (14, 15), (14, 22), (14, 23), (15, 16), (15, 23), (16, 17),
    (16, 23), (16, 24), (17, 18), (17, 24), (18, 19), (18, 24), (18, 25), (19, 20),
    (19, 25), (20, 21), (20, 25), (21, 22), (21, 25), (22, 23), (22, 24), (22, 25),
    (23, 24), (24, 25)])
t5c25_15p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 3), (2, 6), (2, 7),
    (2, 8), (2, 9), (3, 4), (3, 9), (3, 10), (4, 5), (4, 10), (4, 11), (4, 12), (5,
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    (23, 24), (24, 25)])
t5c25_16p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (1, 7), (2, 3), (2, 7),
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    (16, 23), (17, 18), (17, 23), (17, 24), (18, 19), (18, 24), (18, 25), (19, 20),
    (19, 25), (20, 21), (20, 25), (21, 22), (21, 24), (21, 25), (22, 23), (22, 24),
    (23, 24), (24, 25)])
t5c25_17p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (1, 7), (1, 8), (2, 3),
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    (10, 19), (11, 12), (11, 19), (11, 20), (12, 13), (12, 20), (12, 21), (13, 14),
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    (16, 23), (17, 18), (17, 23), (17, 24), (17, 25), (18, 19), (18, 25), (19, 20),
    (19, 25), (20, 21), (20, 25), (21, 22), (21, 24), (21, 25), (22, 23), (22, 24),
    (23, 24), (24, 25)])
t5c25_18p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (1, 7), (2, 3), (2, 7),
    (2, 8), (2, 9), (3, 4), (3, 9), (3, 10), (4, 5), (4, 10), (4, 11), (4, 12), (5,
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    14), (13, 22), (14, 15), (14, 22), (14, 23), (15, 16), (15, 23), (16, 17), (16,
    23), (16, 24), (17, 18), (17, 24), (18, 19), (18, 20), (18, 24), (19, 20), (20,
    21), (20, 24), (20, 25), (21, 22), (21, 25), (22, 23), (22, 25), (23, 24), (23,
    25), (24, 25)])
t5c26_1p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (1, 7), (2, 3), (2, 7),
    (2, 8), (2, 9), (2, 10), (3, 4), (3, 10), (3, 11), (4, 5), (4, 11), (4, 12), (4,
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    (16, 23), (17, 18), (17, 23), (17, 24), (18, 19), (18, 24), (18, 25), (19, 20),
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t5c26_2p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (1, 7), (2, 3), (2, 7),
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    (19, 25), (20, 21), (20, 25), (20, 26), (21, 22), (21, 26), (22, 23), (22, 24),
    (22, 26), (23, 24), (24, 25), (24, 26), (25, 26)])
t5c26_3p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (1, 7), (2, 3), (2, 7),
    (2, 8), (2, 9), (2, 10), (3, 4), (3, 10), (3, 11), (4, 5), (4, 11), (4, 12), (4,
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    (16, 23), (17, 18), (17, 23), (17, 24), (18, 19), (18, 24), (18, 25), (19, 20),
    (19, 25), (20, 21), (20, 25), (20, 26), (21, 22), (21, 26), (22, 23), (22, 26),
    (23, 24), (23, 26), (24, 25), (24, 26), (25, 26)])
t5c27_1p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (1, 7), (2, 3), (2, 7),
    (2, 8), (2, 9), (2, 10), (3, 4), (3, 10), (3, 11), (4, 5), (4, 11), (4, 12), (4,
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    (22, 27), (23, 24), (23, 27), (24, 25), (24, 27), (25, 26), (25, 27), (26, 27)])
t5c27_2p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 3), (2, 6), (2, 7),
    (2, 8), (2, 9), (3, 4), (3, 9), (3, 10), (3, 11), (4, 5), (4, 11), (4, 12), (5,
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    16), (8, 9), (8, 16), (8, 17), (8, 18), (9, 10), (9, 18), (10, 11), (10, 18),
    (10, 19), (10, 20), (11, 12), (11, 20), (11, 21), (12, 13), (12, 21), (12, 22),
    (13, 14), (13, 22), (13, 23), (13, 24), (14, 15), (14, 24), (15, 16), (15, 24),
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    (22, 27), (23, 24), (23, 25), (23, 26), (23, 27), (24, 25), (25, 26), (26, 27)])
t5c27_3p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (1, 7), (2, 3), (2, 7),
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    , (8, 15), (8, 16), (8, 17), (9, 10), (9, 17), (9, 18), (10, 11), (10, 18), (10,
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     24), (22, 27), (23, 24), (24, 25), (24, 26), (24, 27), (25, 26), (26, 27)])
t5c28_1p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (1, 7), (2, 3), (2, 7),
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t5c28_2p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 3), (2, 6), (2, 7),
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    (26, 27), (26, 28), (27, 28)])
t5c28_3p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (1, 7), (2, 3), (2, 7),
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t5c29_1p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (1, 7), (2, 3), (2, 7),
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t5c29_2p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 3), (2, 6), (2, 7),
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t5c30_1p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (1, 7), (2, 3), (2, 7),
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t5c30_2p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 3), (2, 6), (2, 7),
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    (29, 30)])
t5c30_3p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 3), (2, 6), (2, 7),
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     26), (24, 30), (25, 26), (26, 27), (26, 30), (27, 28), (27, 30), (28, 29), (28,
     30)])
t5c30_4p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 3), (2, 6), (2, 7),
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    (29, 30)])
t5c30_5p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (1, 7), (2, 3), (2, 7),
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    (21, 28), (21, 29), (22, 23), (23, 24), (23, 29), (23, 30), (24, 25), (24, 30),
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    (29, 30)])
t5c30_6p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (1, 7), (2, 3), (2, 7),
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    (24, 30), (25, 26), (26, 27), (26, 30), (27, 28), (27, 30), (28, 29), (28, 30),
    (29, 30)])
t5c30_7p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 3), (2, 6), (2, 7),
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    (29, 30)])
t5c30_8p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 3), (2, 6), (2, 7),
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    (25, 29), (25, 30), (26, 27), (26, 30), (27, 28), (27, 30), (28, 29), (28, 30),
    (29, 30)])
t5c30_9p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (1, 7), (2, 3), (2, 7),
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     30), (25, 26), (26, 27), (26, 30), (27, 28), (27, 30), (28, 29), (28, 30), (29,
     30)])
t5c30_10p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 3), (2, 6), (2, 7),
    (2, 8), (2, 9), (2, 10), (3, 4), (3, 10), (3, 11), (3, 12), (4, 5), (4, 12), (4,
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t5c30_11p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (1, 7), (2, 3), (2, 7),
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t5c30_12p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (1, 7), (2, 3), (2, 7),
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     17), (15, 25), (16, 17), (17, 18), (17, 25), (17, 26), (18, 19), (18, 26), (18,
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     26), (25, 30), (26, 27), (26, 30), (27, 28), (27, 29), (27, 30), (28, 29), (29,
     30)])
t5c30_13p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (1, 7), (2, 3), (2, 7),
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    (29, 30)])
t5c30_14p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (1, 7), (2, 3), (2, 7),
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    30)])
t5c30_15p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (1, 7), (2, 3), (2, 7),
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    26), (25, 30), (26, 27), (26, 30), (27, 28), (27, 30), (28, 29), (28, 30), (29,
    30)])
t5c30_16p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 3), (2, 6), (2, 7),
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    (29, 30)])
t5c30_17p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (1, 7), (2, 3), (2, 7),
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    , (8, 15), (8, 16), (8, 17), (9, 10), (9, 17), (9, 18), (10, 11), (10, 18), (10,
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    26), (25, 30), (26, 27), (26, 30), (27, 28), (27, 30), (28, 29), (28, 30), (29,
    30)])
t5c30_18p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 3), (2, 6), (2, 7),
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     25), (25, 26), (25, 27), (26, 27), (27, 28), (27, 30), (28, 29), (28, 30), (29,
     30)])
t5c30_19p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (1, 7), (2, 3), (2, 7),
    (2, 8), (2, 9), (2, 10), (3, 4), (3, 10), (3, 11), (4, 5), (4, 11), (4, 12), (4,
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    (29, 30)])
t5c30_20p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (1, 7), (2, 3), (2, 7),
    (2, 8), (2, 9), (2, 10), (3, 4), (3, 10), (3, 11), (4, 5), (4, 11), (4, 12), (4,
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     27), (19, 20), (19, 27), (19, 28), (20, 21), (20, 28), (21, 22), (21, 28), (22,
     23), (22, 28), (22, 29), (23, 24), (23, 29), (23, 30), (24, 25), (24, 30), (25,
     26), (25, 30), (26, 27), (26, 29), (26, 30), (27, 28), (27, 29), (28, 29), (29,
     30)])
t5c30_21p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 3), (2, 6), (2, 7),
    (2, 8), (3, 4), (3, 8), (3, 9), (3, 10), (4, 5), (4, 10), (4, 11), (4, 12), (5,
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    (25, 26), (25, 30), (26, 27), (26, 30), (27, 28), (27, 30), (28, 29), (28, 30),
    (29, 30)])
t5c30_22p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 3), (2, 6), (2, 7),
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    (29, 30)])
t5c30_23p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (1, 7), (2, 3), (2, 7),
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     30)])
t5c30_24p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (1, 7), (2, 3), (2, 7),
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     26), (25, 27), (25, 30), (26, 27), (27, 28), (27, 29), (27, 30), (28, 29), (29,
     30)])
t5c30_25p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 3), (2, 6), (2, 7),
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t5c30_26p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 3), (2, 6), (2, 7),
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    (29, 30)])
t5c30_27p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 3), (2, 6), (2, 7),
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t5c30_28p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 3), (2, 6), (2, 7),
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     26), (25, 30), (26, 27), (26, 30), (27, 28), (27, 29), (27, 30), (28, 29), (29,
     30)])
t5c30_29p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 3), (2, 6), (2, 7),
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    (25, 26), (25, 28), (25, 30), (26, 27), (26, 28), (27, 28), (28, 29), (28, 30),
    (29, 30)])
t5c30_30p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (1, 7), (2, 3), (2, 7),
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    (25, 26), (25, 30), (26, 27), (26, 30), (27, 28), (27, 29), (27, 30), (28, 29),
    (29, 30)])
t5c30_31p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 3), (2, 6), (2, 7),
    (2, 8), (2, 9), (3, 4), (3, 9), (3, 10), (4, 5), (4, 10), (4, 11), (4, 12), (5,
    6), (5, 12), (5, 13), (5, 14), (6, 7), (6, 14), (6, 15), (7, 8), (7, 15), (7,
    16), (7, 17), (8, 9), (8, 17), (8, 18), (9, 10), (9, 18), (9, 19), (10, 11),
    (10, 19), (10, 20), (11, 12), (11, 20), (11, 21), (11, 22), (12, 13), (12, 22),
    (13, 14), (13, 22), (13, 23), (13, 24), (14, 15), (14, 24), (14, 25), (15, 16),
    (15, 25), (16, 17), (16, 25), (16, 26), (16, 27), (17, 18), (17, 27), (18, 19),
    (18, 27), (18, 28), (19, 20), (19, 28), (19, 29), (20, 21), (20, 29), (21, 22),
    (21, 23), (21, 29), (21, 30), (22, 23), (23, 24), (23, 26), (23, 30), (24, 25),
    (24, 26), (25, 26), (26, 27), (26, 30), (27, 28), (27, 30), (28, 29), (28, 30),
    (29, 30)])
t5c30_32p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 3), (2, 6), (2, 7),
    (2, 8), (2, 9), (3, 4), (3, 9), (3, 10), (3, 11), (4, 5), (4, 11), (4, 12), (4,
    13), (4, 14), (5, 6), (5, 14), (5, 15), (6, 7), (6, 15), (6, 16), (7, 8), (7,
    16), (7, 17), (8, 9), (8, 17), (8, 18), (8, 19), (9, 10), (9, 19), (10, 11),
    (10, 19), (10, 20), (11, 12), (11, 20), (11, 21), (12, 13), (12, 21), (12, 22),
    (13, 14), (13, 22), (13, 23), (14, 15), (14, 23), (15, 16), (15, 23), (15, 24),
    (16, 17), (16, 24), (16, 25), (17, 18), (17, 25), (18, 19), (18, 25), (18, 26),
    (18, 27), (19, 20), (19, 27), (19, 28), (20, 21), (20, 28), (21, 22), (21, 28),
    (21, 29), (22, 23), (22, 29), (22, 30), (23, 24), (23, 30), (24, 25), (24, 30),
    (25, 26), (25, 30), (26, 27), (26, 29), (26, 30), (27, 28), (27, 29), (28, 29),
    (29, 30)])
t5c30_33p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 3), (2, 6), (2, 7),
    (2, 8), (2, 9), (3, 4), (3, 9), (3, 10), (3, 11), (4, 5), (4, 11), (4, 12), (4,
    13), (5, 6), (5, 13), (5, 14), (6, 7), (6, 14), (6, 15), (7, 8), (7, 15), (7,
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    16), (8, 9), (8, 16), (8, 17), (8, 18), (9, 10), (9, 18), (9, 19), (10, 11),
    (10, 19), (10, 20), (10, 21), (11, 12), (11, 21), (12, 13), (12, 21), (12, 22),
    (12, 23), (12, 24), (13, 14), (13, 24), (14, 15), (14, 24), (14, 25), (15, 16),
    (15, 25), (15, 26), (16, 17), (16, 26), (17, 18), (17, 26), (17, 27), (17, 28),
    (18, 19), (18, 28), (19, 20), (19, 28), (20, 21), (20, 28), (20, 29), (21, 22),
    (21, 29), (22, 23), (22, 29), (22, 30), (23, 24), (23, 25), (23, 30), (24, 25),
    (25, 26), (25, 30), (26, 27), (26, 30), (27, 28), (27, 29), (27, 30), (28, 29),
    (29, 30)])
t5c30_34p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 3), (2, 6), (2, 7),
    (2, 8), (2, 9), (3, 4), (3, 9), (3, 10), (4, 5), (4, 10), (4, 11), (4, 12), (5,
    6), (5, 12), (5, 13), (6, 7), (6, 13), (6, 14), (7, 8), (7, 14), (7, 15), (7,
    16), (8, 9), (8, 16), (8, 17), (8, 18), (9, 10), (9, 18), (9, 19), (10, 11),
    (10, 19), (10, 20), (11, 12), (11, 20), (11, 21), (11, 22), (12, 13), (12, 22),
    (13, 14), (13, 22), (13, 23), (14, 15), (14, 23), (14, 24), (15, 16), (15, 24),
    (15, 25), (16, 17), (16, 25), (17, 18), (17, 25), (17, 26), (17, 27), (18, 19),
    (18, 27), (19, 20), (19, 27), (20, 21), (20, 27), (20, 28), (21, 22), (21, 28),
    (21, 29), (22, 23), (22, 29), (23, 24), (23, 29), (23, 30), (24, 25), (24, 30),
    (25, 26), (25, 30), (26, 27), (26, 28), (26, 30), (27, 28), (28, 29), (28, 30),
    (29, 30)])
t5c30_35p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (1, 7), (2, 3), (2, 7),
    (2, 8), (2, 9), (2, 10), (3, 4), (3, 10), (3, 11), (4, 5), (4, 11), (4, 12), (4,
     13), (5, 6), (5, 13), (5, 14), (6, 7), (6, 14), (6, 15), (6, 16), (6, 17), (7,
    8), (7, 17), (8, 9), (8, 17), (8, 18), (8, 19), (9, 10), (9, 19), (9, 20), (9,
    21), (10, 11), (10, 21), (11, 12), (11, 21), (11, 22), (12, 13), (12, 22), (12,
    23), (13, 14), (13, 23), (13, 24), (14, 15), (14, 24), (14, 25), (15, 16), (15,
    25), (15, 26), (16, 17), (16, 18), (16, 26), (17, 18), (18, 19), (18, 26), (18,
    27), (19, 20), (19, 27), (19, 28), (20, 21), (20, 22), (20, 28), (21, 22), (22,
    23), (22, 28), (22, 29), (23, 24), (23, 29), (23, 30), (24, 25), (24, 30), (25,
    26), (25, 30), (26, 27), (26, 30), (27, 28), (27, 29), (27, 30), (28, 29), (29,
    30)])
t5c30_36p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (1, 7), (2, 3), (2, 7),
    (2, 8), (2, 9), (2, 10), (3, 4), (3, 10), (3, 11), (3, 12), (4, 5), (4, 12), (4,
     13), (4, 14), (5, 6), (5, 14), (5, 15), (6, 7), (6, 15), (6, 16), (7, 8), (7,
    16), (8, 9), (8, 16), (8, 17), (8, 18), (9, 10), (9, 18), (9, 19), (10, 11),
    (10, 19), (11, 12), (11, 19), (11, 20), (11, 21), (12, 13), (12, 21), (13, 14),
    (13, 21), (13, 22), (14, 15), (14, 22), (14, 23), (15, 16), (15, 23), (15, 24),
    (16, 17), (16, 24), (17, 18), (17, 24), (17, 25), (17, 26), (18, 19), (18, 26),
    (18, 27), (19, 20), (19, 27), (20, 21), (20, 27), (20, 28), (20, 29), (21, 22),
    (21, 29), (22, 23), (22, 29), (22, 30), (23, 24), (23, 25), (23, 30), (24, 25),
    (25, 26), (25, 30), (26, 27), (26, 28), (26, 30), (27, 28), (28, 29), (28, 30),
    (29, 30)])
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t5c30_37p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (1, 7), (2, 3), (2, 7),
    (2, 8), (2, 9), (2, 10), (3, 4), (3, 10), (3, 11), (4, 5), (4, 11), (4, 12), (5,
     6), (5, 12), (5, 13), (5, 14), (6, 7), (6, 14), (6, 15), (7, 8), (7, 15), (7,
    16), (8, 9), (8, 16), (8, 17), (8, 18), (9, 10), (9, 18), (9, 19), (9, 20), (10,
     11), (10, 20), (11, 12), (11, 20), (11, 21), (12, 13), (12, 21), (12, 22), (13,
     14), (13, 22), (13, 23), (14, 15), (14, 23), (14, 24), (15, 16), (15, 24), (15,
     25), (16, 17), (16, 25), (17, 18), (17, 25), (17, 26), (17, 27), (18, 19), (18,
     27), (19, 20), (19, 27), (19, 28), (20, 21), (20, 28), (21, 22), (21, 28), (22,
     23), (22, 28), (22, 29), (23, 24), (23, 29), (23, 30), (24, 25), (24, 30), (25,
     26), (25, 30), (26, 27), (26, 29), (26, 30), (27, 28), (27, 29), (28, 29), (29,
     30)])
t5c30_38p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (1, 7), (2, 3), (2, 7),
    (2, 8), (2, 9), (2, 10), (3, 4), (3, 10), (3, 11), (3, 12), (4, 5), (4, 12), (4,
     13), (5, 6), (5, 13), (5, 14), (5, 15), (6, 7), (6, 15), (6, 16), (7, 8), (7,
    16), (8, 9), (8, 16), (8, 17), (8, 18), (9, 10), (9, 18), (9, 19), (9, 20), (10,
     11), (10, 20), (11, 12), (11, 20), (11, 21), (11, 22), (12, 13), (12, 22), (13,
     14), (13, 22), (13, 23), (14, 15), (14, 23), (14, 24), (15, 16), (15, 24), (15,
     25), (16, 17), (16, 25), (17, 18), (17, 25), (17, 26), (17, 27), (18, 19), (18,
     27), (19, 20), (19, 27), (19, 28), (20, 21), (20, 28), (21, 22), (21, 28), (21,
     29), (22, 23), (22, 29), (23, 24), (23, 29), (23, 30), (24, 25), (24, 26), (24,
     30), (25, 26), (26, 27), (26, 28), (26, 30), (27, 28), (28, 29), (28, 30), (29,
     30)])
t5c30_39p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 3), (2, 6), (2, 7),
    (2, 8), (2, 9), (2, 10), (3, 4), (3, 10), (3, 11), (4, 5), (4, 11), (4, 12), (4,
     13), (5, 6), (5, 13), (5, 14), (5, 15), (6, 7), (6, 15), (7, 8), (7, 15), (7,
    16), (7, 17), (8, 9), (8, 17), (8, 18), (9, 10), (9, 18), (9, 19), (10, 11),
    (10, 19), (10, 20), (11, 12), (11, 20), (11, 21), (12, 13), (12, 21), (12, 22),
    (13, 14), (13, 22), (13, 23), (14, 15), (14, 23), (14, 24), (15, 16), (15, 24),
    (16, 17), (16, 24), (16, 25), (17, 18), (17, 25), (18, 19), (18, 25), (18, 26),
    (19, 20), (19, 26), (19, 27), (20, 21), (20, 27), (21, 22), (21, 27), (21, 28),
    (22, 23), (22, 28), (23, 24), (23, 28), (23, 29), (24, 25), (24, 29), (25, 26),
    (25, 29), (25, 30), (26, 27), (26, 30), (27, 28), (27, 30), (28, 29), (28, 30),
    (29, 30)])
t5c30_40p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (1, 7), (2, 3), (2, 7),
    (2, 8), (2, 9), (2, 10), (3, 4), (3, 10), (3, 11), (3, 12), (4, 5), (4, 12), (4,
     13), (4, 14), (5, 6), (5, 14), (5, 15), (6, 7), (6, 15), (6, 16), (6, 17), (7,
    8), (7, 17), (8, 9), (8, 17), (8, 18), (9, 10), (9, 18), (9, 19), (9, 20), (10,
    11), (10, 20), (11, 12), (11, 20), (11, 21), (11, 22), (12, 13), (12, 22), (13,
    14), (13, 22), (13, 23), (14, 15), (14, 23), (14, 24), (15, 16), (15, 24), (15,
    25), (16, 17), (16, 25), (16, 26), (17, 18), (17, 26), (18, 19), (18, 26), (18,
    27), (19, 20), (19, 27), (19, 28), (20, 21), (20, 28), (21, 22), (21, 28), (21,
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    26), (25, 30), (26, 27), (26, 30), (27, 28), (27, 29), (27, 30), (28, 29), (29,
    30)])
t5c30_41p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (1, 7), (2, 3), (2, 7),
    (2, 8), (2, 9), (2, 10), (3, 4), (3, 10), (3, 11), (4, 5), (4, 11), (4, 12), (4,
     13), (5, 6), (5, 13), (5, 14), (6, 7), (6, 14), (6, 15), (6, 16), (7, 8), (7,
    16), (8, 9), (8, 16), (8, 17), (8, 18), (9, 10), (9, 18), (9, 19), (9, 20), (10,
     11), (10, 20), (10, 21), (11, 12), (11, 21), (12, 13), (12, 21), (12, 22), (12,
     23), (13, 14), (13, 23), (14, 15), (14, 23), (14, 24), (15, 16), (15, 24), (15,
     25), (16, 17), (16, 25), (17, 18), (17, 25), (17, 26), (18, 19), (18, 26), (19,
     20), (19, 26), (19, 27), (19, 28), (20, 21), (20, 28), (21, 22), (21, 28), (22,
     23), (22, 28), (22, 29), (23, 24), (23, 29), (24, 25), (24, 29), (24, 30), (25,
     26), (25, 30), (26, 27), (26, 30), (27, 28), (27, 29), (27, 30), (28, 29), (29,
     30)])
t5c30_42p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (1, 7), (2, 3), (2, 7),
    (2, 8), (2, 9), (2, 10), (3, 4), (3, 10), (3, 11), (4, 5), (4, 11), (4, 12), (4,
     13), (5, 6), (5, 13), (5, 14), (6, 7), (6, 14), (6, 15), (6, 16), (7, 8), (7,
    16), (8, 9), (8, 16), (8, 17), (8, 18), (9, 10), (9, 18), (9, 19), (9, 20), (10,
     11), (10, 20), (10, 21), (11, 12), (11, 21), (12, 13), (12, 21), (12, 22), (12,
     23), (13, 14), (13, 23), (13, 24), (14, 15), (14, 24), (15, 16), (15, 24), (15,
     25), (16, 17), (16, 25), (17, 18), (17, 25), (17, 26), (18, 19), (18, 26), (18,
     27), (19, 20), (19, 27), (19, 28), (20, 21), (20, 28), (21, 22), (21, 28), (22,
     23), (22, 28), (22, 29), (23, 24), (23, 29), (23, 30), (24, 25), (24, 30), (25,
     26), (25, 30), (26, 27), (26, 30), (27, 28), (27, 29), (27, 30), (28, 29), (29,
     30)])
t5c30_43p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (1, 7), (2, 3), (2, 7),
    (2, 8), (2, 9), (3, 4), (3, 9), (3, 10), (3, 11), (4, 5), (4, 11), (4, 12), (4,
    13), (5, 6), (5, 13), (5, 14), (5, 15), (6, 7), (6, 15), (6, 16), (6, 17), (7,
    8), (7, 17), (8, 9), (8, 17), (8, 18), (8, 19), (9, 10), (9, 19), (9, 20), (10,
    11), (10, 20), (10, 21), (11, 12), (11, 21), (12, 13), (12, 21), (12, 22), (12,
    23), (13, 14), (13, 23), (14, 15), (14, 23), (14, 24), (15, 16), (15, 24), (15,
    25), (16, 17), (16, 18), (16, 25), (16, 26), (17, 18), (18, 19), (18, 26), (19,
    20), (19, 26), (19, 27), (20, 21), (20, 27), (20, 28), (21, 22), (21, 28), (22,
    23), (22, 28), (22, 29), (23, 24), (23, 29), (24, 25), (24, 29), (24, 30), (25,
    26), (25, 30), (26, 27), (26, 30), (27, 28), (27, 30), (28, 29), (28, 30), (29,
    30)])
t5c30_44p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 3), (2, 6), (2, 7),
    (2, 8), (2, 9), (3, 4), (3, 9), (3, 10), (3, 11), (4, 5), (4, 11), (4, 12), (4,
    13), (5, 6), (5, 13), (5, 14), (6, 7), (6, 14), (6, 15), (7, 8), (7, 15), (7,
    16), (7, 17), (8, 9), (8, 17), (8, 18), (8, 19), (9, 10), (9, 19), (10, 11),
    (10, 19), (10, 20), (11, 12), (11, 20), (11, 21), (12, 13), (12, 21), (12, 22),
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    (16, 17), (16, 24), (16, 25), (17, 18), (17, 25), (18, 19), (18, 25), (18, 26),
    (18, 27), (19, 20), (19, 27), (20, 21), (20, 27), (20, 28), (21, 22), (21, 28),
    (22, 23), (22, 28), (22, 29), (23, 24), (23, 29), (23, 30), (24, 25), (24, 30),
    (25, 26), (25, 30), (26, 27), (26, 28), (26, 29), (26, 30), (27, 28), (28, 29),
    (29, 30)])
t5c30_45p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 3), (2, 6), (2, 7),
    (2, 8), (3, 4), (3, 8), (3, 9), (3, 10), (4, 5), (4, 10), (4, 11), (4, 12), (5,
    6), (5, 12), (5, 13), (6, 7), (6, 13), (6, 14), (7, 8), (7, 14), (7, 15), (7,
    16), (7, 17), (8, 9), (8, 17), (8, 18), (9, 10), (9, 18), (9, 19), (10, 11),
    (10, 19), (10, 20), (11, 12), (11, 20), (11, 21), (12, 13), (12, 21), (12, 22),
    (13, 14), (13, 22), (13, 23), (14, 15), (14, 23), (15, 16), (15, 23), (15, 24),
    (16, 17), (16, 24), (16, 25), (17, 18), (17, 25), (18, 19), (18, 25), (18, 26),
    (19, 20), (19, 26), (19, 27), (20, 21), (20, 27), (21, 22), (21, 27), (21, 28),
    (22, 23), (22, 28), (22, 29), (23, 24), (23, 29), (24, 25), (24, 29), (24, 30),
    (25, 26), (25, 30), (26, 27), (26, 30), (27, 28), (27, 30), (28, 29), (28, 30),
    (29, 30)])
t5c30_46p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 3), (2, 6), (2, 7),
    (2, 8), (3, 4), (3, 8), (3, 9), (3, 10), (4, 5), (4, 10), (4, 11), (4, 12), (5,
    6), (5, 12), (5, 13), (6, 7), (6, 13), (6, 14), (7, 8), (7, 14), (7, 15), (7,
    16), (7, 17), (8, 9), (8, 17), (8, 18), (9, 10), (9, 18), (9, 19), (10, 11),
    (10, 19), (10, 20), (11, 12), (11, 20), (11, 21), (12, 13), (12, 21), (12, 22),
    (13, 14), (13, 22), (13, 23), (14, 15), (14, 23), (15, 16), (15, 23), (15, 24),
    (16, 17), (16, 24), (16, 25), (17, 18), (17, 25), (18, 19), (18, 25), (18, 26),
    (19, 20), (19, 26), (19, 27), (20, 21), (20, 27), (20, 28), (21, 22), (21, 28),
    (22, 23), (22, 28), (22, 29), (23, 24), (23, 29), (24, 25), (24, 29), (24, 30),
    (25, 26), (25, 30), (26, 27), (26, 30), (27, 28), (27, 30), (28, 29), (28, 30),
    (29, 30)])
t5c30_47p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 3), (2, 6), (2, 7),
    (2, 8), (2, 9), (3, 4), (3, 9), (3, 10), (3, 11), (4, 5), (4, 11), (4, 12), (4,
    13), (5, 6), (5, 13), (5, 14), (6, 7), (6, 14), (6, 15), (7, 8), (7, 15), (7,
    16), (7, 17), (8, 9), (8, 17), (8, 18), (8, 19), (9, 10), (9, 19), (10, 11),
    (10, 19), (10, 20), (11, 12), (11, 20), (11, 21), (12, 13), (12, 21), (12, 22),
    (13, 14), (13, 22), (13, 23), (14, 15), (14, 23), (14, 24), (15, 16), (15, 24),
    (16, 17), (16, 24), (16, 25), (17, 18), (17, 25), (18, 19), (18, 25), (18, 26),
    (18, 27), (19, 20), (19, 27), (20, 21), (20, 27), (20, 28), (21, 22), (21, 28),
    (21, 29), (22, 23), (22, 29), (23, 24), (23, 29), (23, 30), (24, 25), (24, 30),
    (25, 26), (25, 30), (26, 27), (26, 28), (26, 29), (26, 30), (27, 28), (28, 29),
    (29, 30)])
t5c30_48p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 3), (2, 6), (2, 7),
    (2, 8), (2, 9), (3, 4), (3, 9), (3, 10), (3, 11), (4, 5), (4, 11), (4, 12), (5,
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    6), (5, 12), (5, 13), (5, 14), (5, 15), (6, 7), (6, 15), (6, 16), (7, 8), (7,
    16), (7, 17), (8, 9), (8, 17), (8, 18), (8, 19), (9, 10), (9, 19), (9, 20), (10,
     11), (10, 20), (10, 21), (11, 12), (11, 21), (11, 22), (12, 13), (12, 22), (12,
     23), (13, 14), (13, 23), (13, 24), (14, 15), (14, 24), (14, 25), (15, 16), (15,
     25), (16, 17), (16, 25), (17, 18), (17, 25), (17, 26), (18, 19), (18, 26), (18,
     27), (19, 20), (19, 27), (19, 28), (20, 21), (20, 28), (21, 22), (21, 28), (21,
     29), (22, 23), (22, 29), (23, 24), (23, 29), (23, 30), (24, 25), (24, 26), (24,
     30), (25, 26), (26, 27), (26, 30), (27, 28), (27, 29), (27, 30), (28, 29), (29,
     30)])
t5c30_49p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 3), (2, 6), (2, 7),
    (2, 8), (2, 9), (3, 4), (3, 9), (3, 10), (3, 11), (4, 5), (4, 11), (4, 12), (4,
    13), (5, 6), (5, 13), (5, 14), (6, 7), (6, 14), (6, 15), (7, 8), (7, 15), (7,
    16), (7, 17), (8, 9), (8, 17), (8, 18), (8, 19), (9, 10), (9, 19), (10, 11),
    (10, 19), (10, 20), (10, 21), (11, 12), (11, 21), (12, 13), (12, 21), (12, 22),
    (12, 23), (13, 14), (13, 23), (14, 15), (14, 23), (14, 24), (15, 16), (15, 24),
    (16, 17), (16, 24), (16, 25), (16, 26), (17, 18), (17, 26), (18, 19), (18, 26),
    (18, 27), (19, 20), (19, 27), (20, 21), (20, 27), (20, 28), (20, 29), (21, 22),
    (21, 29), (22, 23), (22, 29), (22, 30), (23, 24), (23, 30), (24, 25), (24, 30),
    (25, 26), (25, 28), (25, 30), (26, 27), (26, 28), (27, 28), (28, 29), (28, 30),
    (29, 30)])
t5c30_50p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (1, 7), (2, 3), (2, 7),
    (2, 8), (2, 9), (2, 10), (3, 4), (3, 10), (3, 11), (3, 12), (4, 5), (4, 12), (4,
     13), (5, 6), (5, 13), (5, 14), (5, 15), (6, 7), (6, 15), (6, 16), (7, 8), (7,
    16), (7, 17), (8, 9), (8, 17), (8, 18), (9, 10), (9, 18), (9, 19), (10, 11),
    (10, 19), (10, 20), (11, 12), (11, 20), (11, 21), (11, 22), (12, 13), (12, 22),
    (13, 14), (13, 22), (13, 23), (14, 15), (14, 23), (14, 24), (15, 16), (15, 24),
    (15, 25), (16, 17), (16, 25), (17, 18), (17, 25), (17, 26), (18, 19), (18, 26),
    (18, 27), (19, 20), (19, 27), (19, 28), (20, 21), (20, 28), (21, 22), (21, 28),
    (21, 29), (22, 23), (22, 29), (23, 24), (23, 29), (24, 25), (24, 29), (24, 30),
    (25, 26), (25, 30), (26, 27), (26, 30), (27, 28), (27, 30), (28, 29), (28, 30),
    (29, 30)])
t5c30_51p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (1, 7), (2, 3), (2, 7),
    (2, 8), (2, 9), (2, 10), (3, 4), (3, 10), (3, 11), (4, 5), (4, 11), (4, 12), (4,
     13), (5, 6), (5, 13), (5, 14), (6, 7), (6, 14), (6, 15), (6, 16), (7, 8), (7,
    16), (7, 17), (8, 9), (8, 17), (8, 18), (9, 10), (9, 18), (9, 19), (10, 11),
    (10, 19), (11, 12), (11, 19), (11, 20), (12, 13), (12, 20), (12, 21), (13, 14),
    (13, 21), (13, 22), (14, 15), (14, 22), (14, 23), (15, 16), (15, 23), (15, 24),
    (15, 25), (16, 17), (16, 25), (17, 18), (17, 25), (17, 26), (18, 19), (18, 26),
    (18, 27), (19, 20), (19, 27), (20, 21), (20, 27), (20, 28), (21, 22), (21, 28),
    (21, 29), (22, 23), (22, 29), (23, 24), (23, 29), (24, 25), (24, 29), (24, 30),
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    (25, 26), (25, 30), (26, 27), (26, 30), (27, 28), (27, 30), (28, 29), (28, 30),
    (29, 30)])
t5c30_52p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (1, 7), (2, 3), (2, 7),
    (2, 8), (2, 9), (2, 10), (3, 4), (3, 10), (3, 11), (4, 5), (4, 11), (4, 12), (4,
     13), (5, 6), (5, 13), (5, 14), (6, 7), (6, 14), (6, 15), (6, 16), (7, 8), (7,
    16), (7, 17), (8, 9), (8, 17), (8, 18), (9, 10), (9, 18), (9, 19), (9, 20), (10,
     11), (10, 20), (11, 12), (11, 20), (11, 21), (11, 22), (12, 13), (12, 22), (12,
     23), (13, 14), (13, 23), (14, 15), (14, 23), (14, 24), (15, 16), (15, 24), (15,
     25), (16, 17), (16, 25), (16, 26), (17, 18), (17, 26), (18, 19), (18, 26), (18,
     27), (19, 20), (19, 27), (19, 28), (20, 21), (20, 28), (21, 22), (21, 28), (21,
     29), (22, 23), (22, 29), (23, 24), (23, 29), (24, 25), (24, 29), (24, 30), (25,
     26), (25, 30), (26, 27), (26, 30), (27, 28), (27, 30), (28, 29), (28, 30), (29,
     30)])
t5c30_53p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 3), (2, 6), (2, 7),
    (2, 8), (2, 9), (3, 4), (3, 9), (3, 10), (4, 5), (4, 10), (4, 11), (5, 6), (5,
    11), (5, 12), (5, 13), (6, 7), (6, 13), (6, 14), (7, 8), (7, 14), (7, 15), (7,
    16), (8, 9), (8, 16), (8, 17), (9, 10), (9, 17), (9, 18), (10, 11), (10, 18),
    (10, 19), (11, 12), (11, 19), (11, 20), (12, 13), (12, 20), (12, 21), (12, 22),
    (13, 14), (13, 22), (14, 15), (14, 22), (14, 23), (15, 16), (15, 23), (15, 24),
    (16, 17), (16, 24), (16, 25), (17, 18), (17, 25), (17, 26), (18, 19), (18, 26),
    (18, 27), (19, 20), (19, 27), (20, 21), (20, 27), (20, 28), (21, 22), (21, 28),
    (21, 29), (22, 23), (22, 29), (23, 24), (23, 29), (24, 25), (24, 29), (24, 30),
    (25, 26), (25, 30), (26, 27), (26, 28), (26, 30), (27, 28), (28, 29), (28, 30),
    (29, 30)])
t5c30_54p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (1, 7), (2, 3), (2, 7),
    (2, 8), (2, 9), (2, 10), (3, 4), (3, 10), (3, 11), (3, 12), (4, 5), (4, 12), (4,
     13), (4, 14), (5, 6), (5, 14), (5, 15), (6, 7), (6, 15), (6, 16), (7, 8), (7,
    16), (7, 17), (8, 9), (8, 17), (8, 18), (8, 19), (9, 10), (9, 19), (9, 20), (10,
     11), (10, 20), (11, 12), (11, 20), (11, 21), (11, 22), (12, 13), (12, 22), (13,
     14), (13, 22), (13, 23), (14, 15), (14, 23), (14, 24), (15, 16), (15, 24), (15,
     25), (16, 17), (16, 25), (16, 26), (17, 18), (17, 26), (18, 19), (18, 26), (18,
     27), (18, 28), (19, 20), (19, 28), (20, 21), (20, 28), (21, 22), (21, 28), (21,
     29), (22, 23), (22, 29), (23, 24), (23, 29), (23, 30), (24, 25), (24, 30), (25,
     26), (25, 30), (26, 27), (26, 30), (27, 28), (27, 29), (27, 30), (28, 29), (29,
     30)])
t5c30_55p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (1, 7), (2, 3), (2, 7),
    (2, 8), (2, 9), (2, 10), (3, 4), (3, 10), (3, 11), (3, 12), (4, 5), (4, 12), (4,
     13), (5, 6), (5, 13), (5, 14), (6, 7), (6, 14), (6, 15), (7, 8), (7, 15), (7,
    16), (8, 9), (8, 16), (8, 17), (9, 10), (9, 17), (9, 18), (10, 11), (10, 18),
    (10, 19), (11, 12), (11, 19), (11, 20), (12, 13), (12, 20), (12, 21), (13, 14),
    (13, 21), (13, 22), (14, 15), (14, 22), (14, 23), (15, 16), (15, 23), (15, 24),
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    (16, 17), (16, 24), (16, 25), (17, 18), (17, 25), (17, 26), (18, 19), (18, 26),
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    (22, 23), (22, 28), (22, 29), (23, 24), (23, 29), (24, 25), (24, 29), (25, 26),
    (25, 29), (25, 30), (26, 27), (26, 30), (27, 28), (27, 30), (28, 29), (28, 30),
    (29, 30)])
t5c30_56p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 3), (2, 6), (2, 7),
    (2, 8), (2, 9), (3, 4), (3, 9), (3, 10), (3, 11), (4, 5), (4, 11), (4, 12), (5,
    6), (5, 12), (5, 13), (5, 14), (6, 7), (6, 14), (6, 15), (7, 8), (7, 15), (7,
    16), (7, 17), (8, 9), (8, 17), (8, 18), (9, 10), (9, 18), (9, 19), (10, 11),
    (10, 19), (10, 20), (11, 12), (11, 20), (11, 21), (12, 13), (12, 21), (12, 22),
    (13, 14), (13, 22), (13, 23), (14, 15), (14, 23), (14, 24), (15, 16), (15, 24),
    (16, 17), (16, 24), (16, 25), (17, 18), (17, 25), (17, 26), (18, 19), (18, 26),
    (18, 27), (19, 20), (19, 27), (20, 21), (20, 27), (20, 28), (21, 22), (21, 28),
    (22, 23), (22, 28), (22, 29), (23, 24), (23, 29), (24, 25), (24, 29), (25, 26),
    (25, 29), (25, 30), (26, 27), (26, 30), (27, 28), (27, 30), (28, 29), (28, 30),
    (29, 30)])
t5c30_57p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (1, 7), (2, 3), (2, 7),
    (2, 8), (2, 9), (2, 10), (3, 4), (3, 10), (3, 11), (3, 12), (4, 5), (4, 12), (4,
     13), (5, 6), (5, 13), (5, 14), (6, 7), (6, 14), (6, 15), (6, 16), (7, 8), (7,
    16), (8, 9), (8, 16), (8, 17), (9, 10), (9, 17), (9, 18), (9, 19), (10, 11),
    (10, 19), (10, 20), (11, 12), (11, 20), (11, 21), (12, 13), (12, 21), (12, 22),
    (13, 14), (13, 22), (14, 15), (14, 22), (14, 23), (15, 16), (15, 23), (15, 24),
    (15, 25), (16, 17), (16, 25), (17, 18), (17, 25), (17, 26), (18, 19), (18, 26),
    (18, 27), (19, 20), (19, 27), (19, 28), (20, 21), (20, 28), (21, 22), (21, 28),
    (21, 29), (22, 23), (22, 29), (23, 24), (23, 29), (24, 25), (24, 29), (24, 30),
    (25, 26), (25, 30), (26, 27), (26, 30), (27, 28), (27, 30), (28, 29), (28, 30),
    (29, 30)])
t5c30_58p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 3), (2, 6), (2, 7),
    (2, 8), (3, 4), (3, 8), (3, 9), (3, 10), (4, 5), (4, 10), (4, 11), (4, 12), (5,
    6), (5, 12), (5, 13), (5, 14), (6, 7), (6, 14), (6, 15), (7, 8), (7, 15), (7,
    16), (7, 17), (8, 9), (8, 17), (8, 18), (9, 10), (9, 18), (9, 19), (9, 20), (10,
     11), (10, 20), (11, 12), (11, 20), (11, 21), (12, 13), (12, 21), (12, 22), (13,
     14), (13, 22), (13, 23), (13, 24), (14, 15), (14, 24), (15, 16), (15, 24), (15,
     25), (16, 17), (16, 25), (16, 26), (17, 18), (17, 26), (18, 19), (18, 26), (18,
     27), (19, 20), (19, 27), (19, 28), (20, 21), (20, 28), (21, 22), (21, 28), (21,
     29), (22, 23), (22, 29), (23, 24), (23, 25), (23, 29), (23, 30), (24, 25), (25,
     26), (25, 30), (26, 27), (26, 30), (27, 28), (27, 30), (28, 29), (28, 30), (29,
     30)])
t5c30_59p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (1, 7), (2, 3), (2, 7),
    (2, 8), (2, 9), (2, 10), (3, 4), (3, 10), (3, 11), (3, 12), (4, 5), (4, 12), (4,
     13), (5, 6), (5, 13), (5, 14), (6, 7), (6, 14), (6, 15), (7, 8), (7, 15), (7,
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    16), (8, 9), (8, 16), (8, 17), (9, 10), (9, 17), (9, 18), (10, 11), (10, 18),
    (10, 19), (11, 12), (11, 19), (11, 20), (12, 13), (12, 20), (12, 21), (13, 14),
    (13, 21), (13, 22), (14, 15), (14, 22), (14, 23), (15, 16), (15, 23), (15, 24),
    (16, 17), (16, 24), (17, 18), (17, 24), (17, 25), (17, 26), (18, 19), (18, 26),
    (18, 27), (19, 20), (19, 27), (20, 21), (20, 27), (20, 28), (21, 22), (21, 28),
    (22, 23), (22, 28), (22, 29), (23, 24), (23, 29), (24, 25), (24, 29), (25, 26),
    (25, 29), (25, 30), (26, 27), (26, 30), (27, 28), (27, 30), (28, 29), (28, 30),
    (29, 30)])
t5c31_1p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 3), (2, 6), (2, 7),
    (2, 8), (2, 9), (3, 4), (3, 9), (3, 10), (4, 5), (4, 10), (4, 11), (4, 12), (5,
    6), (5, 12), (5, 13), (5, 14), (6, 7), (6, 14), (6, 15), (7, 8), (7, 15), (7,
    16), (7, 17), (8, 9), (8, 17), (8, 18), (8, 19), (9, 10), (9, 19), (9, 20), (10,
     11), (10, 20), (10, 21), (11, 12), (11, 21), (11, 22), (12, 13), (12, 22), (12,
     23), (13, 14), (13, 23), (13, 24), (13, 25), (14, 15), (14, 25), (15, 16), (15,
     25), (16, 17), (16, 25), (16, 26), (16, 27), (17, 18), (17, 27), (18, 19), (18,
     27), (18, 28), (18, 29), (19, 20), (19, 29), (20, 21), (20, 29), (21, 22), (21,
     29), (21, 30), (22, 23), (22, 30), (23, 24), (23, 30), (23, 31), (24, 25), (24,
     26), (24, 31), (25, 26), (26, 27), (26, 28), (26, 31), (27, 28), (28, 29), (28,
     30), (28, 31), (29, 30), (30, 31)])
t5c31_2p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (1, 7), (2, 3), (2, 7),
    (2, 8), (2, 9), (2, 10), (3, 4), (3, 10), (3, 11), (4, 5), (4, 11), (4, 12), (4,
     13), (5, 6), (5, 13), (5, 14), (6, 7), (6, 14), (6, 15), (6, 16), (7, 8), (7,
    16), (8, 9), (8, 16), (8, 17), (8, 18), (9, 10), (9, 18), (9, 19), (9, 20), (10,
     11), (10, 20), (11, 12), (11, 20), (11, 21), (12, 13), (12, 21), (12, 22), (13,
     14), (13, 22), (13, 23), (14, 15), (14, 23), (14, 24), (15, 16), (15, 17), (15,
     24), (15, 25), (16, 17), (17, 18), (17, 25), (17, 26), (18, 19), (18, 26), (18,
     27), (19, 20), (19, 27), (19, 28), (20, 21), (20, 28), (21, 22), (21, 28), (21,
     29), (22, 23), (22, 29), (22, 30), (23, 24), (23, 30), (24, 25), (24, 30), (24,
     31), (25, 26), (25, 31), (26, 27), (26, 31), (27, 28), (27, 29), (27, 31), (28,
     29), (29, 30), (29, 31), (30, 31)])
t5c32p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (1, 7), (2, 3), (2, 7), (2,
    8), (2, 9), (3, 4), (3, 9), (3, 10), (3, 11), (4, 5), (4, 11), (4, 12), (5, 6),
    (5, 12), (5, 13), (5, 14), (6, 7), (6, 14), (6, 15), (7, 8), (7, 15), (7, 16),
    (8, 9), (8, 16), (8, 17), (8, 18), (9, 10), (9, 18), (9, 19), (10, 11), (10, 19)
    , (10, 20), (11, 12), (11, 20), (11, 21), (12, 13), (12, 21), (12, 22), (13, 14)
    , (13, 22), (13, 23), (14, 15), (14, 23), (14, 24), (15, 16), (15, 24), (15, 25)
    , (16, 17), (16, 25), (17, 18), (17, 25), (17, 26), (17, 27), (18, 19), (18, 27)
    , (19, 20), (19, 27), (19, 28), (20, 21), (20, 28), (20, 29), (21, 22), (21, 29)
    , (22, 23), (22, 29), (22, 30), (23, 24), (23, 30), (23, 31), (24, 25), (24, 31)
    , (25, 26), (25, 31), (26, 27), (26, 31), (26, 32), (27, 28), (27, 32), (28, 29)
    , (28, 32), (29, 30), (29, 32), (30, 31), (30, 32), (31, 32)])
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Appendix B

SageMath Quadrangulations Graph Database

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#general quadrangulations which maximize the WI
q4maxwi = Graph([(1, 2), (1, 3), (2, 4), (3, 4)])
q5maxwi = Graph([(1, 2), (1, 3), (1, 4), (2, 5), (3, 5), (4, 5)])
q6maxwi = Graph([(1, 2), (1, 3), (1, 4), (2, 5), (3, 5), (3, 6), (4, 5), (4, 6)])
q7_1maxwi = Graph([(1, 2), (1, 3), (1, 4), (2, 5), (3, 5), (3, 6), (3, 7), (4, 5),
    (4, 6), (4, 7)])
q7_2maxwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 6), (3, 6), (3, 7), (4, 7),
    (5, 6), (5, 7)])
q8maxwi = Graph([(1, 2), (1, 3), (1, 4), (2, 5), (2, 6), (2, 7), (3, 7), (4, 5), (4,
     6), (4, 7), (5, 8), (6, 8)])
q9maxwi = Graph([(1, 2), (1, 3), (1, 4), (2, 5), (2, 6), (2, 7), (3, 7), (4, 5), (4,
     6), (4, 7), (5, 8), (5, 9), (6, 8), (6, 9)])
q10maxwi = Graph([(1, 2), (1, 3), (1, 4), (2, 5), (2, 6), (2, 7), (3, 7), (4, 5),
    (4, 6), (4, 7), (5, 8), (5, 9), (6, 8), (6, 9), (8, 10), (9, 10)])
q11maxwi = Graph([(1, 2), (1, 3), (1, 4), (2, 5), (2, 6), (2, 7), (3, 5), (3, 6),
    (3, 7), (4, 5), (6, 8), (6, 9), (7, 8), (7, 9), (8, 10), (8, 11), (9, 10), (9,
    11)])
q12maxwi = Graph([(1, 2), (1, 3), (1, 4), (2, 5), (2, 6), (2, 7), (3, 5), (3, 6),
    (3, 7), (4, 5), (6, 8), (6, 9), (7, 8), (7, 9), (8, 10), (8, 11), (9, 10), (9,
    11), (10, 12), (11, 12)])
q13maxwi = Graph([(1, 2), (1, 3), (1, 4), (2, 5), (2, 6), (2, 7), (3, 5), (3, 6),
    (3, 7), (4, 5), (6, 8), (6, 9), (7, 8), (7, 9), (8, 10), (8, 11), (9, 10), (9,
    11), (10, 12), (10, 13), (11, 12), (11, 13)])
q14maxwi = Graph([(1, 2), (1, 3), (1, 4), (2, 5), (2, 6), (2, 7), (3, 5), (3, 6),
    (3, 7), (4, 5), (6, 8), (6, 9), (7, 8), (7, 9), (8, 10), (8, 11), (9, 10), (9,
    11), (10, 12), (10, 13), (11, 12), (11, 13), (12, 14), (13, 14)])
q15maxwi = Graph([(1, 2), (1, 3), (1, 4), (2, 5), (2, 6), (2, 7), (3, 5), (3, 6),
    (3, 7), (4, 5), (6, 8), (6, 9), (7, 8), (7, 9), (8, 10), (8, 11), (9, 10), (9,
    11), (10, 12), (10, 13), (11, 12), (11, 13), (12, 14), (12, 15), (13, 14), (13,
    15)])
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```
q16maxwi = Graph([(1, 2), (1, 3), (1, 4), (2, 5), (2, 6), (2, 7), (3, 5), (3, 6),
    (3, 7), (4, 5), (6, 8), (6, 9), (7, 8), (7, 9), (8, 10), (8, 11), (9, 10), (9,
    11), (10, 12), (10, 13), (11, 12), (11, 13), (12, 14), (12, 15), (13, 14), (13,
    15), (14, 16), (15, 16)])
q17maxwi = Graph([(1, 2), (1, 3), (1, 4), (2, 5), (2, 6), (2, 7), (3, 5), (3, 6),
    (3, 7), (4, 5), (6, 8), (6, 9), (7, 8), (7, 9), (8, 10), (8, 11), (9, 10), (9,
    11), (10, 12), (10, 13), (11, 12), (11, 13), (12, 14), (12, 15), (13, 14), (13,
    15), (14, 16), (14, 17), (15, 16), (15, 17)])
q18maxwi = Graph([(1, 2), (1, 3), (1, 4), (2, 5), (2, 6), (2, 7), (3, 5), (3, 6),
    (3, 7), (4, 5), (6, 8), (6, 9), (7, 8), (7, 9), (8, 10), (8, 11), (9, 10), (9,
    11), (10, 12), (10, 13), (11, 12), (11, 13), (12, 14), (12, 15), (13, 14), (13,
    15), (14, 16), (14, 17), (15, 16), (15, 17), (16, 18), (17, 18)])
q19maxwi = Graph([(1, 2), (1, 3), (1, 4), (2, 5), (2, 6), (2, 7), (3, 5), (3, 6),
    (3, 7), (4, 5), (6, 8), (6, 9), (7, 8), (7, 9), (8, 10), (8, 11), (9, 10), (9,
    11), (10, 12), (10, 13), (11, 12), (11, 13), (12, 14), (12, 15), (13, 14), (13,
    15), (14, 16), (14, 17), (15, 16), (15, 17), (16, 18), (16, 19), (17, 18), (17,
    19)])
q20maxwi = Graph([(1, 2), (1, 3), (1, 4), (2, 5), (2, 6), (2, 7), (3, 5), (3, 6),
    (3, 7), (4, 5), (6, 8), (6, 9), (7, 8), (7, 9), (8, 10), (8, 11), (9, 10), (9,
    11), (10, 12), (10, 13), (11, 12), (11, 13), (12, 14), (12, 15), (13, 14), (13,
    15), (14, 16), (14, 17), (15, 16), (15, 17), (16, 18), (16, 19), (17, 18), (17,
    19), (18, 20), (19, 20)])

#general quadrangulations which minimize the WI
q4minwi = Graph([(1, 2), (1, 3), (2, 4), (3, 4)])
q5minwi = Graph([(1, 2), (1, 3), (1, 4), (2, 5), (3, 5), (4, 5)])
q6minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 6), (3, 6), (4, 6), (5, 6)])
q7minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 7), (3, 7), (4, 7), (5,
     7), (6, 7)])
q8minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (1, 7), (2, 8), (3, 8), (4,
     8), (5, 8), (6, 8), (7, 8)])
q9minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (1, 7), (1, 8), (2, 9), (3,
     9), (4, 9), (5, 9), (6, 9), (7, 9), (8, 9)])
q10minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (1, 7), (1, 8), (1, 9),
    (2, 10), (3, 10), (4, 10), (5, 10), (6, 10), (7, 10), (8, 10), (9, 10)])
q11minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (1, 7), (1, 8), (1, 9),
    (1, 10), (2, 11), (3, 11), (4, 11), (5, 11), (6, 11), (7, 11), (8, 11), (9, 11),
     (10, 11)])
q12minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (1, 7), (1, 8), (1, 9),
    (1, 10), (1, 11), (2, 12), (3, 12), (4, 12), (5, 12), (6, 12), (7, 12), (8, 12),
     (9, 12), (10, 12), (11, 12)])
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q13minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (1, 7), (1, 8), (1, 9),
    (1, 10), (1, 11), (1, 12), (2, 13), (3, 13), (4, 13), (5, 13), (6, 13), (7, 13),
     (8, 13), (9, 13), (10, 13), (11, 13), (12, 13)])
q14minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (1, 7), (1, 8), (1, 9),
    (1, 10), (1, 11), (1, 12), (1, 13), (2, 14), (3, 14), (4, 14), (5, 14), (6, 14),
     (7, 14), (8, 14), (9, 14), (10, 14), (11, 14), (12, 14), (13, 14)])
q15minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (1, 7), (1, 8), (1, 9),
    (1, 10), (1, 11), (1, 12), (1, 13), (1, 14), (2, 15), (3, 15), (4, 15), (5, 15),
     (6, 15), (7, 15), (8, 15), (9, 15), (10, 15), (11, 15), (12, 15), (13, 15),
    (14, 15)])
q16minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (1, 7), (1, 8), (1, 9),
    (1, 10), (1, 11), (1, 12), (1, 13), (1, 14), (1, 15), (2, 16), (3, 16), (4, 16),
     (5, 16), (6, 16), (7, 16), (8, 16), (9, 16), (10, 16), (11, 16), (12, 16), (13,
     16), (14, 16), (15, 16)])
q17minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (1, 7), (1, 8), (1, 9),
    (1, 10), (1, 11), (1, 12), (1, 13), (1, 14), (1, 15), (1, 16), (2, 17), (3, 17),
     (4, 17), (5, 17), (6, 17), (7, 17), (8, 17), (9, 17), (10, 17), (11, 17), (12,
    17), (13, 17), (14, 17), (15, 17), (16, 17)])
q18minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (1, 7), (1, 8), (1, 9),
    (1, 10), (1, 11), (1, 12), (1, 13), (1, 14), (1, 15), (1, 16), (1, 17), (2, 18),
     (3, 18), (4, 18), (5, 18), (6, 18), (7, 18), (8, 18), (9, 18), (10, 18), (11,
    18), (12, 18), (13, 18), (14, 18), (15, 18), (16, 18), (17, 18)])
q19minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (1, 7), (1, 8), (1, 9),
    (1, 10), (1, 11), (1, 12), (1, 13), (1, 14), (1, 15), (1, 16), (1, 17), (1, 18),
     (2, 19), (3, 19), (4, 19), (5, 19), (6, 19), (7, 19), (8, 19), (9, 19), (10,
    19), (11, 19), (12, 19), (13, 19), (14, 19), (15, 19), (16, 19), (17, 19), (18,
    19)])
q20minwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (1, 7), (1, 8), (1, 9),
    (1, 10), (1, 11), (1, 12), (1, 13), (1, 14), (1, 15), (1, 16), (1, 17), (1, 18),
     (1, 19), (2, 20), (3, 20), (4, 20), (5, 20), (6, 20), (7, 20), (8, 20), (9, 20)
    , (10, 20), (11, 20), (12, 20), (13, 20), (14, 20), (15, 20), (16, 20), (17, 20)
    , (18, 20), (19, 20)])

#3-connected quadrangulations which maximize the WI
q3c8maxwi = Graph([(1, 2), (1, 3), (1, 4), (2, 5), (2, 6), (3, 6), (3, 7), (4, 5),
    (4, 7), (5, 8), (6, 8), (7, 8)])
q3c10maxwi = Graph([(1, 2), (1, 3), (1, 4), (2, 5), (2, 6), (2, 7), (3, 7), (3, 8),
    (4, 5), (4, 8), (5, 9), (6, 9), (6, 10), (7, 10), (8, 9), (8, 10)])
q3c11maxwi = Graph([(1, 2), (1, 3), (1, 4), (2, 5), (2, 6), (2, 7), (3, 7), (3, 8),
    (4, 5), (4, 8), (4, 9), (5, 10), (6, 10), (6, 11), (7, 11), (8, 11), (9, 10),
    (9, 11)])
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q3c12maxwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 6), (2, 7), (2, 8), (3, 8),
    (3, 9), (4, 7), (4, 9), (4, 10), (5, 6), (5, 10), (6, 11), (7, 11), (7, 12), (8,
     12), (9, 12), (10, 11)])
q3c13maxwi = Graph([(1, 2), (1, 3), (1, 4), (2, 5), (2, 6), (2, 7), (2, 8), (3, 6),
    (3, 8), (3, 9), (3, 10), (4, 5), (4, 10), (5, 11), (6, 11), (6, 12), (7, 12),
    (7, 13), (8, 13), (9, 12), (9, 13), (10, 11)])
q3c14maxwi = Graph([(1, 2), (1, 3), (1, 4), (2, 5), (2, 6), (2, 7), (2, 8), (3, 6),
    (3, 8), (3, 9), (4, 5), (4, 9), (5, 10), (6, 10), (6, 11), (6, 12), (7, 12), (7,
     13), (8, 11), (8, 13), (9, 10), (11, 14), (12, 14), (13, 14)])
q3c15maxwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 6), (2, 7), (2, 8), (3, 8),
    (3, 9), (4, 7), (4, 9), (5, 6), (5, 7), (5, 10), (6, 11), (6, 12), (7, 12), (7,
    13), (7, 14), (8, 14), (9, 14), (10, 11), (10, 13), (11, 15), (12, 15), (13, 15)
    ])
q3c16_1maxwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 6), (2, 7), (3, 7), (3, 8)
    , (3, 9), (3, 10), (4, 7), (4, 10), (4, 11), (5, 6), (5, 11), (6, 12), (7, 12),
    (7, 13), (8, 13), (8, 14), (9, 14), (9, 15), (10, 13), (10, 15), (11, 12), (13,
    16), (14, 16), (15, 16)])
q3c16_2maxwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 6), (2, 7), (2, 8), (3, 8)
    , (3, 9), (4, 7), (4, 9), (4, 10), (5, 6), (5, 10), (5, 11), (6, 12), (6, 13),
    (7, 13), (7, 14), (8, 14), (9, 14), (10, 13), (10, 15), (11, 12), (11, 15), (12,
     16), (13, 16), (15, 16)])
q3c17maxwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 6), (2, 7), (2, 8), (3, 7),
    (3, 8), (3, 9), (4, 7), (4, 10), (5, 6), (5, 10), (6, 11), (7, 11), (7, 12), (8,
     12), (8, 13), (8, 14), (9, 12), (9, 14), (9, 15), (10, 11), (12, 16), (13, 16),
     (13, 17), (14, 17), (15, 16), (15, 17)])
q3c18maxwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 6), (2, 7), (2, 8), (2, 9),
    (3, 7), (3, 9), (3, 10), (4, 6), (4, 7), (4, 11), (5, 6), (5, 11), (5, 12), (6,
    13), (6, 14), (7, 15), (8, 15), (8, 16), (9, 16), (10, 15), (10, 16), (11, 14),
    (11, 17), (12, 13), (12, 17), (13, 18), (14, 18), (17, 18)])
q3c19maxwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 7), (2, 8), (2, 9),
    (3, 9), (3, 10), (4, 8), (4, 10), (5, 7), (5, 8), (5, 11), (6, 7), (6, 11), (6,
    12), (7, 13), (8, 14), (9, 14), (10, 14), (11, 13), (11, 15), (11, 16), (12, 13)
    , (12, 16), (12, 17), (13, 18), (15, 18), (15, 19), (16, 19), (17, 18), (17, 19)
    ])
q3c20maxwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 6), (2, 7), (2, 8), (2, 9),
    (3, 7), (3, 9), (3, 10), (4, 6), (4, 7), (4, 11), (5, 6), (5, 11), (5, 12), (6,
    13), (7, 14), (8, 14), (8, 15), (9, 15), (10, 14), (10, 15), (11, 13), (11, 16),
     (11, 17), (12, 13), (12, 17), (12, 18), (13, 19), (16, 19), (16, 20), (17, 20),
     (18, 19), (18, 20)])
q3c21maxwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 6), (2, 7), (2, 8), (3, 8),
    (3, 9), (3, 10), (4, 9), (4, 10), (4, 11), (5, 6), (5, 8), (5, 9), (5, 12), (6,
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    13), (7, 13), (7, 14), (8, 14), (9, 15), (10, 15), (10, 16), (11, 15), (11, 16),
     (11, 17), (12, 13), (12, 14), (15, 18), (15, 19), (16, 19), (16, 20), (17, 18),
     (17, 20), (18, 21), (19, 21), (20, 21)])
q3c22maxwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 6), (2, 7), (2, 8), (3, 7),
    (3, 8), (3, 9), (4, 6), (4, 7), (4, 10), (5, 6), (5, 10), (5, 11), (6, 12), (7,
    13), (7, 14), (8, 14), (8, 15), (9, 13), (9, 15), (10, 12), (10, 16), (11, 12),
    (11, 16), (11, 17), (12, 18), (12, 19), (13, 20), (14, 20), (15, 20), (16, 19),
    (16, 21), (17, 18), (17, 21), (18, 22), (19, 22), (21, 22)])
q3c23maxwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 6), (2, 7), (2, 8), (3, 8),
    (3, 9), (3, 10), (4, 9), (4, 10), (4, 11), (5, 6), (5, 8), (5, 9), (6, 12), (6,
    13), (7, 13), (7, 14), (8, 12), (8, 14), (9, 15), (10, 15), (10, 16), (11, 15),
    (11, 16), (11, 17), (11, 18), (12, 19), (13, 19), (14, 19), (15, 20), (16, 20),
    (16, 21), (17, 21), (17, 22), (18, 20), (18, 22), (20, 23), (21, 23), (22, 23)])
q3c24maxwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 6), (2, 7), (2, 8), (3, 7),
    (3, 8), (3, 9), (4, 6), (4, 7), (4, 10), (5, 6), (5, 10), (5, 11), (6, 12), (7,
    13), (7, 14), (8, 14), (8, 15), (9, 13), (9, 15), (10, 12), (10, 16), (11, 12),
    (11, 16), (11, 17), (12, 18), (13, 19), (14, 19), (15, 19), (16, 18), (16, 20),
    (16, 21), (17, 18), (17, 21), (17, 22), (18, 23), (20, 23), (20, 24), (21, 24),
    (22, 23), (22, 24)])
q3c25maxwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 6), (2, 7), (2, 8), (3, 7),
    (3, 8), (3, 9), (4, 6), (4, 7), (4, 10), (5, 6), (5, 10), (5, 11), (6, 12), (7,
    13), (7, 14), (8, 14), (8, 15), (9, 13), (9, 15), (10, 12), (10, 16), (11, 12),
    (11, 16), (11, 17), (12, 18), (13, 19), (14, 19), (15, 19), (16, 18), (16, 20),
    (17, 18), (17, 20), (17, 21), (18, 22), (18, 23), (20, 23), (20, 24), (21, 22),
    (21, 24), (22, 25), (23, 25), (24, 25)])
q3c26maxwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 6), (2, 7), (2, 8), (3, 7),
    (3, 8), (3, 9), (3, 10), (4, 6), (4, 7), (4, 11), (5, 6), (5, 11), (5, 12), (6,
    13), (7, 14), (8, 14), (8, 15), (9, 15), (9, 16), (10, 14), (10, 16), (11, 13),
    (11, 17), (12, 13), (12, 17), (12, 18), (13, 19), (14, 20), (15, 20), (16, 20),
    (17, 19), (17, 21), (18, 19), (18, 21), (18, 22), (19, 23), (19, 24), (21, 24),
    (21, 25), (22, 23), (22, 25), (23, 26), (24, 26), (25, 26)])
q3c27maxwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 6), (2, 7), (2, 8), (3, 7),
    (3, 8), (3, 9), (4, 6), (4, 7), (4, 10), (5, 6), (5, 10), (5, 11), (6, 12), (7,
    13), (8, 13), (8, 14), (8, 15), (9, 13), (9, 15), (9, 16), (10, 12), (10, 17),
    (11, 12), (11, 17), (11, 18), (12, 19), (13, 20), (14, 20), (14, 21), (15, 21),
    (16, 20), (16, 21), (17, 19), (17, 22), (18, 19), (18, 22), (18, 23), (19, 24),
    (19, 25), (22, 25), (22, 26), (23, 24), (23, 26), (24, 27), (25, 27), (26, 27)])
q3c28maxwi = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 6), (2, 7), (2, 8), (3, 7),
    (3, 8), (3, 9), (4, 6), (4, 7), (4, 10), (5, 6), (5, 10), (5, 11), (6, 12), (7,
    13), (8, 13), (8, 14), (9, 13), (9, 14), (9, 15), (10, 12), (10, 16), (11, 12),
    (11, 16), (11, 17), (12, 18), (13, 19), (13, 20), (14, 20), (14, 21), (15, 19),
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    (15, 21), (16, 18), (16, 22), (17, 18), (17, 22), (17, 23), (18, 24), (18, 25),
    (19, 26), (20, 26), (21, 26), (22, 25), (22, 27), (23, 24), (23, 27), (24, 28),
    (25, 28), (27, 28)])

#3-connected quadrangulations which minimize the WI
q3c8minwi = Graph([(1, 2), (1, 3), (1, 4), (2, 5), (2, 6), (3, 6), (3, 7), (4, 5),
    (4, 7), (5, 8), (6, 8), (7, 8)])
q3c10minwi = Graph([(1, 2), (1, 3), (1, 4), (2, 5), (2, 6), (2, 7), (3, 7), (3, 8),
    (4, 5), (4, 8), (5, 9), (6, 9), (6, 10), (7, 10), (8, 9), (8, 10)])
q3c11minwi = Graph([(1, 2), (1, 3), (1, 4), (2, 5), (2, 6), (2, 7), (3, 7), (3, 8),
    (4, 5), (4, 8), (4, 9), (5, 10), (6, 10), (6, 11), (7, 11), (8, 11), (9, 10),
    (9, 11)])
q3c12minwi = Graph([(1, 2), (1, 3), (1, 4), (2, 5), (2, 6), (2, 7), (2, 8), (3, 8),
    (3, 9), (4, 5), (4, 9), (5, 10), (6, 10), (6, 11), (7, 11), (7, 12), (8, 12),
    (9, 10), (9, 11), (9, 12)])
q3c13minwi = Graph([(1, 2), (1, 3), (1, 4), (2, 5), (2, 6), (2, 7), (2, 8), (3, 8),
    (3, 9), (3, 10), (4, 5), (4, 10), (5, 11), (6, 11), (6, 12), (7, 12), (7, 13),
    (8, 13), (9, 12), (9, 13), (10, 11), (10, 12)])
q3c14minwi = Graph([(1, 2), (1, 3), (1, 4), (2, 5), (2, 6), (2, 7), (2, 8), (2, 9),
    (3, 9), (3, 10), (4, 5), (4, 10), (5, 11), (6, 11), (6, 12), (7, 12), (7, 13),
    (8, 13), (8, 14), (9, 14), (10, 11), (10, 12), (10, 13), (10, 14)])
q3c15minwi = Graph([(1, 2), (1, 3), (1, 4), (2, 5), (2, 6), (2, 7), (2, 8), (3, 8),
    (3, 9), (4, 5), (4, 9), (5, 10), (6, 10), (6, 11), (7, 11), (7, 12), (7, 13),
    (8, 13), (8, 14), (9, 10), (9, 11), (9, 12), (9, 14), (12, 15), (13, 15), (14,
    15)])
q3c16minwi = Graph([(1, 2), (1, 3), (1, 4), (2, 5), (2, 6), (2, 7), (2, 8), (2, 9),
    (2, 10), (3, 10), (3, 11), (4, 5), (4, 11), (5, 12), (6, 12), (6, 13), (7, 13),
    (7, 14), (8, 14), (8, 15), (9, 15), (9, 16), (10, 16), (11, 12), (11, 13), (11,
    14), (11, 15), (11, 16)])
q3c17minwi = Graph([(1, 2), (1, 3), (1, 4), (2, 5), (2, 6), (3, 6), (3, 7), (3, 8),
    (3, 9), (3, 10), (3, 11), (4, 5), (4, 11), (5, 12), (5, 13), (5, 14), (5, 15),
    (6, 15), (7, 14), (7, 15), (8, 13), (8, 14), (9, 13), (9, 16), (10, 12), (10,
    16), (11, 12), (12, 17), (13, 17), (16, 17)])
q3c18minwi = Graph([(1, 2), (1, 3), (1, 4), (2, 5), (2, 6), (2, 7), (2, 8), (2, 9),
    (2, 10), (2, 11), (3, 11), (3, 12), (4, 5), (4, 12), (5, 13), (6, 13), (6, 14),
    (7, 14), (7, 15), (8, 15), (8, 16), (9, 16), (9, 17), (10, 17), (10, 18), (11,
    18), (12, 13), (12, 14), (12, 15), (12, 16), (12, 17), (12, 18)])
q3c19minwi = Graph([(1, 2), (1, 3), (1, 4), (2, 5), (2, 6), (3, 6), (3, 7), (3, 8),
    (3, 9), (3, 10), (3, 11), (4, 5), (4, 11), (5, 12), (5, 13), (5, 14), (5, 15),
    (5, 16), (5, 17), (6, 17), (7, 16), (7, 17), (8, 15), (8, 16), (9, 14), (9, 15),
     (9, 18), (10, 12), (10, 13), (10, 18), (11, 12), (13, 19), (14, 19), (18, 19)])
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q3c20minwi = Graph([(1, 2), (1, 3), (1, 4), (2, 5), (2, 6), (2, 7), (2, 8), (2, 9),
    (2, 10), (2, 11), (2, 12), (3, 12), (3, 13), (4, 5), (4, 13), (5, 14), (6, 14),
    (6, 15), (7, 15), (7, 16), (8, 16), (8, 17), (9, 17), (9, 18), (10, 18), (10,
    19), (11, 19), (11, 20), (12, 20), (13, 14), (13, 15), (13, 16), (13, 17), (13,
    18), (13, 19), (13, 20)])
q3c21minwi = Graph([(1, 2), (1, 3), (1, 4), (2, 5), (2, 6), (3, 6), (3, 7), (4, 5),
    (4, 7), (4, 8), (4, 9), (4, 10), (4, 11), (4, 12), (4, 13), (5, 14), (6, 14),
    (6, 15), (6, 16), (6, 17), (6, 18), (6, 19), (7, 19), (8, 18), (8, 19), (9, 17),
     (9, 18), (10, 16), (10, 17), (11, 16), (11, 20), (12, 15), (12, 20), (13, 14),
    (13, 15), (15, 21), (16, 21), (20, 21)])
q3c22minwi = Graph([(1, 2), (1, 3), (1, 4), (2, 5), (2, 6), (2, 7), (2, 8), (2, 9),
    (2, 10), (2, 11), (2, 12), (2, 13), (3, 13), (3, 14), (4, 5), (4, 14), (5, 15),
    (6, 15), (6, 16), (7, 16), (7, 17), (8, 17), (8, 18), (9, 18), (9, 19), (10, 19)
    , (10, 20), (11, 20), (11, 21), (12, 21), (12, 22), (13, 22), (14, 15), (14, 16)
    , (14, 17), (14, 18), (14, 19), (14, 20), (14, 21), (14, 22)])
q3c23minwi = Graph([(1, 2), (1, 3), (1, 4), (2, 5), (2, 6), (3, 6), (3, 7), (3, 8),
    (3, 9), (3, 10), (3, 11), (3, 12), (3, 13), (4, 5), (4, 13), (5, 14), (5, 15),
    (5, 16), (5, 17), (5, 18), (5, 19), (5, 20), (5, 21), (6, 21), (7, 20), (7, 21),
     (8, 19), (8, 20), (9, 18), (9, 19), (10, 17), (10, 18), (10, 22), (11, 15),
    (11, 16), (11, 22), (12, 14), (12, 15), (13, 14), (16, 23), (17, 23), (22, 23)])
q3c24minwi = Graph([(1, 2), (1, 3), (1, 4), (2, 5), (2, 6), (2, 7), (2, 8), (2, 9),
    (2, 10), (2, 11), (2, 12), (2, 13), (2, 14), (3, 14), (3, 15), (4, 5), (4, 15),
    (5, 16), (6, 16), (6, 17), (7, 17), (7, 18), (8, 18), (8, 19), (9, 19), (9, 20),
     (10, 20), (10, 21), (11, 21), (11, 22), (12, 22), (12, 23), (13, 23), (13, 24),
     (14, 24), (15, 16), (15, 17), (15, 18), (15, 19), (15, 20), (15, 21), (15, 22),
     (15, 23), (15, 24)])
q3c25minwi = Graph([(1, 2), (1, 3), (1, 4), (2, 5), (2, 6), (3, 6), (3, 7), (3, 8),
    (3, 9), (3, 10), (3, 11), (3, 12), (3, 13), (3, 14), (3, 15), (4, 5), (4, 15),
    (5, 16), (5, 17), (5, 18), (5, 19), (5, 20), (5, 21), (5, 22), (5, 23), (6, 23),
     (7, 22), (7, 23), (8, 21), (8, 22), (9, 20), (9, 21), (10, 19), (10, 20), (11,
    19), (11, 24), (12, 18), (12, 24), (13, 17), (13, 18), (14, 16), (14, 17), (15,
    16), (18, 25), (19, 25), (24, 25)])
q3c26minwi = Graph([(1, 2), (1, 3), (1, 4), (2, 5), (2, 6), (2, 7), (2, 8), (2, 9),
    (2, 10), (2, 11), (2, 12), (2, 13), (2, 14), (2, 15), (3, 15), (3, 16), (4, 5),
    (4, 16), (5, 17), (6, 17), (6, 18), (7, 18), (7, 19), (8, 19), (8, 20), (9, 20),
     (9, 21), (10, 21), (10, 22), (11, 22), (11, 23), (12, 23), (12, 24), (13, 24),
    (13, 25), (14, 25), (14, 26), (15, 26), (16, 17), (16, 18), (16, 19), (16, 20),
    (16, 21), (16, 22), (16, 23), (16, 24), (16, 25), (16, 26)])
q3c27minwi = Graph([(1, 2), (1, 3), (1, 4), (2, 5), (2, 6), (3, 6), (3, 7), (4, 5),
    (4, 7), (4, 8), (4, 9), (4, 10), (4, 11), (4, 12), (4, 13), (4, 14), (4, 15),
    (5, 16), (6, 16), (6, 17), (6, 18), (6, 19), (6, 20), (6, 21), (6, 22), (6, 23),
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     (6, 24), (6, 25), (7, 25), (8, 24), (8, 25), (9, 23), (9, 24), (10, 22), (10,
    23), (11, 21), (11, 22), (12, 20), (12, 21), (12, 26), (13, 18), (13, 19), (13,
    26), (14, 17), (14, 18), (15, 16), (15, 17), (19, 27), (20, 27), (26, 27)])
q3c28minwi = Graph([(1, 2), (1, 3), (1, 4), (2, 5), (2, 6), (2, 7), (2, 8), (2, 9),
    (2, 10), (2, 11), (2, 12), (2, 13), (2, 14), (2, 15), (2, 16), (3, 16), (3, 17),
     (4, 5), (4, 17), (5, 18), (6, 18), (6, 19), (7, 19), (7, 20), (8, 20), (8, 21),
     (9, 21), (9, 22), (10, 22), (10, 23), (11, 23), (11, 24), (12, 24), (12, 25),
    (13, 25), (13, 26), (14, 26), (14, 27), (15, 27), (15, 28), (16, 28), (17, 18),
    (17, 19), (17, 20), (17, 21), (17, 22), (17, 23), (17, 24), (17, 25), (17, 26),
    (17, 27), (17, 28)])

#general quadrangulations which maximize the prox
q4p = Graph([(1, 2), (1, 3), (2, 4), (3, 4)])
q5p = Graph([(1, 2), (1, 3), (1, 4), (2, 5), (3, 5), (4, 5)])
q6p = Graph([(1, 2), (1, 3), (1, 4), (2, 5), (3, 5), (3, 6), (4, 5), (4, 6)])
q7_1p = Graph([(1, 2), (1, 3), (1, 4), (2, 5), (3, 5), (3, 6), (3, 7), (4, 5), (4,
    6), (4, 7)])
q7_2p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 6), (3, 6), (3, 7), (4, 7), (5,
    6), (5, 7)])
q8p = Graph([(1, 2), (1, 3), (1, 4), (2, 5), (2, 6), (3, 6), (3, 7), (4, 5), (4, 7),
     (5, 8), (6, 8), (7, 8)])
q9p = Graph([(1, 2), (1, 3), (1, 4), (2, 5), (2, 6), (3, 6), (3, 7), (3, 8), (4, 5),
     (4, 8), (5, 9), (6, 9), (7, 9), (8, 9)])
q10p = Graph([(1, 2), (1, 3), (1, 4), (2, 5), (2, 6), (2, 7), (3, 7), (3, 8), (3, 9)
    , (4, 5), (4, 6), (4, 9), (6, 10), (7, 10), (8, 10), (9, 10)])
q11p = Graph([(1, 2), (1, 3), (1, 4), (2, 5), (2, 6), (3, 6), (3, 7), (3, 8), (4, 5)
    , (4, 8), (4, 9), (5, 10), (6, 10), (7, 10), (7, 11), (8, 11), (9, 10), (9, 11)
    ])
q12_1p = Graph([(1, 2), (1, 3), (1, 4), (2, 5), (2, 6), (2, 7), (3, 5), (3, 6), (3,
    7), (4, 5), (6, 8), (6, 9), (7, 8), (7, 9), (8, 10), (8, 11), (9, 10), (9, 11),
    (10, 12), (11, 12)])
q12_2p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 6), (2, 7), (2, 8), (3, 6), (3,
    7), (3, 8), (4, 6), (5, 6), (7, 9), (7, 10), (8, 9), (8, 10), (9, 11), (9, 12),
    (10, 11), (10, 12)])
q12_3p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 6), (2, 7), (3, 7), (3, 8), (3,
    9), (4, 6), (4, 7), (4, 9), (5, 6), (7, 10), (8, 10), (8, 11), (9, 10), (9, 11),
     (10, 12), (11, 12)])
q12_4p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 6), (2, 7), (3, 7), (3, 8), (3,
    9), (4, 8), (4, 9), (4, 10), (5, 6), (5, 10), (6, 11), (7, 11), (7, 12), (8, 11)
    , (8, 12), (10, 11)])
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q12_5p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 6), (2, 7), (3, 7), (3, 8), (3,
    9), (4, 8), (4, 9), (4, 10), (5, 6), (5, 10), (6, 11), (7, 11), (8, 11), (8, 12)
    , (9, 12), (10, 11)])
q12_6p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 6), (2, 7), (3, 7), (3, 8), (3,
    9), (4, 8), (4, 9), (4, 10), (5, 6), (5, 10), (6, 11), (6, 12), (7, 12), (8, 12)
    , (10, 11), (10, 12)])
q12_7p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 6), (2, 7), (3, 7), (3, 8), (4,
    8), (4, 9), (4, 10), (5, 6), (5, 10), (6, 11), (7, 11), (7, 12), (8, 12), (9,
    11), (9, 12), (10, 11)])
q12_8p = Graph([(1, 2), (1, 3), (1, 4), (2, 5), (2, 6), (3, 6), (3, 7), (4, 5), (4,
    7), (4, 8), (5, 9), (5, 10), (6, 10), (7, 10), (7, 11), (8, 9), (8, 11), (9, 12)
    , (10, 12), (11, 12)])
q12_9p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 6), (2, 7), (2, 8), (3, 8), (3,
    9), (4, 7), (4, 9), (4, 10), (5, 6), (5, 7), (7, 11), (8, 11), (9, 11), (9, 12),
     (10, 11), (10, 12)])
q12_10p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 6), (2, 7), (2, 8), (3, 7), (3,
     8), (4, 7), (4, 9), (5, 6), (5, 9), (5, 10), (6, 11), (7, 11), (9, 11), (9, 12)
    , (10, 11), (10, 12)])
q12_11p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 6), (2, 7), (3, 7), (3, 8), (4,
     7), (4, 8), (4, 9), (5, 6), (5, 9), (5, 10), (6, 11), (7, 11), (9, 11), (9, 12)
    , (10, 11), (10, 12)])
q12_12p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 6), (2, 7), (2, 8), (3, 8), (3,
     9), (3, 10), (4, 6), (4, 7), (4, 10), (5, 6), (7, 11), (8, 11), (8, 12), (9,
    11), (9, 12), (10, 11)])
q12_13p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 6), (2, 7), (2, 8), (3, 7), (3,
     8), (3, 9), (4, 6), (4, 9), (4, 10), (5, 6), (6, 11), (7, 11), (9, 11), (9, 12)
    , (10, 11), (10, 12)])
q12_14p = Graph([(1, 2), (1, 3), (1, 4), (2, 5), (2, 6), (2, 7), (3, 7), (3, 8), (3,
     9), (4, 5), (4, 8), (4, 9), (5, 10), (5, 11), (6, 10), (6, 11), (6, 12), (7,
    10), (7, 12), (8, 10)])
q12_15p = Graph([(1, 2), (1, 3), (1, 4), (2, 5), (2, 6), (2, 7), (3, 7), (3, 8), (4,
     5), (4, 8), (4, 9), (5, 10), (6, 10), (6, 11), (7, 11), (8, 11), (8, 12), (9,
    10), (9, 11), (9, 12)])
q12_16p = Graph([(1, 2), (1, 3), (1, 4), (2, 5), (2, 6), (2, 7), (3, 7), (3, 8), (4,
     5), (4, 8), (4, 9), (5, 10), (6, 10), (6, 11), (7, 11), (7, 12), (8, 11), (8,
    12), (9, 10), (9, 11)])
q12_17p = Graph([(1, 2), (1, 3), (1, 4), (2, 5), (2, 6), (2, 7), (2, 8), (3, 6), (3,
     7), (3, 8), (3, 9), (4, 5), (4, 9), (4, 10), (4, 11), (5, 12), (6, 12), (9, 12)
    , (10, 12), (11, 12)])
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q12_18p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 6), (2, 7), (2, 8), (3, 7), (3,
     8), (3, 9), (4, 6), (4, 9), (4, 10), (5, 6), (6, 11), (7, 11), (7, 12), (9, 11)
    , (9, 12), (10, 11)])
q12_19p = Graph([(1, 2), (1, 3), (1, 4), (2, 5), (2, 6), (2, 7), (2, 8), (3, 8), (3,
     9), (3, 10), (3, 11), (4, 5), (4, 6), (4, 10), (4, 11), (6, 12), (7, 12), (8,
    12), (9, 12), (10, 12)])
q13p = Graph([(1, 2), (1, 3), (1, 4), (2, 5), (2, 6), (2, 7), (3, 5), (3, 6), (3, 7)
    , (4, 5), (6, 8), (6, 9), (7, 8), (7, 9), (8, 10), (8, 11), (9, 10), (9, 11),
    (10, 12), (10, 13), (11, 12), (11, 13)])
q14p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 6), (2, 7), (3, 7), (3, 8), (4, 8)
    , (4, 9), (5, 6), (5, 9), (6, 10), (6, 11), (7, 11), (7, 12), (8, 12), (8, 13),
    (9, 10), (9, 13), (10, 14), (11, 14), (12, 14), (13, 14)])
q15p = Graph([(1, 2), (1, 3), (1, 4), (2, 5), (2, 6), (2, 7), (3, 5), (3, 6), (3, 7)
    , (4, 5), (6, 8), (6, 9), (7, 8), (7, 9), (8, 10), (8, 11), (9, 10), (9, 11),
    (10, 12), (10, 13), (11, 12), (11, 13), (12, 14), (12, 15), (13, 14), (13, 15)])
q16_1p = Graph([(1, 2), (1, 3), (1, 4), (2, 5), (2, 6), (2, 7), (3, 5), (3, 6), (3,
    7), (4, 5), (6, 8), (6, 9), (7, 8), (7, 9), (8, 10), (8, 11), (9, 10), (9, 11),
    (10, 12), (10, 13), (11, 12), (11, 13), (12, 14), (12, 15), (13, 14), (13, 15),
    (14, 16), (15, 16)])
q16_2p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 6), (2, 7), (2, 8), (3, 6), (3,
    7), (3, 8), (4, 6), (5, 6), (7, 9), (7, 10), (8, 9), (8, 10), (9, 11), (9, 12),
    (10, 11), (10, 12), (11, 13), (11, 14), (12, 13), (12, 14), (13, 15), (13, 16),
    (14, 15), (14, 16)])
q17p = Graph([(1, 2), (1, 3), (1, 4), (2, 5), (2, 6), (2, 7), (3, 5), (3, 6), (3, 7)
    , (4, 5), (6, 8), (6, 9), (7, 8), (7, 9), (8, 10), (8, 11), (9, 10), (9, 11),
    (10, 12), (10, 13), (11, 12), (11, 13), (12, 14), (12, 15), (13, 14), (13, 15),
    (14, 16), (14, 17), (15, 16), (15, 17)])
q18p = Graph([(1, 2), (1, 3), (1, 4), (2, 5), (2, 6), (2, 7), (3, 5), (3, 6), (3, 7)
    , (4, 5), (6, 8), (6, 9), (7, 8), (7, 9), (8, 10), (8, 11), (9, 10), (9, 11),
    (10, 12), (10, 13), (11, 12), (11, 13), (12, 14), (12, 15), (13, 14), (13, 15),
    (14, 16), (14, 17), (15, 16), (15, 17), (16, 18), (17, 18)])
q19p = Graph([(1, 2), (1, 3), (1, 4), (2, 5), (2, 6), (2, 7), (3, 5), (3, 6), (3, 7)
    , (4, 5), (6, 8), (6, 9), (7, 8), (7, 9), (8, 10), (8, 11), (9, 10), (9, 11),
    (10, 12), (10, 13), (11, 12), (11, 13), (12, 14), (12, 15), (13, 14), (13, 15),
    (14, 16), (14, 17), (15, 16), (15, 17), (16, 18), (16, 19), (17, 18), (17, 19)])
q20_1p = Graph([(1, 2), (1, 3), (1, 4), (2, 5), (2, 6), (2, 7), (3, 5), (3, 6), (3,
    7), (4, 5), (6, 8), (6, 9), (7, 8), (7, 9), (8, 10), (8, 11), (9, 10), (9, 11),
    (10, 12), (10, 13), (11, 12), (11, 13), (12, 14), (12, 15), (13, 14), (13, 15),
    (14, 16), (14, 17), (15, 16), (15, 17), (16, 18), (16, 19), (17, 18), (17, 19),
    (18, 20), (19, 20)])
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q20_2p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 6), (2, 7), (2, 8), (3, 6), (3,
    7), (3, 8), (4, 6), (5, 6), (7, 9), (7, 10), (8, 9), (8, 10), (9, 11), (9, 12),
    (10, 11), (10, 12), (11, 13), (11, 14), (12, 13), (12, 14), (13, 15), (13, 16),
    (14, 15), (14, 16), (15, 17), (15, 18), (16, 17), (16, 18), (17, 19), (17, 20),
    (18, 19), (18, 20)])

#3-connected quadrangulations which maximize the prox
q3c8p = Graph([(1, 2), (1, 3), (1, 4), (2, 5), (2, 6), (3, 6), (3, 7), (4, 5), (4,
    7), (5, 8), (6, 8), (7, 8)])
q3c10p = Graph([(1, 2), (1, 3), (1, 4), (2, 5), (2, 6), (2, 7), (3, 7), (3, 8), (4,
    5), (4, 8), (5, 9), (6, 9), (6, 10), (7, 10), (8, 9), (8, 10)])
q3c11p = Graph([(1, 2), (1, 3), (1, 4), (2, 5), (2, 6), (2, 7), (3, 7), (3, 8), (4,
    5), (4, 8), (4, 9), (5, 10), (6, 10), (6, 11), (7, 11), (8, 11), (9, 10), (9,
    11)])
q3c12_1p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 6), (2, 7), (2, 8), (3, 8),
    (3, 9), (4, 9), (4, 10), (5, 6), (5, 10), (6, 11), (7, 11), (7, 12), (8, 12),
    (9, 12), (10, 11), (10, 12)])
q3c12_1p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 6), (2, 7), (2, 8), (3, 8),
    (3, 9), (4, 7), (4, 9), (4, 10), (5, 6), (5, 10), (6, 11), (7, 11), (7, 12), (8,
     12), (9, 12), (10, 11)])
q3c13p = Graph([(1, 2), (1, 3), (1, 4), (2, 5), (2, 6), (2, 7), (3, 7), (3, 8), (3,
    9), (4, 5), (4, 9), (4, 10), (5, 11), (6, 11), (6, 12), (7, 12), (8, 12), (8,
    13), (9, 13), (10, 11), (10, 12), (10, 13)])
q3c14p = Graph([(1, 2), (1, 3), (1, 4), (2, 5), (2, 6), (2, 7), (3, 7), (3, 8), (3,
    9), (4, 5), (4, 9), (4, 10), (5, 11), (6, 11), (6, 12), (7, 12), (8, 12), (8,
    13), (9, 13), (10, 11), (10, 13), (11, 14), (12, 14), (13, 14)])
q3c15p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 6), (2, 7), (3, 7), (3, 8), (4,
    8), (4, 9), (5, 6), (5, 9), (6, 10), (6, 11), (7, 11), (7, 12), (8, 12), (8, 13)
    , (9, 10), (9, 13), (10, 14), (11, 14), (11, 15), (12, 15), (13, 14), (13, 15)])
q3c16_1p = Graph([(1, 2), (1, 3), (1, 4), (2, 5), (2, 6), (2, 7), (3, 7), (3, 8),
    (3, 9), (4, 5), (4, 9), (5, 10), (5, 11), (6, 11), (6, 12), (6, 13), (7, 13),
    (8, 12), (8, 13), (8, 14), (9, 10), (9, 14), (10, 15), (11, 15), (11, 16), (12,
    16), (14, 15), (14, 16)])
q3c16_1p = Graph([(1, 2), (1, 3), (1, 4), (2, 5), (2, 6), (2, 7), (3, 7), (3, 8),
    (4, 5), (4, 8), (4, 9), (5, 10), (6, 10), (6, 11), (6, 12), (7, 12), (7, 13),
    (8, 13), (8, 14), (9, 10), (9, 11), (9, 14), (11, 15), (11, 16), (12, 16), (13,
    15), (13, 16), (14, 15)])
q3c16_1p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 6), (2, 7), (3, 7), (3, 8),
    (4, 8), (4, 9), (4, 10), (5, 6), (5, 10), (6, 11), (6, 12), (7, 12), (7, 13),
    (8, 13), (9, 13), (9, 14), (10, 11), (10, 14), (11, 15), (12, 15), (12, 16),
    (13, 16), (14, 15), (14, 16)])
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q3c16_1p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 6), (2, 7), (2, 8), (3, 8),
    (3, 9), (4, 7), (4, 9), (4, 10), (5, 6), (5, 10), (5, 11), (6, 12), (6, 13), (7,
     13), (7, 14), (8, 14), (9, 14), (10, 13), (10, 15), (11, 12), (11, 15), (12,
    16), (13, 16), (15, 16)])
q3c17p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 6), (2, 7), (3, 7), (3, 8), (3,
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    14), (8, 13), (8, 14), (8, 15), (9, 15), (10, 15), (10, 16), (11, 12), (11, 16),
     (12, 17), (13, 17), (15, 17), (16, 17)])
q3c18p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 6), (2, 7), (3, 7), (3, 8), (4,
    8), (4, 9), (5, 6), (5, 9), (6, 10), (6, 11), (7, 11), (7, 12), (8, 12), (8, 13)
    , (9, 10), (9, 13), (10, 14), (10, 15), (11, 15), (11, 16), (12, 16), (12, 17),
    (13, 14), (13, 17), (14, 18), (15, 18), (16, 18), (17, 18)])
q3c19_1p = Graph([(1, 2), (1, 3), (1, 4), (2, 5), (2, 6), (3, 6), (3, 7), (3, 8),
    (4, 5), (4, 8), (4, 9), (5, 10), (5, 11), (6, 11), (6, 12), (7, 12), (7, 13),
    (8, 13), (8, 14), (9, 10), (9, 14), (10, 15), (10, 16), (11, 16), (11, 17), (12,
     17), (12, 18), (13, 18), (14, 15), (14, 18), (15, 19), (16, 19), (17, 19), (18,
     19)])
q3c19_2p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 6), (2, 7), (3, 7), (3, 8),
    (4, 8), (4, 9), (4, 10), (5, 6), (5, 10), (5, 11), (6, 12), (6, 13), (7, 13),
    (7, 14), (8, 14), (8, 15), (9, 15), (9, 16), (10, 16), (10, 17), (11, 12), (11,
    17), (12, 18), (12, 19), (13, 19), (14, 19), (15, 18), (15, 19), (16, 18), (17,
    18)])
q3c19_3p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 6), (2, 7), (3, 7), (3, 8),
    (4, 8), (4, 9), (4, 10), (5, 6), (5, 10), (5, 11), (6, 12), (6, 13), (7, 13),
    (7, 14), (8, 14), (8, 15), (9, 15), (9, 16), (9, 17), (10, 17), (11, 12), (11,
    16), (11, 17), (12, 18), (13, 18), (13, 19), (14, 19), (15, 18), (15, 19), (16,
    18)])
q3c20_1p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 6), (2, 7), (3, 7), (3, 8),
    (4, 8), (4, 9), (4, 10), (5, 6), (5, 10), (6, 11), (6, 12), (7, 12), (7, 13),
    (8, 13), (8, 14), (9, 14), (9, 15), (10, 11), (10, 15), (11, 16), (12, 16), (12,
     17), (13, 17), (13, 18), (14, 18), (14, 19), (15, 16), (15, 19), (16, 20), (17,
     20), (18, 20), (19, 20)])
q3c20_2p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 7), (2, 8), (3, 8),
    (3, 9), (4, 9), (4, 10), (5, 10), (5, 11), (6, 7), (6, 11), (6, 12), (7, 13),
    (8, 13), (8, 14), (9, 14), (9, 15), (9, 16), (10, 16), (10, 17), (11, 17), (12,
    13), (12, 17), (13, 18), (13, 19), (14, 19), (15, 18), (15, 19), (15, 20), (16,
    20), (17, 18), (17, 20)])
q3c20_3p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 6), (2, 7), (3, 7), (3, 8),
    (3, 9), (4, 9), (4, 10), (5, 6), (5, 10), (5, 11), (6, 12), (7, 12), (7, 13),
    (8, 13), (8, 14), (8, 15), (9, 15), (10, 15), (10, 16), (11, 12), (11, 16), (11,
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     17), (12, 18), (13, 18), (13, 19), (14, 19), (14, 20), (15, 20), (16, 19), (16,
     20), (17, 18), (17, 19)])
q3c20_4p = Graph([(1, 2), (1, 3), (1, 4), (2, 5), (2, 6), (2, 7), (3, 7), (3, 8),
    (3, 9), (4, 5), (4, 9), (5, 10), (5, 11), (6, 11), (6, 12), (6, 13), (7, 13),
    (8, 13), (8, 14), (8, 15), (9, 10), (9, 15), (10, 16), (10, 17), (11, 17), (11,
    18), (12, 18), (12, 19), (13, 19), (14, 16), (14, 18), (14, 19), (15, 16), (16,
    20), (17, 20), (18, 20)])
q3c20_5p = Graph([(1, 2), (1, 3), (1, 4), (2, 5), (2, 6), (2, 7), (3, 7), (3, 8),
    (3, 9), (4, 5), (4, 9), (5, 10), (5, 11), (6, 11), (6, 12), (7, 12), (7, 13),
    (8, 13), (8, 14), (8, 15), (9, 10), (9, 15), (10, 16), (10, 17), (11, 17), (11,
    18), (12, 18), (12, 19), (13, 19), (14, 16), (14, 18), (14, 19), (15, 16), (16,
    20), (17, 20), (18, 20)])
q3c20_6p = Graph([(1, 2), (1, 3), (1, 4), (2, 5), (2, 6), (3, 6), (3, 7), (4, 5),
    (4, 7), (4, 8), (5, 9), (5, 10), (6, 10), (6, 11), (7, 11), (7, 12), (8, 9), (8,
     12), (8, 13), (9, 14), (9, 15), (10, 15), (10, 16), (11, 16), (11, 17), (12,
    17), (12, 18), (13, 14), (13, 18), (14, 19), (15, 19), (15, 20), (16, 20), (17,
    19), (17, 20), (18, 19)])
q3c20_7p = Graph([(1, 2), (1, 3), (1, 4), (2, 5), (2, 6), (3, 6), (3, 7), (4, 5),
    (4, 7), (4, 8), (5, 9), (5, 10), (6, 10), (6, 11), (7, 11), (7, 12), (8, 9), (8,
     12), (8, 13), (9, 14), (9, 15), (10, 15), (10, 16), (11, 16), (11, 17), (12,
    17), (12, 18), (13, 14), (13, 18), (14, 19), (14, 20), (15, 20), (16, 20), (17,
    19), (17, 20), (18, 19)])
q3c20_8p = Graph([(1, 2), (1, 3), (1, 4), (2, 5), (2, 6), (2, 7), (3, 7), (3, 8),
    (4, 5), (4, 8), (4, 9), (4, 10), (5, 11), (6, 11), (6, 12), (6, 13), (7, 13),
    (7, 14), (8, 14), (9, 14), (9, 15), (9, 16), (10, 11), (10, 16), (11, 17), (12,
    17), (12, 18), (13, 18), (13, 19), (14, 19), (15, 17), (15, 19), (16, 17), (17,
    20), (18, 20), (19, 20)])
q3c20_9p = Graph([(1, 2), (1, 3), (1, 4), (2, 5), (2, 6), (2, 7), (3, 7), (3, 8),
    (4, 5), (4, 8), (4, 9), (5, 10), (5, 11), (6, 11), (6, 12), (7, 12), (7, 13),
    (8, 13), (9, 10), (9, 13), (9, 14), (10, 15), (11, 15), (11, 16), (12, 16), (12,
     17), (13, 17), (14, 15), (14, 17), (14, 18), (15, 19), (16, 19), (16, 20), (17,
     20), (18, 19), (18, 20)])
q3c20_10p = Graph([(1, 2), (1, 3), (1, 4), (2, 5), (2, 6), (2, 7), (3, 7), (3, 8),
    (4, 5), (4, 8), (4, 9), (4, 10), (5, 11), (6, 11), (6, 12), (6, 13), (7, 13),
    (7, 14), (7, 15), (8, 15), (9, 14), (9, 15), (9, 16), (10, 11), (10, 16), (11,
    17), (11, 18), (12, 18), (12, 19), (13, 19), (14, 19), (16, 17), (16, 19), (17,
    20), (18, 20), (19, 20)])
q3c20_11p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 6), (2, 7), (2, 8), (2, 9),
    (3, 7), (3, 9), (3, 10), (4, 6), (4, 7), (4, 11), (5, 6), (5, 11), (5, 12), (6,
    13), (7, 14), (8, 14), (8, 15), (9, 15), (10, 14), (10, 15), (11, 13), (11, 16),
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     (18, 19), (18, 20)])
q3c20_12p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 6), (2, 7), (3, 7), (3, 8),
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     20), (18, 20), (19, 20)])
q3c20_13p = Graph([(1, 2), (1, 3), (1, 4), (2, 5), (2, 6), (2, 7), (3, 7), (3, 8),
    (3, 9), (4, 5), (4, 9), (5, 10), (5, 11), (6, 11), (6, 12), (6, 13), (7, 13),
    (7, 14), (8, 14), (8, 15), (9, 10), (9, 15), (10, 16), (10, 17), (11, 17), (11,
    18), (12, 16), (12, 18), (12, 19), (13, 19), (14, 19), (15, 16), (15, 19), (16,
    20), (17, 20), (18, 20)])
q3c20_14p = Graph([(1, 2), (1, 3), (1, 4), (2, 5), (2, 6), (2, 7), (3, 7), (3, 8),
    (4, 5), (4, 8), (4, 9), (5, 10), (5, 11), (6, 11), (6, 12), (7, 12), (7, 13),
    (8, 13), (8, 14), (9, 10), (9, 14), (9, 15), (10, 16), (11, 16), (11, 17), (12,
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    20), (18, 20), (19, 20)])
q3c20_15p = Graph([(1, 2), (1, 3), (1, 4), (2, 5), (2, 6), (2, 7), (3, 7), (3, 8),
    (3, 9), (4, 5), (4, 9), (4, 10), (4, 11), (5, 12), (6, 12), (6, 13), (6, 14),
    (7, 14), (7, 15), (8, 15), (8, 16), (8, 17), (9, 17), (10, 16), (10, 17), (10,
    18), (11, 12), (11, 18), (12, 19), (13, 19), (13, 20), (14, 20), (15, 20), (16,
    20), (18, 19), (18, 20)])
q3c20_16p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 6), (2, 7), (2, 8), (3, 8),
    (3, 9), (4, 9), (4, 10), (4, 11), (5, 6), (5, 11), (5, 12), (6, 13), (7, 13),
    (7, 14), (7, 15), (8, 15), (8, 16), (9, 16), (9, 17), (10, 17), (10, 18), (11,
    18), (12, 13), (12, 14), (12, 18), (14, 19), (15, 19), (15, 20), (16, 20), (17,
    19), (17, 20), (18, 19)])
q3c20_17p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 6), (2, 7), (3, 7), (3, 8),
    (3, 9), (4, 9), (4, 10), (4, 11), (5, 6), (5, 11), (6, 12), (6, 13), (7, 13),
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     17), (12, 18), (13, 18), (13, 19), (14, 19), (14, 20), (15, 20), (16, 19), (16,
     20), (17, 18), (17, 19)])
q3c20_18p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 6), (2, 7), (2, 8), (3, 8),
    (3, 9), (4, 9), (4, 10), (5, 6), (5, 10), (5, 11), (6, 12), (6, 13), (7, 13),
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     17), (13, 17), (13, 18), (14, 18), (15, 18), (15, 19), (16, 17), (16, 19), (17,
     20), (18, 20), (19, 20)])
q3c20_19p = Graph([(1, 2), (1, 3), (1, 4), (2, 5), (2, 6), (3, 6), (3, 7), (3, 8),
    (4, 5), (4, 8), (4, 9), (5, 10), (5, 11), (6, 11), (6, 12), (7, 12), (7, 13),
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q3c20_20p = Graph([(1, 2), (1, 3), (1, 4), (2, 5), (2, 6), (3, 6), (3, 7), (3, 8),
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    20), (18, 20), (19, 20)])
q3c20_21p = Graph([(1, 2), (1, 3), (1, 4), (2, 5), (2, 6), (3, 6), (3, 7), (3, 8),
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    (17, 20), (18, 20), (19, 20)])
q3c20_22p = Graph([(1, 2), (1, 3), (1, 4), (2, 5), (2, 6), (2, 7), (2, 8), (3, 8),
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    19), (16, 20), (17, 18)])
q3c20_23p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 6), (2, 7), (2, 8), (3, 8),
    (3, 9), (3, 10), (4, 7), (4, 10), (4, 11), (5, 6), (5, 11), (5, 12), (6, 13),
    (6, 14), (7, 14), (7, 15), (8, 15), (8, 16), (9, 16), (9, 17), (10, 15), (10,
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    20), (17, 20), (18, 19)])
q3c21_1p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 6), (2, 7), (2, 8), (3, 8),
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    12), (11, 13), (11, 18), (13, 19), (14, 19), (14, 20), (15, 20), (16, 19), (16,
    20), (16, 21), (17, 21), (18, 19), (18, 21)])
q3c21_1p = Graph([(1, 2), (1, 3), (1, 4), (2, 5), (2, 6), (2, 7), (3, 7), (3, 8),
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    20), (15, 21), (16, 20), (18, 19), (18, 20)])
q3c21_1p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 6), (2, 7), (2, 8), (3, 8),
    (3, 9), (4, 9), (4, 10), (5, 6), (5, 10), (5, 11), (6, 12), (7, 12), (7, 13),
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    17), (12, 18), (13, 18), (13, 19), (13, 20), (14, 20), (15, 19), (15, 20), (15,
    21), (16, 21), (17, 18), (17, 19), (17, 21)])
q3c21_1p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 6), (2, 7), (3, 7), (3, 8),
    (3, 9), (3, 10), (4, 10), (4, 11), (4, 12), (5, 6), (5, 12), (6, 13), (6, 14),
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    , (15, 21), (17, 21), (19, 20), (19, 21)])
q3c21_1p = Graph([(1, 2), (1, 3), (1, 4), (2, 5), (2, 6), (2, 7), (3, 7), (3, 8),
    (3, 9), (3, 10), (4, 5), (4, 10), (4, 11), (5, 12), (5, 13), (6, 13), (6, 14),
    (7, 14), (8, 14), (8, 15), (9, 15), (9, 16), (10, 16), (10, 17), (11, 12), (11,
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    21), (17, 21), (18, 19), (18, 20), (18, 21)])
q3c21_1p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 6), (2, 7), (3, 7), (3, 8),
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     14), (8, 14), (8, 15), (9, 15), (9, 16), (10, 11), (10, 16), (10, 17), (11, 18)
    , (12, 18), (13, 18), (13, 19), (14, 19), (15, 19), (15, 20), (16, 20), (17, 18)
    , (17, 20), (18, 21), (19, 21), (20, 21)])
q3c21_1p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 6), (2, 7), (2, 8), (3, 8),
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    (7, 14), (8, 14), (8, 15), (9, 15), (9, 16), (9, 17), (10, 17), (10, 18), (11,
    12), (11, 18), (12, 19), (13, 19), (13, 20), (13, 21), (14, 21), (15, 21), (16,
    20), (16, 21), (17, 20), (18, 19), (18, 20)])
q3c22p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 6), (2, 7), (3, 7), (3, 8), (3,
    9), (4, 9), (4, 10), (4, 11), (5, 6), (5, 11), (6, 12), (6, 13), (6, 14), (7,
    14), (7, 15), (8, 15), (8, 16), (9, 16), (10, 16), (10, 17), (11, 12), (11, 17),
     (12, 18), (13, 18), (13, 19), (14, 19), (15, 19), (15, 20), (16, 20), (16, 21),
     (17, 18), (17, 21), (18, 22), (19, 22), (20, 22), (21, 22)])
q3c23p = Graph([(1, 2), (1, 3), (1, 4), (2, 5), (2, 6), (2, 7), (3, 7), (3, 8), (3,
    9), (4, 5), (4, 9), (4, 10), (5, 11), (5, 12), (6, 12), (6, 13), (7, 13), (7,
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     (16, 22), (17, 18), (17, 21), (17, 22), (18, 23), (19, 23), (20, 23), (21, 23)
    ])
q3c24_1p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 6), (2, 7), (2, 8), (3, 8),
    (3, 9), (3, 10), (4, 10), (4, 11), (5, 6), (5, 11), (5, 12), (6, 13), (6, 14),
    (7, 14), (7, 15), (7, 16), (8, 16), (9, 16), (9, 17), (10, 17), (10, 18), (11,
    18), (12, 13), (12, 18), (12, 19), (13, 20), (14, 20), (14, 21), (15, 21), (15,
    22), (16, 22), (17, 22), (17, 23), (18, 23), (19, 20), (19, 21), (19, 23), (21,
    24), (22, 24), (23, 24)])
q3c24_2p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 6), (2, 7), (2, 8), (3, 8),
    (3, 9), (3, 10), (4, 10), (4, 11), (5, 6), (5, 11), (5, 12), (6, 13), (6, 14),
    (7, 14), (7, 15), (7, 16), (8, 16), (9, 16), (9, 17), (10, 17), (10, 18), (11,
    18), (12, 13), (12, 18), (12, 19), (13, 20), (14, 20), (14, 21), (15, 21), (15,
    22), (16, 22), (17, 22), (17, 23), (18, 23), (19, 20), (19, 23), (20, 24), (21,
    24), (22, 24), (23, 24)])
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q3c24_3p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 7), (2, 8), (3, 8),
    (3, 9), (4, 9), (4, 10), (4, 11), (5, 11), (5, 12), (6, 7), (6, 12), (7, 13),
    (7, 14), (8, 14), (8, 15), (9, 15), (10, 15), (10, 16), (11, 16), (11, 17), (12,
     13), (12, 17), (12, 18), (13, 19), (14, 19), (14, 20), (15, 20), (15, 21), (16,
     21), (16, 22), (17, 22), (17, 23), (18, 19), (18, 23), (19, 24), (20, 24), (21,
     24), (22, 24), (23, 24)])
q3c24_4p = Graph([(1, 2), (1, 3), (1, 4), (2, 5), (2, 6), (2, 7), (2, 8), (3, 8),
    (3, 9), (3, 10), (4, 5), (4, 10), (4, 11), (4, 12), (5, 13), (6, 13), (6, 14),
    (6, 15), (7, 15), (7, 16), (8, 16), (9, 16), (9, 17), (9, 18), (10, 18), (10,
    19), (11, 19), (11, 20), (12, 13), (12, 20), (13, 21), (14, 21), (14, 22), (15,
    22), (16, 22), (17, 22), (17, 23), (18, 23), (19, 23), (20, 21), (20, 23), (21,
    24), (22, 24), (23, 24)])
q3c24_5p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 6), (2, 7), (2, 8), (3, 8),
    (3, 9), (3, 10), (4, 7), (4, 8), (4, 10), (4, 11), (5, 6), (5, 11), (6, 12), (7,
     12), (8, 13), (8, 14), (9, 14), (9, 15), (9, 16), (10, 13), (10, 16), (11, 12),
     (13, 17), (14, 17), (15, 17), (15, 18), (15, 19), (15, 20), (16, 17), (16, 20),
     (17, 21), (18, 21), (18, 22), (19, 22), (19, 23), (20, 21), (20, 23), (21, 24),
     (22, 24), (23, 24)])
q3c24_6p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 6), (2, 7), (2, 8), (3, 7),
    (3, 8), (3, 9), (4, 6), (4, 7), (4, 10), (5, 6), (5, 10), (5, 11), (6, 12), (7,
    13), (7, 14), (8, 14), (8, 15), (9, 13), (9, 15), (10, 12), (10, 16), (11, 12),
    (11, 16), (11, 17), (12, 18), (13, 19), (14, 19), (15, 19), (16, 18), (16, 20),
    (16, 21), (17, 18), (17, 21), (17, 22), (18, 23), (20, 23), (20, 24), (21, 24),
    (22, 23), (22, 24)])
q3c24_7p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 6), (2, 7), (2, 8), (3, 7),
    (3, 8), (3, 9), (3, 10), (4, 6), (4, 7), (4, 11), (5, 6), (5, 11), (5, 12), (6,
    13), (7, 14), (7, 15), (8, 15), (9, 15), (9, 16), (10, 14), (10, 16), (11, 13),
    (11, 17), (12, 13), (12, 17), (12, 18), (12, 19), (13, 20), (14, 21), (15, 21),
    (16, 21), (17, 20), (17, 22), (18, 22), (18, 23), (19, 20), (19, 23), (20, 24),
    (22, 24), (23, 24)])
q3c24_8p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 6), (2, 7), (2, 8), (3, 7),
    (3, 8), (3, 9), (3, 10), (4, 6), (4, 7), (4, 11), (5, 6), (5, 11), (5, 12), (6,
    13), (7, 14), (7, 15), (8, 15), (9, 15), (9, 16), (10, 14), (10, 16), (11, 13),
    (11, 17), (12, 13), (12, 17), (12, 18), (12, 19), (13, 20), (13, 21), (14, 22),
    (15, 22), (16, 22), (17, 21), (18, 21), (18, 23), (19, 20), (19, 23), (20, 24),
    (21, 24), (23, 24)])
q3c24_9p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 6), (2, 7), (2, 8), (3, 8),
    (3, 9), (4, 9), (4, 10), (4, 11), (5, 6), (5, 11), (5, 12), (6, 13), (6, 14),
    (7, 14), (7, 15), (8, 15), (8, 16), (9, 16), (10, 16), (10, 17), (10, 18), (11,
    18), (11, 19), (12, 13), (12, 19), (13, 20), (14, 20), (14, 21), (15, 21), (15,
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    22), (16, 22), (17, 22), (17, 23), (18, 23), (19, 20), (19, 23), (20, 24), (21,
    24), (22, 24), (23, 24)])
q3c24_10p = Graph([(1, 2), (1, 3), (1, 4), (2, 5), (2, 6), (2, 7), (3, 7), (3, 8),
    (3, 9), (4, 5), (4, 9), (4, 10), (4, 11), (5, 12), (6, 12), (6, 13), (6, 14),
    (7, 14), (7, 15), (8, 15), (8, 16), (8, 17), (9, 17), (9, 18), (10, 18), (10,
    19), (11, 12), (11, 19), (12, 20), (13, 20), (13, 21), (13, 22), (14, 22), (15,
    21), (15, 22), (16, 21), (16, 23), (17, 23), (18, 23), (19, 20), (19, 23), (20,
    24), (21, 24), (23, 24)])
q3c24_11p = Graph([(1, 2), (1, 3), (1, 4), (2, 5), (2, 6), (2, 7), (3, 7), (3, 8),
    (4, 5), (4, 8), (4, 9), (4, 10), (5, 11), (6, 11), (6, 12), (6, 13), (7, 13),
    (7, 14), (7, 15), (8, 15), (8, 16), (9, 16), (9, 17), (10, 11), (10, 17), (11,
    18), (11, 19), (12, 19), (12, 20), (13, 20), (14, 20), (14, 21), (15, 21), (16,
    21), (16, 22), (17, 18), (17, 22), (18, 23), (19, 23), (20, 23), (20, 24), (21,
    24), (22, 23), (22, 24)])
q3c24_12p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 6), (2, 7), (3, 7), (3, 8),
    (4, 8), (4, 9), (5, 6), (5, 9), (5, 10), (6, 11), (6, 12), (7, 12), (7, 13), (7,
     14), (8, 14), (8, 15), (8, 16), (9, 16), (9, 17), (10, 11), (10, 17), (10, 18),
     (11, 19), (11, 20), (12, 20), (13, 20), (13, 21), (14, 21), (15, 21), (15, 22),
     (16, 22), (17, 22), (18, 19), (18, 22), (19, 23), (19, 24), (20, 24), (21, 23),
     (21, 24), (22, 23)])
q3c24_13p = Graph([(1, 2), (1, 3), (1, 4), (2, 5), (2, 6), (2, 7), (3, 7), (3, 8),
    (3, 9), (4, 5), (4, 9), (4, 10), (5, 11), (5, 12), (6, 12), (6, 13), (6, 14),
    (7, 14), (7, 15), (8, 15), (8, 16), (8, 17), (9, 17), (10, 11), (10, 17), (10,
    18), (11, 19), (12, 19), (12, 20), (13, 20), (13, 21), (13, 22), (14, 22), (15,
    22), (16, 21), (16, 22), (16, 23), (17, 23), (18, 19), (18, 23), (19, 24), (20,
    24), (21, 24), (23, 24)])
q3c24_14p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 6), (2, 7), (2, 8), (3, 8),
    (3, 9), (4, 9), (4, 10), (4, 11), (5, 6), (5, 11), (6, 12), (6, 13), (7, 13),
    (7, 14), (8, 14), (8, 15), (9, 15), (9, 16), (10, 16), (10, 17), (11, 12), (11,
    17), (12, 18), (12, 19), (13, 19), (13, 20), (14, 20), (14, 21), (15, 21), (16,
    21), (16, 22), (17, 18), (17, 22), (18, 23), (19, 23), (20, 23), (20, 24), (21,
    24), (22, 23), (22, 24)])
q3c24_15p = Graph([(1, 2), (1, 3), (1, 4), (2, 5), (2, 6), (2, 7), (3, 7), (3, 8),
    (4, 5), (4, 8), (4, 9), (5, 10), (5, 11), (6, 11), (6, 12), (6, 13), (7, 13),
    (7, 14), (8, 14), (8, 15), (9, 10), (9, 15), (9, 16), (10, 17), (11, 17), (11,
    18), (12, 18), (12, 19), (13, 19), (13, 20), (14, 20), (14, 21), (15, 21), (15,
    22), (16, 17), (16, 18), (16, 22), (18, 23), (19, 23), (20, 23), (20, 24), (21,
    24), (22, 23), (22, 24)])
q3c24_16p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 6), (2, 7), (2, 8), (3, 8),
    (3, 9), (3, 10), (4, 7), (4, 10), (4, 11), (5, 6), (5, 11), (5, 12), (6, 13),
    (6, 14), (7, 14), (7, 15), (8, 15), (8, 16), (9, 16), (9, 17), (9, 18), (10, 15)
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    , (10, 18), (11, 14), (11, 19), (12, 13), (12, 19), (13, 20), (14, 20), (15, 21)
    , (16, 21), (16, 22), (17, 22), (17, 23), (18, 21), (18, 23), (19, 20), (21, 24)
    , (22, 24), (23, 24)])
q3c25p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 6), (2, 7), (2, 8), (3, 7), (3,
    8), (3, 9), (4, 6), (4, 7), (4, 10), (5, 6), (5, 10), (5, 11), (6, 12), (7, 13),
     (7, 14), (8, 14), (8, 15), (9, 13), (9, 15), (10, 12), (10, 16), (11, 12), (11,
     16), (11, 17), (12, 18), (13, 19), (14, 19), (15, 19), (16, 18), (16, 20), (17,
     18), (17, 20), (17, 21), (18, 22), (18, 23), (20, 23), (20, 24), (21, 22), (21,
     24), (22, 25), (23, 25), (24, 25)])
q3c26p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 6), (2, 7), (2, 8), (3, 7), (3,
    8), (3, 9), (3, 10), (4, 6), (4, 7), (4, 11), (5, 6), (5, 11), (5, 12), (6, 13),
     (7, 14), (8, 14), (8, 15), (9, 15), (9, 16), (10, 14), (10, 16), (11, 13), (11,
     17), (12, 13), (12, 17), (12, 18), (13, 19), (14, 20), (15, 20), (16, 20), (17,
     19), (17, 21), (18, 19), (18, 21), (18, 22), (19, 23), (19, 24), (21, 24), (21,
     25), (22, 23), (22, 25), (23, 26), (24, 26), (25, 26)])
q3c27_1p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 6), (2, 7), (2, 8), (3, 7),
    (3, 8), (3, 9), (4, 6), (4, 7), (4, 10), (5, 6), (5, 10), (5, 11), (6, 12), (7,
    13), (8, 13), (8, 14), (8, 15), (9, 13), (9, 15), (9, 16), (10, 12), (10, 17),
    (11, 12), (11, 17), (11, 18), (12, 19), (13, 20), (14, 20), (14, 21), (15, 21),
    (16, 20), (16, 21), (17, 19), (17, 22), (18, 19), (18, 22), (18, 23), (19, 24),
    (19, 25), (22, 25), (22, 26), (23, 24), (23, 26), (24, 27), (25, 27), (26, 27)])
q3c27_2p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 6), (2, 7), (2, 8), (3, 7),
    (3, 8), (3, 9), (3, 10), (4, 6), (4, 7), (4, 11), (5, 6), (5, 11), (5, 12), (6,
    13), (7, 14), (8, 14), (8, 15), (8, 16), (9, 16), (9, 17), (10, 14), (10, 15),
    (10, 17), (11, 13), (11, 18), (12, 13), (12, 18), (12, 19), (13, 20), (15, 21),
    (16, 21), (17, 21), (18, 20), (18, 22), (19, 20), (19, 22), (19, 23), (20, 24),
    (20, 25), (22, 25), (22, 26), (23, 24), (23, 26), (24, 27), (25, 27), (26, 27)])
q3c27_3p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 6), (2, 7), (2, 8), (3, 7),
    (3, 8), (3, 9), (3, 10), (4, 6), (4, 7), (4, 11), (5, 6), (5, 11), (5, 12), (6,
    13), (7, 14), (7, 15), (8, 15), (9, 15), (9, 16), (10, 14), (10, 16), (11, 13),
    (11, 17), (12, 13), (12, 17), (12, 18), (13, 19), (14, 20), (15, 20), (16, 20),
    (17, 19), (17, 21), (18, 19), (18, 21), (18, 22), (18, 23), (19, 24), (19, 25),
    (21, 25), (22, 25), (22, 26), (23, 24), (23, 26), (24, 27), (25, 27), (26, 27)])
q3c28_1p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 6), (2, 7), (2, 8), (3, 7),
    (3, 8), (3, 9), (4, 6), (4, 7), (4, 10), (5, 6), (5, 10), (5, 11), (6, 12), (7,
    13), (8, 13), (8, 14), (9, 13), (9, 14), (9, 15), (10, 12), (10, 16), (11, 12),
    (11, 16), (11, 17), (12, 18), (13, 19), (13, 20), (14, 20), (14, 21), (15, 19),
    (15, 21), (16, 18), (16, 22), (17, 18), (17, 22), (17, 23), (18, 24), (18, 25),
    (19, 26), (20, 26), (21, 26), (22, 25), (22, 27), (23, 24), (23, 27), (24, 28),
    (25, 28), (27, 28)])
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q3c28_2p = Graph([(1, 2), (1, 3), (1, 4), (1, 5), (2, 6), (2, 7), (2, 8), (3, 7),
    (3, 8), (3, 9), (3, 10), (4, 6), (4, 7), (4, 11), (5, 6), (5, 11), (5, 12), (6,
    13), (7, 14), (8, 14), (8, 15), (9, 15), (9, 16), (10, 14), (10, 16), (10, 17),
    (11, 13), (11, 18), (12, 13), (12, 18), (12, 19), (13, 20), (14, 21), (15, 21),
    (15, 22), (16, 22), (17, 21), (17, 22), (18, 20), (18, 23), (19, 20), (19, 23),
    (19, 24), (20, 25), (20, 26), (23, 26), (23, 27), (24, 25), (24, 27), (25, 28),
    (26, 28), (27, 28)])
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